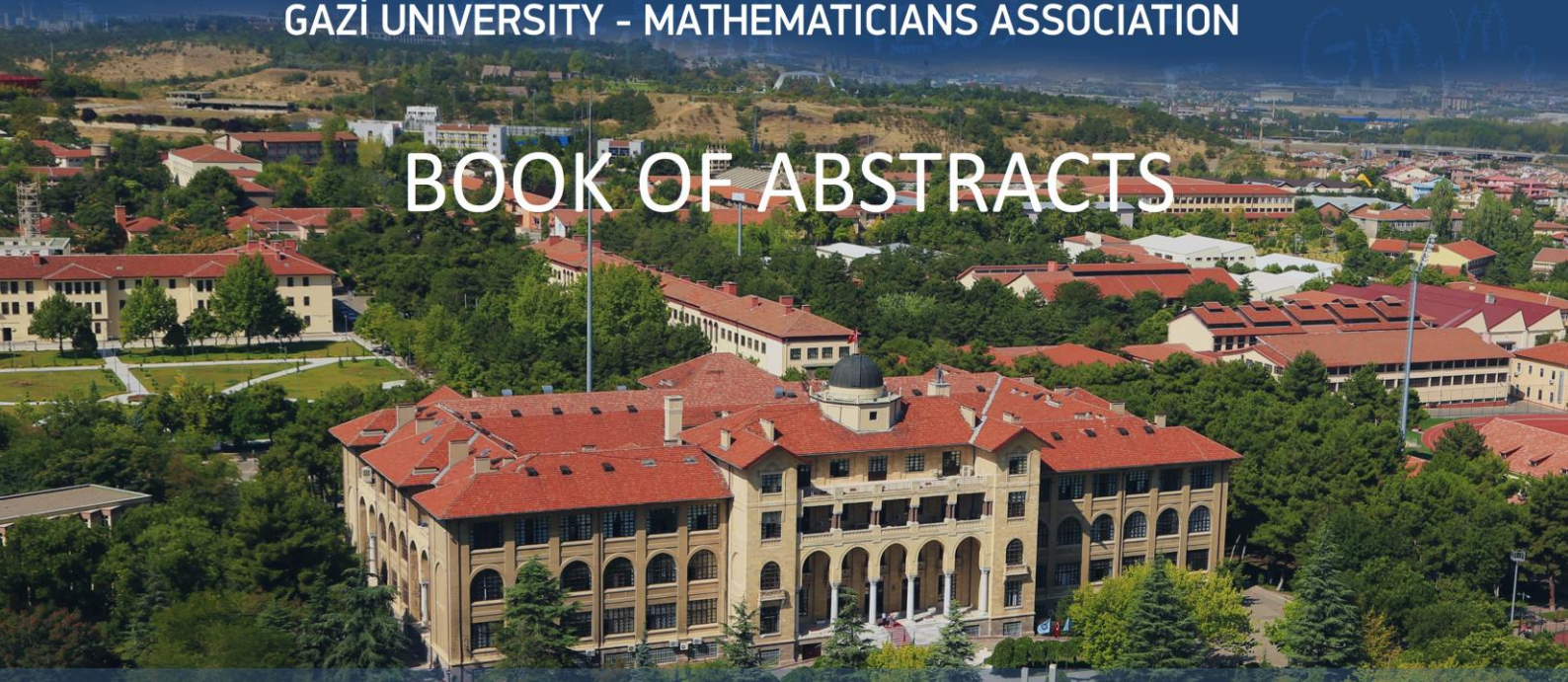
ICMME 2021

INTERNATIONAL CONFERENCE ON MATHEMATICS AND MATHEMATICS EDUCATION

MATHEMATICS *in the footsteps of Gazi*

GAZİ UNIVERSITY - MATHEMATICIANS ASSOCIATION

BOOK OF ABSTRACTS



16-18 SEPTEMBER 2021

www.matder.org.tr / www.theicmme.org.

International Conference on Mathematics and Mathematics Education
(ICMME - 2021)

Gazi University, Ankara, Turkey, 16-18 September 2021

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PREFACE

International Conference on Mathematics and Mathematics Education (ICMME-2021) was held on 16-18 September 2021 in Ankara, Turkey, entirely online via the zoom platform.

MATDER-Association of Mathematicians is an association founded in 1995 by mathematicians in Turkey. Up to now 14 national and 2 international mathematics symposium were organized by MATDER.

These meetings have been one of the main national symposiums. Since the talks in the meetings covers almost all areas of mathematics, mathematics education and engineering mathematics, the conferences have been well attended by mathematicians from academia, Ministry of Education and engineers as well. The last five conferences have been held in Konya (ICMME-2019), Ordu (ICMME-2018), Şanlıurfa (ICMME-2017), Elazig (ICMME-2016) and Niğde (2015). This year ICMME-2021 has been held at Gazi University in Ankara/Turkey on 16-18 September 2021 as an international conference.

The main aim of this conference is to contribute to the development of mathematical sciences, mathematical education, and their applications and to bring together the members of the mathematics community, interdisciplinary researchers, educators, mathematicians, and statisticians from all over the world. The conference will present new results and future challenges, in series of invited and short talks, poster presentations, workshops, and exhibitions. All presented paper's abstracts will be published in the conference proceeding. Moreover, selected and peer review articles will be published in the following journals:

- Turkish Journal of Mathematics & Computer Science (TJMCS)
- MATDER Matematik Eğitim Dergisi

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CONTENTS

INVITED SPEAKERS

AN ASSESSMENT ON THE PLACE OF MATHEMATICAL KNOWLEDGE BETWEEN FORM AND METAPHYSICS

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ABSTRACT

It is known that the source of knowledge, which is the subject of thinking, is the realm of concrete or intangible existence. For this reason, the issue of how knowledge is produced from existence or how existence is the subject of knowledge has been one of the main subjects of philosophy throughout the history of thought. The approach of philosophy on this subject has led to finding the unchanging principles of existence, that is to seek universal truths. These orientations and pursuits have become an alliance in the fact that the knowledge of God is unchangeable and imperative, especially Aristotle and then Ibn Sînâ. The fact that the knowledge of God is logically necessary and unchangeable rather than ontological has necessitated the search for unchangeable principles on existence. Immutable principles on existence are possible through the forms possessed by the being. The lexical meaning of the word form is "shape", but it has been described as "the nature of something, the substance that makes it that thing" [1]. Depending on the philosophical definition of the form, the generally accepted definition given by Ibn Sînâ is "the substance that takes the thing it is from into the realm of action" [2]. According to Aristotle, who accepts the principle of existence as matter (heyula) and form, existence consists of the form realized in matter [3]. The first matter, which is considered as a potential force and a possibility for the emergence of existence, comes into existence only after it takes a form. The entity that takes on a form becomes available for classification as the same genus, the same species or the same chapter, and becomes perceptible by being divided into categories. Man has divided the realm of forms into different groups by using the cognitive features he has from his creation. The first group is the physical world of objects, in which the entity is open to sensory description and definitions of physical objects can be made with the forms obtained from physical objects. According to Aristotle's view adopted in Islamic

Peripatetic philosophy, in order to understand what a thing is physically, it is necessary to know the reason for it and go as far as the first causes. In that case, four reasons must be sought for the realization of all kinds of phenomena and events that occur in the universe. These are matter, form, agent and objective causes. For obtaining information from non-physical entities, the form reason constitutes an important basis. The most general field of knowledge that is not physical and uses form is mathematical knowledge. Mathematical objects have form but are non-physical entities closed to the senses. In this study, how the being becomes the subject of mathematics by gaining form and the relationship between mathematical entities and the form will be examined. How mathematical entities correspond to counting and measuring through form and how we do mathematics through this relationship will be evaluated both in terms of classical philosophy and through modern approaches. Due to the dependence of mathematical and physical knowledge on logically necessary existence knowledge, the possibilities of metaphysical knowledge will also be briefly mentioned. In particular, it will be investigated whether the quantitative approach of Descartes in natural philosophy and the commitment to intuition in Kant's philosophy have a relationship with corporeal form ([4],[5]).

Key Words: Knowledge, Mathematical object, Mathematical form

An efficient algorithm to solve nonlinear reaction-diffusion problems with non-integer order derivatives

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ABSTRACT

In this talk present a numerical analysis of two types of nonlinear reaction-diffusion problems with periodic conditions and initial conditions, where the order of differentiation is arbitrary. Two iterative schemes are constructed using the upper and lower solutions method, which converges monotonically towards a maximum solution, or a minimum solution of the problem considered when the mesh decreases to zero, depending on whether the initial iteration is an upper solution or a lower solution. Besides, a result for comparing the different monotonic sequences is stated proved and theoretical results are supported by some numerical simulations.

Key Words: Caputo derivative; Conformable derivative; Monotone iterative method; Nonlinear parabolic PDE, Upper and lower solutions.

Flows past cylinders: Are the transitions between different flow regimes caused by a continuous evolution or by bifurcations?

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ABSTRACT

Solutions to the Navier-Stokes equations often go through a sequence of distinct regimes, with the flow field becoming more "complicated" as the Reynolds number increases. These changes may occur via (i) bifurcations of the underlying solutions of the Navier-Stokes equations, or (ii) a continuous evolution of the "complicated" flow field (with quantifiable, discrete changes to its topology).

We analyse the interplay between these two, in principle distinct, mechanisms in the context of flows past circular cylinders. If the cylinder is stationary the flow undergoes a Hopf bifurcation at a Reynolds number of approximately 46, resulting in the formation of the famous von Kármán vortex street -- a time-periodic flow in which vortices are shed downstream. While this suggests that the change to the flow topology arises via mechanism

(i) we show that the transition from steady to time-periodic flow (through the Hopf bifurcation) and the formation of individual vortices are in fact distinct events that occur at slightly different Reynolds numbers.

When the cylinder performs forced oscillations transverse to the flow direction, the vortex-shedding pattern becomes significantly more complex, leading to the formation of so-called "exotic wakes" whose character is controlled by the Reynolds number as well as the period and amplitude of the cylinder's motion. While it has generally been assumed that the transition between different wake patterns in response to changes in the amplitude occurs via mechanism

(ii) we show that they are actually associated with a spatio-temporal symmetry-breaking bifurcation of the time-periodic flow.

Sayı Duyusunun Gelişimi ve Matematiksel Yeterlikler

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ABSTRACT

Bireylerde sayı duyusu erken çocukluk, hatta bebeklik döneminden itibaren başlar ve okul hayatı boyunca gelişir. İlkokuldan itibaren sayı duyusunun matematik derslerinde desteklenmesi ve geliştirilmesi öğretim programının temel amaçlardan birisidir. Fakat ilkokulun ilk yıllarında matematik öğretimindeki eksiklikler sonraki yıllarda karşılaştığımız matematik öğrenme sorunlarının temelini oluşturmaktadır. Matematikte işlemleri doğru yapabilmek ve kavramlara hakim olmak önemli olsa da matematikte yetkin bireyler yetiştirebilmek için yeterli değildir. Matematikte sağlam bir bilgi alt yapısı oluşturabilmek için bireylerin kazanması gereken birtakım yeterlikler bulunmaktadır. NRC (2001) tarafından önerilen bu yeterlikler (1) kavramsal anlama, (2) işlemsel akıcılık, (3) stratejik yetkinlik, (4) uyarlanabilir muhakeme ve (5) üretken eğilim olmak üzere beş ana başlık altında tanımlanmaktadır. Matematik derslerinde bu 5 yeterliğin birbirine paralel olarak eş zamanlı ele alınması ve öğrencilerin bu yeterlikleri kazanarak matematiği öğrenmesi daha güçlü bir matematiksel temel oluşmasına yardımcı olmaktadır. İlk bakışta bu yeterlikleri öğrencilere kazandırmak ütopik gibi görünse de kazanılan her bir yeterliğin diğerini de beslediği ve bunların sınıflarda uygulanabilir olduğu çeşitli araştırmalarda gösterilmiştir. Bu çalışmada matematiksel yeterliklerin çocuklarda sayı duyusunun gelişimine nasıl katkı sağladığı hem kuramsal hem de uygulamalı olarak açıklanacaktır. Ayrıca her bir yeterliğin diğerlerinin gelişimine nasıl katkı sağladığı araştırma bulgularından örneklerle paylaşılacaktır. Ayrıca konuyla ilgili bir TÜBİTAK projesinin araştırma çıktılarından örnekler de paylaşılacaktır.

SELF-SMALL PRODUCTS OF ABELIAN GROUPS

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ABSTRACT

Let A and B be two abelian groups. The group A is called B-small if the covariant functor Hom(A,-) commutes with all direct sums of the form B(k) and A is a self-small group provided it is A-small. The main aim of the talk is to characterize self-small products applying developed closure properties of the classes of relatively small groups. In particular, we show that a product of a system of abelian groups is self small if and only if it relatively small over a direct sum of the system.

As a consequence of the theory of relatively small groups and the well-known fact that powers Zk of the group Z of all integers is slender for any nonmeasurable cardinal k, we characterize self-small products of finitely generated abelian groups. Namely, the product M of finitely generated groups is self-small if and only if either M is isomorphic to power Zk for some cardinal k, or M is isomorphic to a direct sum of a finitely generated free group F and finite abelian p-groups for each prime number p.

Finally, we also discuss possible application of the developed tools for description of self-compact objects in context of general additive and abelian categories.

Time-dependent source identification problem for Schrodinger differential and difference equations

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ABSTRACT

Source identification problems(SIPs) have the significant role in natural science, applied sciences, engineering, quantum mechanics, diffusion equations, heat equations (see, e.g., [1]-[4]). This is a discuss, the time-dependent SIP for the Schrödinger equation

$$\begin{cases} i\dfrac{du(t)}{dt} + Au(t) = p(t)q + f(t), t \in (0,T) \\ u(0) = \varphi, B[u(t)] = \tau(t), t \in [0,T] \end{cases} \quad (1)$$

In a Hilbert space H with the self-adjoint positive definite operator A with the dense domain $D(A)$ in H is studied. Here, $B: H \to \mathbb{R}$ is a given linear bounded functional and $\tau(t): [0,T] \to \mathbb{R}$ is a given smooth function and $q \in D(A), Bq \neq 0.$

Single-step absolute stable di§erence schemes for the numerical solution of time-dependent SIP are presented. The main theorems on stability estimates for the solutions of these problems are established. In practice, the stability estimates for solution of four problems for Schrˆdinger differential and difference equations are proved. Numerical results and explanatory illustrations are presented show the validation of the theoretical results.

ABSTRACTS OF ORAL PRESENTATIONS

ALGEBRA AND NUMBER THEORY

A Generalization of Gaussian Pell-Lucas Polynomials

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ABSTRACT

Number sequences and their polynomials have attracted the attention of many scientists for many years, as they find application in nature and in many sciences. Of course, the Fibonacci numbers are the best known of the sequences of numbers [3-5,7]. Many generalizations of number sequences were then described and studied [3-7]. One of the most important of these generalizations is those about Gaussian. Özkan et al. defined Gauss Fibonacci polynomials, Gauss Lucas polynomials and gave their applications in [4].

Now, let us give some basic definitions for this paper in this section.
We know well that the Pell numbers P_n are defined by

$$P_n = 2P_{n-1} + P_{n-2}, \text{ for } n \geq 3$$

with $P_1 = 1$ and $P_2 = 2$. [2]
Similarly, the Pell-Lucas numbers L_n,

$$Q_n = 2Q_{n-1} + Q_{n-2}, \text{ for } n \geq 3$$

with $Q_1 = 1$ and $Q_2 = 3$. [2]

Pell polynomials are defined by respectively,

$$P_{n+2}(x) = 2xP_{n+1}(x) + P_n(x)$$

with $P_0(x) = 0$ and $P_1(x) = 1$. [2]

Pell-Lucas polynomials are defined by respectively,

$$Q_{n+2}(x) = 2xQ_{n+1}(x) + Q_n(x)$$

with $Q_0(x) = 2$ and $Q_1(x) = 2x$. [2]

In [1], the authors introduced the Gaussian Pell polynomials and examined their properties. Later, Gaussian Pell-Lucas polynomials are defined in [8] and given their properties.

One of the latest works in this area is [7] where it is introduced d-Fibonacci and d-Lucas polynomials.

We define d −Gaussian Pell-Lucas polynomials. Then we present the sum, generating functions and Binet formulas of these polynomials. We give the matrix representations of d −Gaussian Pell-Lucas polynomials.

Key Words: d −Gaussian Pell-Lucas polynomials, Generating Function, Binet Formula, d −Gaussian Pell-Lucas polynomials matrix.

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A Note on Sum Formulas of Generalized Hexanacci Numbers: Closed Forms of the Sum Formulas $\sum_{k=0}^{n} kx^k W_k$ and $\sum_{k=1}^{n} kx^k W_{-k}$

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ABSTRACT

In this paper, closed forms of the sum formulas $\sum_{k=0}^{n} kx^k W_k$ and $\sum_{k=1}^{n} kx^k W_{-k}$ for generalized Hexanacci numbers are presented. As special cases, we give summation formulas of Hexanacci, Hexanacci-Lucas, and other sixth-order recurrence sequences.

Key Words: Hexanacci numbers, Hexanacci-Lucas numbers, sum formulas, summing formulas.

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A Note on Sum Formulas of Generalized Pentanacci Sequence: Closed Forms of the Sum Formulas $\sum_{k=0}^{n} kx^k W_k$ and $\sum_{k=1}^{n} kx^k W_{-k}$

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ABSTRACT

In this paper, closed forms of the sum formulas $\sum_{k=0}^{n} kx^k W_k$ and $\sum_{k=1}^{n} kx^k W_{-k}$ for generalized Pentanacci numbers are presented. As special cases, we give summation formulas of Pentanacci, Pentanacci-Lucas, and other fifth-order recurrence sequences.

Key Words: Pentanacci numbers, Pentanacci-Lucas numbers, sum formulas, summing formulas.

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Complex (Gaussian) q-Fibonacci and Complex (Gaussian) q-Lucas Quaternions

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ABSTRACT

Quaternions, introduced by Irish mathematician William Rowan Hamilton, are a four-dimensional non-commutative algebra over the field of real numbers with basis $\{1, i, j, k\}$ [1]. A quaternion q is of the form

$$q = q_0 + q_1 i + q_2 j + q_3 k,$$

where q_0, q_1, q_2, q_3 are real numbers, and i, j, k are quaternionic units satisfy the rules

$$i^2 = j^2 = k^2 = ijk = -1, \quad ij = k = -ji, \quad jk = i = -jk, \quad ki = j = -ik.$$

Horadam [2] defined the n^{th} Fibonacci and Lucas quaternions as

$$Q_n = F_n + F_{n+1} i + F_{n+2} j + F_{n+3} k$$

and

$$\mathcal{V}_n = L_n + L_{n+1} i + L_{n+2} j + L_{n+3} k,$$

respectively, where F_n is the n^{th} Fibonacci number defined by $F_n = F_{n-1} + F_{n-2}$ with $F_0 = 0$, $F_1 = 1$, L_n is the n^{th} Lucas number defined by $L_n = L_{n-1} + L_{n-2}$ with $L_0 = 2$, $L_1 = 1$, and i, j, k are quaternionic units. In [3], Halıcı defined the complex Fibonacci quaternions and gave some algebraic properties for these quaternions. Moreover, in [4], Halıcı and Cerda-Morales studied the Gaussian Fibonacci quaternions, and in [5], Halıcı introduced the Gaussian Lucas quaternions.

More recently, Akkuş and Kızılaslan [6] introduced the q-Fibonacci and q-Lucas quaternions as

$$Q_n = \alpha^{n-1}[n]_q + \alpha^n [n+1]_q i + \alpha^{n+1}[n+2]_q j + \alpha^{n+2}[n+3]_q k$$

and

$$V_n = \alpha^n \frac{[2n]_q}{[n]_q} + \alpha^{n+1} \frac{[2n+2]_q}{[n+1]_q} i + \alpha^{n+2} \frac{[2n+4]_q}{[n+2]_q} j + \alpha^{n+3} \frac{[2n+6]_q}{[n+3]_q} k,$$

respectively, where i, j, k are quaternionic units. These quaternions are generalized the Fibonacci and Lucas quaternions by receiving components from the q-integers

$[n]_q$ defined by $[n]_q = \frac{1-q^n}{1-q}$ (see, [7]). For $\alpha = \frac{1+\sqrt{5}}{2}$ and $q = -\frac{1}{\alpha^2}$, q-Fibonacci and q-Lucas quaternions are reduced the Fibonacci and Lucas quaternions, respectively.

Inspired from the above mentioned studies, in this study, a new generalization for complex (Gaussian) Fibonacci quaternions and complex (Gaussian) Lucas quaternions called complex (Gaussian) q-Fibonacci quaternions and complex (Gaussian) q-Lucas quaternions are defined, respectively, by

$$\mathbb{C}Q_n = Q_n + i\, Q_{n+1}$$

and

$$\mathbb{C}V_n = V_n + i\, V_{n+1},$$

where Q_n and V_n are the n^{th} q-Fibonacci and q-Lucas quaternions, respectively.

Furthermore, Binet's formulas, exponential generating functions, Poisson generating functions, Catalan's identitites, Cassini's identities and d'Ocagne's identities for complex (Gaussian) q-Fibonacci quaternions and complex (Gaussian) q-Lucas quaternions are given.

Key Words: Quaternion, q-integer, complex Fibonacci number.

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Cycle Neighborhood Polynomial of Graphs

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ABSTRACT

Let G be a connected cyclic graph. An induced cycle in G is a cycle such that no two vertices of the cycle are connected by an edge that does not itself belong to the cycle. The neighborrhood of a subgraph H in G is a subset S of the vertex-set of G such that for every element s in S, there exists a vertex h in H for which [s,h] is an edge in G. The cycle neighborhood polynomial of G is a bivariate polynomial with non-negative integral coefficients given by $c(G;x,y)=\sum_{j=0}^{n-i}\sum_{i=3}^{n} c_{ij}(G)x^i y^j$, where $c_{ij}(G)$ is the number of induced cycles of G or order i with neighborhood cardinality equal to j. In this paper, we established the basic properties of the cycle neighborhood polynomial of graphs and examined the relationships between the combinatorial and graph theoretic properties of G and the algebraic properties of the cycle neighborhood polynomial of G.

In addition, we characterized the induced cycles of some special graphs such as cycles, fans, wheels, complete graphs, and complete q-partite graphs; and graphs resulting from some binary operations such as the join and the corona of two connected cyclic graphs. Finally, we established the explicit forms of the induced cycle neighborhood polynomials of these graphs.

Key Words: Induced cycle, cycle neighborhood, cycle neighborhood polynomial.

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Groups Whose Proper Factor Groups Are Fitting

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ABSTRACT

Let G be a group and N be a normal subgroup of G. The factor-group G/N is said to be a proper factor-group if N is non-trivial. The influence of properties of proper factor-groups on properties of groups was the subject of investigation of many authors. Some the results related to groups with many similar proper factor-groups can be found in [2]. Torsion-free groups with every proper homomorphic image an N_1-group in [1], where N_1-group if all subgroup of G are subnormal. N_1-group are certain generalizations of Fitting group. Fitting group is defined to be the subgroup generated by nilpotent normal subgroups of G. Groups, all proper factor-groups of which are Fitting group, are studied in this study. Therefore, in an investigation concerning all proper factor-groups, it is natural to consider groups which include a non-identity normal nilpotent subgroup, that are groups with a non-identity Fitting subgroup.

In this study, we obtained the following results:

Let G be a group which all proper factor-groups of which are Fitting.

(i) If G is not Fitting group, then Fitting subgroup of G is abelian,

(ii) If $Fit(G)$ is torsion-free, then G is torsion free,

(iii) If for all $x \in G$, $C_G(< x >)$ is non-trivial, then G is Fitting.

Key Words: Fitting, homomorphic image, subnormal subgroup

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LINEAR MAPS PRESERVING DRAZIN INVERSES OF MATRICES OVER LOCAL RINGS

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ABSTRACT

Let be R a local ring and suppose that there exists $a \in F^*$ such that $a^6 \neq 1$ and let $T: M_n(R)$ to $M_m(R)$ be a linear map preserving Drazin inverses. Then we prove that $T = 0$; or $n = m$ and T preserves idempotents. We thereby determine the form of linear maps from $M_n(R)$ to $M_m(R)$ preserving Drazin inverses of matrices.

Key Words: Linear map, Drazin inverse, local ring.

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N-Barely Transitive Groups

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ABSTRACT

A group is called barely transitive (BT-group) or it has barely transitive permutation representation if it acts on an infinite set transitively and faithfully and all its proper subgroups have finite orbits. In [4,5] Hartley introduced BT-groups in connection with groups of Heineken-Mohamed type. A description of non-perfect locally finite BT-groups was given in [5] (Love's theorem). Some of the main properties of BT-groups are described by Kuzucuoğlu in [6].

We restrict the definition of BT-groups to normal subgroups and consider the questions “which properties of BT-groups are preserved? Is there such a group which is not a BT-group?”. We call this new form N-barely transitive (NBT-group) i.e., the group G acts on an infinite set transitively and faithfully, and all its proper normal subgroups have finite orbits. Equivalently, G is an NBT-group if G possesses a subgroup H such that $Core_G H = \bigcap_{g \in G} H^g = 1$ and $|N : N \cap H| < \infty$ for every proper normal subgroup N of G. Notice that the subgroup H is the stabilizer of a point. Clearly every infinite simple group is an NBT-group. So, there are NBT-groups of any given infinite cardinality.

A group is called an FC-group if every element has finitely many conjugates. If a group in which every proper subgroup is an FC-group but itself is not, then it is called a minimal non-FC-group ($MNFC$-group). The answer of the question “Does there exist a perfect locally finite $MNFC$-p-group?” is still unknown. If there exists such a group, then this group has a quotient which is a BT-p-group of finitary permutations

on some infinite set by [8]. Recall that a permutation group G is finitary if the support of every element is finite. In [5] it is shown that if there exists a finitary BT-group, then this group is a perfect $MNFC$-p-group where p is a prime. Therefore, the existence of a finitary BT-group will answer the question positively as it is mentioned in [7]. In this presentation, some of the results on NBT-groups in [1,2] are mentioned and it is given a perfect finitary NBT-p-group for any prime p which is not a BT-group. So, the question is answered positively for NBT-groups.

Key Words: Barely transitive group, *N*-barely transitive group, minimal non-*FC*-group, finitary permutation group, perfect group.

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NC-Barely Transitive Groups

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ABSTRACT

Let G be a group acting transitively and faithfully on an infinite set. If for every proper normal subgroup N and element x in G with $\langle N,x\rangle \neq G$, every orbit of $\langle N,x\rangle$ is finite, then G will be called an NC-barely transitive group ($NCBT$-group). Then G has a subgroup H of infinite index such that $Core_G H = 1$ and $|\langle N,x\rangle : \langle N,x\rangle \cap H| < \infty$ for every proper normal subgroup N and x in G with $\langle N,x\rangle \neq G$. Recall that an infinite group is said to be an N-barely transitive group (NBT-group) if it acts transitively and faithfully on an infinite set and every orbit of every proper normal subgroup is finite [1,2]. So, every $NCBT$-group is an NBT-group. As NBT-groups, the concept of $NCBT$-group is a generalization of barely transitive groups (BT-groups) in which every proper subgroup has finite orbits (see [3,4] for locally finite BT-groups).

In this presentation, we consider NBT-groups which are $NCBT$-groups in [1,2]. We give a characterization of NBT-groups with every non-trivial epimorphic image is non-simple. Also, the structure of non-perfect NBT-groups and some results under the restriction of being locally soluble (or locally nilpotent) are given. Furthermore, we provide perfect $NCBT$-group examples and soluble $NCBT$-groups examples of derived length 2 in the context of the groups satisfying minimal condition on normal subgroups. Indeed, it is shown that it is possible to construct soluble $NCBT$-groups of arbitrary derived length [1].

Key Words: Barely transitive group, *N*-barely transitive group, *NC*-barely transitive group, finitary permutation group, perfect group, locally soluble group.

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On The Catalan Transforms of The Incomplete Jacobsthal Numbers

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ABSTRACT

Number sequences have attracted the attention of many scientists for many years, as they find application in nature and in many sciences [2,4,5,6,7,8]. Number sequences, especially Fibonacci sequences, find application in many departments of mathematics as well as in other branches of science [2,3,4,8].

One of these generalizations is the Jacobsthal numbers. Jacobsthal numbers were first defined by Horodam in 1996 as follows.

For $n \geq 2$, the Jacobsthal numbers J_n are defined by the following recurrence relation

$$J_n = J_{n-1} + 2J_{n-2}$$

with $J_0 = 0$ and $J_1 = 1$.

In the same paper, he gave an explicit form of Jacobsthal numbers as follows

$$J_n = \sum_{r=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \binom{n-1-r}{r} 2^r$$

where $\lfloor x \rfloor = max\{m \epsilon \mathbb{Z} : m \leq x\}$.

The Catalan numbers with general term $C(n)$ are defined in [1] by

$$C(n) = \frac{1}{n+1}\binom{2n}{n}.$$

Generating functions of the Catalan numbers $C(n)$ are given in [1] by

$$c(x) = \frac{1-\sqrt{1-4x}}{2x}.$$

Philip defined incomplete Fibonacci and Lucas numbers in 1996 [4]. Djordjevic *et al.* introduced incomplete generalized Jacobsthal and Jacobsthal-Lucas numbers in 2005 [2].

In this work, we give the Catalan transform of the incomplete Jacobsthal numbers and find the generating functions of the Catalan transform of these numbers. Also, we apply the Hankel transform to the Catalan transforms of these numbers. We calculate determinants of matrixes formed with $CJ_{k,n}^{l}$ by using Hankel transform.

Key Words: Binet formula, Generating function, Hankel transform, Catalan transform, Incomplete Jacobsthal numbers.

This work was supported by Erzincan Binali Yıldırım University Scientific Research Projects Coordination Unit. Project Number: FBA-2021-783.

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On the Finitary Permutation Groups

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ABSTRACT

Let G be a group acting on infinity set Ω. The support of x in G is defined by $supp(x) = \{\alpha \in \Omega | \alpha^x \neq \alpha\}$. If $supp(x)$ is finite for all $x \in G$, then G is called finitary symmetric group on Ω. If there exist an equivalence relation ϱ on Ω such that $\alpha \equiv \beta \ (mod\ \varrho)$ if and only if $\alpha^x \equiv \beta^x \ (mod\ \varrho)$ for all $x \in G$, then ϱ is a congruence relation. G is almost primitive on Ω if there is maximal congruence such that $|\varrho|$ is finite; if there is no maximal congruence such that $|\varrho|$ is finite, then G is totally imprimitive. According to the result of Giorgetta [1], if such a group G has finite exponent then all its orbits must be finite. As for the result of Wiegold [3], if such a group G is soluble then all its orbits must be finite.

The main purpose of this study is to generalize these two studies mentioned. It is investigated that all orbits of the group G in Ω are finite for almost primitive and totally imprimitive cases of the group G of finite permutations of a set Ω satisfying some non-trivial laws. For this research, the group G is shown to generates variety $\mathfrak{D}$ of all groups. Also, it is examined that all orbits are finite if the group G is locally solvable group.

Key Words: Finitary permutation groups, almost primitive, totally imprimitive.

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SIP-direct-injective Modules and Rings

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ABSTRACT

Recall that a module *M* over a ring is called a *direct-injective module* (or *C2-module*) if every submodule of *M* is isomorphic to a direct summand of *M*, then it is a direct summand of *M*. A module *M* is called an *SIP-module* if the intersection of every pair of direct summands of *M* is again a direct summand of *M*. In this study, we define that a module *M* over a ring is called an *SIP-direct-injective module* if the intersection of every pair of direct summands of *M* is isomorphic to a direct summand of *M*, then it is a direct summand of *M*. The class of SIP-direct-injective modules unifies SIP-modules and C2-modules. Various basic properties of these modules are studied.

Key Words: summand intersection property, C2-modules, direct-injective-modules, direct summand, isomorphism.

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Some Notes on Soft Modules in Soft Sets

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ABSTRACT

There are several theories, such as the probability theory, the theory of fuzzy sets, the theory of rough sets, the theory of vague sets, the theory of interval mathematics, which can be considered as mathematical tools for dealing with uncertainties. The concept of soft sets was introduced by Molodtsov [1] as a new mathematical tool for dealing with uncertainties in 1999. Maji [2] defined operations on soft sets. According to Maji's definition,

Let A be a set of parameters. Let U be an initial universal set and let P(U) be the power set of U. A function F_A is called as a soft set over U, where F_A: A → P(U) is a function.

The soft set theory has been developed in many fields as soft decision making, soft fuzzy sets, intuitionistic soft set. Next, many researchers have studied algebraic structures over a soft set.

In this study, the operations of soft sets over a module are investigated. Sum of two soft sets, negative of a soft set and multiplied by a scalar of a soft set over a module are defined. Their properties were examined. Then, a soft module is defined as follows.

A soft module F_M is a soft set F_M over U such that

1) $F_M(0_R) = U$
2) $F_M(r.a) \supseteq F_M(a)$ for all $r \in R$ and $a \in M$.
3) $F_M(a+b) \supseteq F_M(a) \cap F_M(b)$ for all $a,b \in M$.

It is shown that multiplied by a scalar of the soft module is a subset of the soft module. Also, it is proved that sum of the soft module is a soft module. And some basic properties of soft modules are discussed.

Key Words: Soft sets, soft product, soft intersection modules.

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Some properties of Apostol Bernoulli Fibonacci and Apostol Euler Fibonacci Polynomials

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ABSTRACT

The generating function for Bernoulli polynomials is defined by Taylor series expansion,

$$\frac{te^{xt}}{e^t-1}=\sum_{n=0}^{\infty}B_n(\mathrm{x})\frac{t^n}{n!}$$

where $B_n(\mathrm{x})$ are the Bernoulli polynomials in x, for all $n>0$.

The Apostol-Bernoulli polynomials are defined by the exponential generating function,

$$\left(\frac{t}{\lambda e^t-1}\right)e^{xt}=\sum_{n=0}^{\infty}B_n(x,\lambda)\frac{t^n}{n!}$$

A famous and important sequence is the Fibonacci sequence, named after the the Italian mathematician known as Leonardo Pisano. This sequence is defined recursively. The Fibonacci sequence is defined by $F_{n+2}=F_{n+1}+F_n$,when $F_0=0$,$F_1=1$

The Golden exponential function is entire analytic function, defined as

$$e_F^{tx}=\sum_{n=0}^{\infty}\frac{t^n}{F_n!}$$

Generating function for Bernoulli-Fibonacci polynomials $B_n^F(x)$ is defined by series expansion,

$$\frac{te_F^{tx}}{e_F^t-1}=\sum_{n=0}^{\infty}B_n^F(\mathrm{x})\frac{t^n}{F_n!}$$

The generating function for Euler polynomials is defined by Taylor series expansion,

$$\frac{2e^{xt}}{e^{t}+1}=\sum_{n=0}^{\infty}E_{n}\left(\mathrm{x}\right)\frac{t^{n}}{n!}$$

The Apostol-Euler polynomials are defined by the exponential generating function,

$$\left(\frac{2}{\lambda e^{t}+1}\right)e^{xt}=\sum_{n=0}^{\infty}E_{n}(x,\lambda)\frac{t^{n}}{n!}$$

Generating function for Euler-Fibonacci polynomials $B_n^F(x)$ is defined by series expansion,

$$\frac{2e_F^{tx}}{e_F^{t}+1}=\sum_{n=0}^{\infty}E_n^F(x)\frac{t^{n}}{F_n!}$$

The Golden derivative, applied to Bernoulli-Fibonacci polynomials $B_n^F(x)$ gives Fibonacci numbers, $D_F^x(B_n^F(x))=F_nB_n^F(x)$

In this study a new F-exponential generating function for Apostol Bernoulli Fibonacci polynomials and various properties of Apostol Bernoulli Fibonacci polynomials are obtained. By identifying Apostol Euler Fibonacci numbers and polynomials are found. In addition, harmonic based F exponential generating function is defined for Apostol Bernoulli Fibonacci numbers and Apostol Euler Fibonacci numbers. We define incomplete Apostol Bernoulli Fibonacci numbers and incomplete Apostol Euler Fibonacci numbers by using Apostol Bernoulli Fibonacci numbers and Apostol Euler Fibonacci numbers.

Key Words: Bernoulli Fibonacci polynomials, Euler Fibonacci polynomials, Apostol polynomials.

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The Universal Difference Property for Generalized Splines over Principal Ideal Domains

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ABSTRACT

We study generalized splines over principal ideal domains and investigate the universal difference property (UDP). The UDP is introduced by K. Anders and her students [3]. They showed that the UDP holds for cycles, paths and trees. They also proved that the UDP holds for theta graphs over Prüfer domains and uniserial rings.

We prove that the UDP holds for any graph over principal ideal domain by using the existence of flow-up classes over PID's.

Key Words: Generalized splines, universal difference property, principal ideal domains.

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On semi-Nekrasov matrices

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ABSTRACT

Nekrasov matrices represent a special subclass in the class of non-singular H-matrices. In this paper, lower semi-Nekrasov matrix class, a generalization of Nekrasov matrix class, is considered. The relation of semi-Nekrasov matrices to semi-strictly diagonally dominant matrices is established. For matrices of this type, an upper bound for the maximum norm of the inverse matrix is proposed. New bound can be applied to Nekrasov matrices as well and numerical examples show that, in some cases, new norm bound for the inverse matrix is tighter than some already known bounds defined for Nekrasov matrices. The corresponding class of block matrices is also considered, as the subclass in the class of block-H matrices. Two different types of block-generalizations are given. Starting from the original matrix and the fixed partition of the index set, we observe two different ways to define the comparison matrix that is of a smaller dimension and using its entries we define upper bounds for the norm of the inverse matrix in the block case as well. As the norm bounds for the inverse can be used in bounding the condition number or in estimating error bounds in linear complementarity problems, we discussed possibilities for applications of the presented results.

Key Words: Nekrasov matrices, maximum norm bound, block matrices.

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ANALYSIS

An Extension of the Adams-type Theorem to Vanishing Generalized Weighted Morrey Spaces

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ABSTRACT

Morrey spaces were introduced by Morrey in [3]. These spaces appeared to be useful in the study of local behavior properties of the solutions of second order elliptic PDEs. The vanishing Morrey space of the classical Morrey spaces was introduced by Vitanza in [7] and applied there to obtain a regularity result for elliptic PDEs. Later in [8] Vitanza proved an existence theorem for a Dirichlet problem and a regularity result assuming that the partial derivatives of the coefficients of the highest and lower order terms belong to vanishing Morrey spaces depending on the dimension. Persson et al. [4] showed the commutators of Hardy operators on vanishing Morrey spaces. Also Ragusa [5] obtained a sufficient condition for commutators of fractional integral operators to belong to vanishing Morrey spaces.

The vanishing generalized Morrey space and vanishing generalized local Morrey space was introduced by Samko in [6]. The boundedness of the multi-dimensional Hardy type operators, maximal, potential and singular operators in these spaces were proved in [6]. Kucukaslan et al. [1] proved the Spanne-type and Adams-type boundedness of generalized fractional integral operators on vanishing generalized local Morrey spaces. Guliyev et al. [2] proved the commutators of Riesz potential operator in the vanishing generalized weighted Morrey spaces with variable exponent.

The generalized fractional maximal operators M_ρ was initially investigated in by Nakai in 1994. Nakai introduced the generalized Morrey spaces and proved the boundedness of the generalized fractional integral operator in these spaces. Nowadays many authors have been culminating important observations about the generalized fractional maximal operators M_ρ especially in connection with Morrey-type spaces.

In this talk, we generalize the Adams-type boundedness of the generalized fractional maximal operator M_ρ from the vanishing generalized weighted Morrey spaces $VM_{p,\varphi^{\frac{1}{p}}}(\omega)$ to the vanishing generalized weighted Morrey spaces $VM_{q,\varphi^{\frac{1}{q}}}(\omega)$ with the weight function ω, and from the vanishing generalized weighted Morrey spaces $VM_{1,\varphi}(\omega)$ to the vanishing generalized weighted weak Morrey spaces $VWM_{q,\varphi^{\frac{1}{q}}}(\omega)$. The all weight functions belong to the Muckenhoupt-Weeden classes $A_{p,q}$.

Key Words: Generalized fractional maximal operator, Vanishing generalized weighted Morrey space, Muckenhoupt-Weeden classes.

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Approximation by Some Kantorovich Type Max-Product Operators

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ABSTRACT

In order to draw attention to a possible interesting new direction in the constructive approximation theory, an open problem was presented by S.G. Gal in [1]. According to this problem a sequence of discrete max-product approximation operators $L_n^{(M)}$ attached to $f: I \subseteq \mathbb{R} \to \mathbb{R}^+$ with the operations "$\vee$" (maximum) and "."(product), can be defined by

$$L_n^{(M)}(f)(x) = \frac{\vee_{k\in I_n} p_{n,k}(x).f(x_{n,k})}{\vee_{k\in I_n} p_{n,k}(x)}, x \in I, n \in \mathbb{N}$$

where $p_{n,k}(x)$ are various kinds of function basis on I, I_n are finite or infinite families of indices and $\{x_{n,k}: k \in I_n\}$ represents a division of I. These operators are nonlinear positive operators having pseudo-linearity property. Moreover, the max-product operators are subadditive, positive homogenous and monotone. In [2], to each max-product operator $L_n^{(M)}$, the authors defined its Kantorovich variant by

$$LK_n^{(M)}(f)(x) = \frac{\vee_{k\in I_n} p_{n,k}(x).\left(1/(x_{n,k+1}-x_{n,k})\right).\int_{x_{n,k}}^{x_{n,k+1}} f(t)dt}{\vee_{k\in I_n} p_{n,k}(x)}, x \in I, n \in \mathbb{N}$$

with $\{x_{n,k}: k \in I_n\}$ a division of the finite or infinite interval I. They studied uniform and pointwise approximation properties, shape preserving properties and localization results for certain well-known operators.

Based on this study we introduce Kantorovich variants of some max-product operators and give quantitative estimates with the help of the quantitative estimates of the original operators. Compared with the approximation order by the original operators, it follows that the approximation order by its Kantorovich variants is no worse. Also, we investigate shape preserving properties for Kantorovich type operators.

Key Words: Max-product operators, max-product operators of Kantorovich type, shape preserving properties.

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APPROXIMATION PROPERTIES OF GENERALIZED JAIN OPERATORS OF TWO VARIABLES

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ABSTRACT

In this study, The deals with the approximation processes of the bivariate Jain operators defined in this paper. We also obtain some rates of not only classical uniform convergence a Vorovskaya type result is also proved for these operators.

In the first chapter, the approximation properties of Jain operators are studied. In the second chapter, the continuity of Jain operators and the approximation speed using functions in the Lipschitz class are given. In the third chapter, the approximation properties of bivariate generalized Jain operators are studied. In the fourth chapter, the approximation speed using continuity of bivariate generalized Jain operators and functions in the Lipschitz class is given. In the fifth chapter, the Voronovskaya Asymptotic Approach of Jain Operators is presented. Finally, in the sixth chapter, the Voronovskaya asymptotic approach of bivariate generalized Jain operators is obtained.

Key Words: Jain operators of the variables, linear positive operators, Lipschitz class, modulus of continuity.

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Generalized Fractional Integral Inequalities on Convex Functions

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ABSTRACT

The Fractional calculus technique is an important subject that has been known for approximately 300 years and is expected to lead to beneficial results. We can understand the importance of fractional derivatives and integrals from the various branches of science to which they have been applied and the useful results that have emerged. Although there are many definitions of the fractional calculus technique, the most well-known form is Riemann-Liouville derivatives and integrals. For this, please see the following references [6, 9].

The Hermite-Hadamard inequality is indispensable for the theory of inequality, especially for convex functions. The number of Hermite-Hadamar type inequalities for fractional derivatives and integrals has increased considerably in recent years. For this, please see the following references [1-5, 7, 8, 10, 11].

In this paper, we have obtained some new fractional integral inequalities for two synchronous functions, where generalized Riemann-Liouville fractional integrals are used. In obtaining these fractional integral inequalities, we extended them with some new parameters. We show that these inequalities we found support the results available in the literature for the specific values.

Key Words: Fractional Integral, Convex Functions, Riemann-Liouville Fractional Integral.

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Matrix Transformations and Some Algebraic and Topological Properties Related to the New Catalan Double Sequence Space

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ABSTRACT

In mathematics, one of the fascinating number sequence is the integer sequence
1, 1, 2, 5, 14, 42, 132, 429, . . .
of Catalan numbers. More recently, a new conservative matrix have been defined by means of the fascinating sequence of Catalan numbers and have been studied the matrix domains of this newly introduced matrix in the classical sequence spaces in [5]. After that, this matrix and its domains have been studied in the papers [1,6]. Each story on single sequence and single sequence space has been experienced over double sequence and double sequence space. In this present paper, firstly we define a new double sequence space by using the 4 dimensional Catalan matrix derived by the sequences of the Catalan numbers and we prove that this newly described double sequence space is a Banach space with its norm. Then, we give a inclusion relation including this newly defined double sequence space. Moreover, we calculate the α-, β(bp)- and γ-duals of this space. Finally, we characterize some new 4 dimensional matrix transformation classes and we complete this work with some significant results.

Key Words: 4 dimensional Catalan matrix, double sequence space, α-, β(bp)- and γ-duals, matrix transformation.

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On Absolute Fibonacci Series Space And Matrix Operators

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ABSTRACT

In a more recent paper, the series space $|F_\theta|(p)$ which is defined as the domain of a matrix corresponding to the absolute Fibonacci summability in the Maddox's space $l(p)$ has been introduced and studied by Gökçe and Sarıgöl [1]. In this study, certain characterizations of matrix operators from the paranormed space $|F_\theta|(p)$ to the classical sequence spaces c, c_0, l_∞ are obtained. Also, we show that the matrix operators between the absolute Fibonacci series space and the spaces c, c_0, l_∞ are bounded operators.

Key Words: Absolute summability, Fibonacci numbers, matrix transformation, bounded linear operators.

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On the Absolute Euler Summability Factors

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ABSTRACT

By (A, B), it is denoted the set of all sequences λ such that the series $\sum a_n\lambda_n$ is summable by the summability method B whenever the series $\sum a_n$ is summable by the summability method A. In the present study, applying the main theorems in [2] to summability factors, we give necessary and sufficient conditions on the sequence λ for $\lambda \in (|E^r,\theta|(p),|E^r,\mu|)$ and $\lambda \in \left(|E^r,\theta|(p),|E^r,\mu|(q)\right)$ where $|E^r,\theta|(p),|E^r,\mu|(q)$ are the absolute Euler summability methods and θ,μ are any sequences of non-negative numbers. Finally, we obtain some results as a special case.

Key Words: Absolute summability, Euler matrix, summability factor, matrix transformations

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On The Multilinear Riesz Bessel Transforms On Morrey Spaces

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ABSTRACT

A multilinear B-maximal operator that acts on the product of m Lebesgue spaces and is smaller that the m-fold product of the B-maximal function is studied. The operator is used to obtain a precise control on multilinear singular integral operators of the Riesz Bessel transforms. A natural variant of the operator which is useful to control certain commutators of multilinear Riesz Bessel transforms with BMO functions is then considered. Finally, the Riesz Bessel transform generated by generalized translate operator on Morrey spaces are studied.

Key Words: B-maximal operator, Riesz Bessel transforms, BMO.

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Rate of Convergence by Bivariate Generalized Kantorovich Type Sampling Series

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ABSTRACT

One of the crucial subfields of electrical engineering is Signal Processing, which focuses on analysing, modifying and synthesizing signals such as sound and images. The analysis of Kantorovich type sampling series in the one-dimensional case has an important place in terms of its application to this theory. In particular, the multivariate versions of this type sampling series include Image Processing applications and allow us to examine the reconstruction of the image. In terms of constructing generalized Kantorovich type sampling operators, it is important to consider an average of f function on a small interval including k/n values instead of using $f(k/n)$ sample values. Therefore, the reduction of the so-called "time-jitter" errors in the neighbourhood of a point rather than a point is studied using Kantorovich type sampling operators.

In [1], a generalized class of Kantorovich type sampling series is studied in the case of one variable and some pointwise convergence theorems are given. In addition, the Voronovskaja type asymptotic formula is obtained. In this study, inspired by the operators in [1], bivariate generalized Kantorovich type sampling series are discussed. Then the degree of approximation in terms of full and partial modulus of continuity is given. Furthermore, the generalized Boolean Sum (GBS) operators of these generalized Kantorovich type sampling series are introduced, a result regarding the rate of convergence to Bögel continuous functions is obtained.

Key Words: Kantorovich type sampling operators, GBS operators, mixed modulus of smoothness.

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Some Mean Ergodic Type Theorems

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ABSTRACT

This talk is based on a joint paper with C.Orhan [5]. A bounded linear operator T on a Banach space X is called mean ergodic if its averages $M_n(T) := \frac{1}{n}\sum_{k=1}^{n} T^k$ converge for every x∈X. The ergodic theorem asserts that if $T: X \to X$ is a bounded linear operator on a reflexive Banach space whose iterates T^j form a bounded sequence of bounded linear operators, then the sequence $\{M_n(T)\}$ converges strongly to a projection onto the kernel of the operator $I - T$.

Cohen [1] determined a class of regular infinite matrices $A = (a_{nk})$ for which

$$L_n := \sum_{k=1}^{\infty} a_{nk} T^k$$

converges strongly to an element invariant under T. He proved that such a sequence $\{L_n\}$ is strongly convergent provided that $\{L_n x: n \in N\}$ is weakly compact and $\lim_{j\to\infty} \sum_{k=j}^{\infty} |a_{n,k+1} - a_{nk}| = 0$, uniformly in n. Recall that, if the matrix $A = (a_{nk})$ maps convergent sequences into the convergent sequences leaving the limit invariant, then A is called a regular matrix. It seems that Cohen's result provides a generalization of the mean ergodic theorems given by J. Von Neumann [4], F. Riesz [6] and K. Yosida [8].

In this talk, we will call an operator $T \in B(X)$ an A-mean ergodic operator if the limit of $\{L_n x\}$ exists. Then we study A-mean ergodic type theorems when $A = (a_{nk})$ is a regular infinite matrix satisfying Cohen's uniformity condition. In particular we get an ergodic decomposition. Using this we also give necessary and sufficient conditions in order that $\{T^n x\}$ is convergent.

Key Words: Ergodic theorem, Mean ergodic theorem, Ergodic decomposition, Regular matrix, Bounded linear operator

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Some Operator Identities on the Umbral Algebra

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ABSTRACT

Let P be the algebra of polynomials in the single variable x over the field complex numbers. Let P^* be the vector space of all linear functionals on P. We use the notation

$$\langle L|p(x)\rangle$$

to denote the action of a linear functional L on a polynomial $p(x)$.

Let F denote the algebra of formal power series in the variable t over C. F will denote both the algebra of formal power series in t and the vector space of all linear functionals on P. Then we have automatically defined an algebra structure on the vector space of all linear functionals on P , namely, the algebra of formal power series. This shall call F the umbral algebra.

An element of F plays three roles in the umbral calculus. It is a formal power series, a linear functional and a linear operator [4].

One can get interesting identities by using actions of some linear functionals and linear operators on some special polynomials, such as Sheffer polynomials.

In this work, we study some operators by using the methods of the umbral algebra. We give the actions of this operators on some special polynomials. In this way, we obtain various properties of these polynomials.

Key Words: Umbral algebra, linear operators, special polynomials.

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Some Results Based on (p,r) −Compact Sets in Banach Spaces

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ABSTRACT

Grothendieck [1] characterized relatively compact subsets of a Banach space as sets contained in the closed convex hull of a sequence converging to zero in the Banach space. This result is well known in the literature as Grothendieck's compactness criterion.

Inspired by this result of Grothendieck [1], which characterizes relatively compact sets in Banach spaces, Sinha and Karn [2] introduced p −compactness notion ($1 \leq p \leq \infty$), which is a stronger form of the compactness.

Later, Ain et al. [3] introduced (p,r) −compactness notion ($1 \leq p \leq \infty, 1 \leq r \leq p^*$), which is a stronger form of the p −compactness. We denote that if $p = \infty$ (thus $r = 1$), then the (p,r) −compactness coincides with the compactness and if $r = p^*$, then the (p,r) −compactness is precisely the p −compactness [3].

The approximation property, which is one of the important properties of Banach spaces theory, has been systematically studied by Grothendieck [1]. Grothendieck obtained various characterizations of the approximation property, defined some versions of the approximation property and examined the relationships between them (see [4,5]).

The p −compactness concept leaded naturally p −approximation property concept. Sinha and Karn [2] defined the p −approximation property concept by replacing compact sets in definition of the approximation property with p −compact sets. The p −compact set and the p −approximation property concepts defined by Sinha and Karn [2] have attracted great attention and been studied by many scientists. We can mention the works done by [5-8].

In this study, some results in the literature based on p −compact sets such as factorizations of p −compact operators, p −approximation property and the representation of a dual space are considered in terms of (p,r) −compact sets.

Key Words: approximation property, p −compact set, (p,r) −compact set.

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Some Results Related to New Jordan Totient Double Sequence Spaces

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ABSTRACT

The Jordan totient function $J_t: N \backslash N, k \angle J_t(k)$ is described as the number of t-tuples of positive integers all less then or equal to k that form a coprime with $(t+1)$-tuples together with k, where $k, t \in N$ and $N = \{1,2,3,\dots\}$. J_t is multiplicative and furthermore it is an arithmetic function and it is one of many generalizations of the famous Euler totient function φ. The 2 dimensional Jordan totient matrix and its domain on the space of l_p of absolutely p-summable single sequences are described and examined by İkhan et al. [5]. After that, compact operators on the matrix domain of the jordan totient matrix have been studied in [6]. 4 dimensional Jordan totient matrix and its associated double sequence spaces have been examined in [4]. In the present paper, first of all we define two new double sequence spaces by using the 4 dimensional Jordan totient matrix derived by the Jordan totient function and we show that this newly described double sequence spaces are Banach spaces with their norms. Then, we give a inclusion relation including this spaces. Moreover, we compute the α-, β(bp)- and γ-duals and finally, we characterize some new 4 dimensional matrix transformation classes and complete this work with some significant results.

Key Words: Jordan totient function, RH-regular matrix, matrix domain, double sequence space, α-, β(bp)- and γ-duals, matrix transformations.

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Spanne-type Result for the Generalized Fractional Maximal Operator in the Vanishing Generalized Weighted Local Morrey Spaces

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ABSTRACT

Morrey spaces were introduced by Morrey in [3]. These spaces appeared to be useful in the study of local behavior properties of the solutions of second order elliptic PDEs. The vanishing Morrey space of the classical Morrey spaces was introduced by Vitanza in [7] and applied there to obtain a regularity result for elliptic PDEs. Later in [8] Vitanza proved an existence theorem for a Dirichlet problem, under weaker assumptions then those introduced by Miranda in [2] and a regularity result assuming that the partial derivatives of the coefficients of the highest and lower order terms belong to vanishing Morrey spaces depending on the dimension. Persson et al. [4] showed the commutators of Hardy operators on vanishing Morrey spaces. Also Ragusa [5] obtained a sufficient condition for commutators of fractional integral operators to belong to vanishing Morrey spaces.

The vanishing generalized Morrey space and vanishing generalized local Morrey space was introduced by Samko in [6]. The boundedness of the multi-dimensional Hardy type operators, maximal, potential and singular operators in these spaces were proved in [6]. Kucukaslan et al. [1] proved the Spanne-type and Adams-type boundedness of generalized fractional integral operators on vanishing generalized local Morrey spaces.

The generalized fractional maximal operators M_ρ was initially investigated in by Nakai in 1994. Nakai introduced the generalized Morrey spaces and proved the boundedness of the generalized fractional integral operator in these spaces. Nowadays many authors have been culminating important observations about the generalized fractional maximal operators M_ρ especially in connection with Morrey-type spaces.

In this talk, we prove the Spanne-type boundedness of the generalized fractional maximal operator M_ρ from the vanishing generalized weighted local Morrey spaces $VLM_{p,\varphi_1}(\omega^p)$ to another vanishing generalized weighted local Morrey spaces $VLM_{q,\varphi_2}(\omega^q)$ with ω^q, and from the vanishing generalized weighted local Morrey spaces $VLM_{1,\varphi_1}(\omega\ )$ to the vanishing generalized weighted weak local Morrey spaces $VWLM_{q,\varphi_2}(\omega^q)$ with ω . Also, we get a new theorem for the Spanne-type result of the generalized fractional maximal operator M_ρ from the vanishing generalized local Morrey spaces VLM_{p,φ_1} to vanishing generalized local Morrey spaces VLM_{q,φ_2}, including weak estimates. The all weight functions belong to Muckenhoupt-Weeden classes $A_{p,q}$.

Key Words: Generalized fractional maximal operator, Vanishing generalized weighted local Morrey space, Vanishing generalized Morrey space, Muckenhoupt-Weeden classes.

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Upper and Lower θp-Continuous Multifunctions

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ABSTRACT

By a multifunction F:X→Y, we mean a point-to-set correspondence from X into Y. The aim of this presentation is to introduce and study the notion of upper (resp. lower) θp-continuous multifunctions. Basic properties of upper (resp. lower) θp-continuous multifunctions and relationships between upper (resp. lower) θp-continuous multifunctions and graphs are also investigated.

Key Words: preopen sets, preclosure, multifunctions, upper (resp. lower) θp-continuous multifunctions.

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Order Boundedness of Ideal Operator

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ABSTRACT

A subspace A of a Riesz space is called ideal whenever $|x|\leq|y|$ and $y\in A$ imply $x\in A$. Let E and F be Riesz spaces. An operator $T:E\to F$ is called ideal operator if $T(A)$ is ideal in F for every ideal A in E and is called order bounded that maps order bounded subsets of E onto order bounded subsets of F. In this talk, we present that not every ideal operator is order bounded by an example and then give conditions when an ideal operator becomes order bounded.

Key Words: Riesz space, ideal operator, order boundedness.

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APPLIED MATHEMATICS

A Mathematical Investigation of COVID-19 in Turkey Using a Susceptible Infectious Recovered (SIR) Type Model

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ABSTRACT

At the end of 2019, a novel coronavirus (abbreviated as COVID-19) was identified in Wuhan, China. It rapidly spreads, resulting in an epidemic throughout China, followed by an increasing number of cases in other countries throughout the World. The World Health Organization (WHO) declared COVID-19 a global pandemic on 11 March 2020 when there were more than 100.000 reported cases and 4.000 deaths across 114 countries.

In [1] we applied the Fuzzyfied Richards Growth Model to understand the dynamic behaviour of the COVID-19 based on the real data and predicted possible future scenarios applying fuzzy approaches for some countries around the world including China, the United States, the top five countries with the highest population in Europe and Turkey.

In this joint work, we investigate the dynamic behaviour of COVID-19 in Turkey using the generalized SEIR (Susceptible Exposed Infectious Recovered) Model [2] that includes seven different states namely susceptible, insusceptible, exposed, infectious, quarantined, recovered and death. After that, by getting help from [1] we extend the generalized SEIR Model to the fuzzy environment. Finally we try to obtain lower and upper solutions by using Zadeh's extension principle [3] in order to approximate the spread of COVID19 among people in Turkey.

Key Words: COVID19, SIR Model, Zadeh's Extension Principle, Fuzzy Environment.

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A NOTE ON MONOTONE TECHNİQUE FOR A CLASS OF FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT

In this paper, we consider the following nonlinear Caputo fractional integro-differential equation

$$^{C}D^{q_1}u(t) = F\left(t, u(t), {}^{C}I^{q_2}u(t)\right)$$

with boundary condition

$$g\left(u(0), u(T)\right) = 0$$

where $F \in C[J \times R \times R_+, R]$, $J = [0, T]$ and $0 < q_2 \leq q_1 < 1$.

We investigate the existence of the solutions of the Caputo fractional integro-differential equation by applying monotone iterative technique with the method of upper and lower solutions. The monotone iterative technique, with the method of upper and lower solutions, offers monotone sequences that converge uniformly and monotonically to minimal and maximal solutions of the problem considered. Since each member of such sequence is the solution of the linear initial value problem of Caputo fractional integro-differential equations which can be explicitly computed, the advantange and the importance of this technique needs no special emphasis. The monotone iterative technique can be applied to many nonlinear fractional differential equations. Moreover, this method can successfully be employed to generate two sided pointwise bounds on solutions of the linear initial value problem of Caputo fractional integro-differential equations, from which qualitative and quantitative behaviour can be investigated. Furthermore, if uniqueness conditions are satisfied the minimal and maximal solutions will converge to the unique solution of the nonlinear Caputo fractional integro-differential equation.

Key Words: Caputo Derivative, Integro-differential equation, Monotone Iterative Technique, Maximal and Minimal Solutions

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A Slough/Wound Interaction Model Accounting for Cellular Diffusion

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ABSTRACT

In this study we consider Slough/Wound interaction model, called SWODE, a nonlinear Ordinary Differential Equation, ODE system, developed by Jones et. al in 2003 [1] that describes evolution of wound and slough area.

We first nondimensionalize the model and estimate a larger domain of attraction for the zero equilibrium solution with an appropriately formulated Liapunov function. Thus, we determine the region where the wound heals and slough tissue disappears in terms of nondimensional model parameters.

Next, we extend the SWODE model to account for one dimensional cellular diffusion resulting in a model, called SWPDE1, a one dimensional nonlinear system of Partial Differential Equations. Evolution of wound and slough interactions is described and equilibrium solutions with lower and upper solutions are determined. The uniqueness of corresponding stationary system is also handled numerically and qualitative behaviour of equilibrium solutions are analysed.

Finally, we further extend SWPDE1 to account for two dimensional cellular diffusion, resulting in a model, SWPDE2, a two dimensional time-dependent nonlinear PDE system. In this model, we consider rectangular wound and slough within a rectangular region ana investigate wound-slough interaction as the healing process goes on. Method of lines is used to obtain a nonlinear system of ODEs and MATLAB ODE solvers [2] are used to integrate the resulting system.

Key Words: Wound healing, nonlinear ordinary differential equation system, nonlinear partial differential equation system, ODE solvers of MATLAB.

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ANALYSIS OF CONTINUOUS DATA ASSIMILATION SCHEME FOR THE NAVIER-STOKES EQUATIONS USING VARIATIONAL MULTISCALE METHOD

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ABSTRACT

In this study, we analyse a continuous data assimilation (CDA) [1] scheme which enables us to combine an observable data with a numerical method to obtain better solutions in which these solutions are also closely similar to the current state of the system. The scheme applied on a Navier-Stokes system which is discretized with two-step Backward Differentiation (BDF2) in time and finite element in space. Also the nonlinear term arises in the system is treated with extrapolation techniques. In order to improve the accuracy and prevent some non-physical oscillations due to the effect of small viscosity and the dominance of convection, a projection based variational multiscale method (VMS) [2] was also cast on the system. The corresponding CDA scheme with VMS is given by,

$$\left(\frac{3y_h^{n+1}-4y_h^n+y_h^{n-1}}{2\Delta t},v_h\right)+\mathrm{Re}^{-1}(\nabla y_h^{n+1},\nabla v_h)+b(2y_h^n-y_h^{n-1},y_h^{n+1},v_h)$$
$$-(p_h^{n+1},\nabla\cdot v_h)+(\sigma(I-P_H)\nabla y_h^{n+1},(I-P_H)\nabla v_h)+\mu(I_H(y_h^{n+1}-u^{n+1}),v_h)=(f^{n+1},v_h)$$
$$(\nabla\cdot y_h^{n+1},q_h)=0$$

where μ is the nudging parameter, I_H is the interpolation operator, σ in VMS term is the eddy viscosity parameter.

The nudging term μ pushes the large spatial scales of the approximating solution toward those of the reference solution while the viscosity stabilizes and dissipates the fine spatial scales and any spillover into the fine scales caused by the nudging term. That's why we added this term to the equation we studied. In the rest of the study, the long-time stability and long-time convergence analyses of the

scheme are presented in detail and several numerical tests are given in order to support theoretical findings and demonstrate the promise of the method.

Key Words: Continuous data assimilation, Navier-Stokes equations, variational multiscale method.

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Analysis of Covid-19 epidemic in Turkey using Generalized SEIR Model

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ABSTRACT

A new type of coronavirus called SARS-CoV-2, or Covid-19 was first identified in Wuhan city, Hubei Province of China at the end of 2019. In the following weeks, Covid-19 has rapidly spread to all over China and to many countries. So, WHO has declared the disease as a pandemic on 11 March 2020. Covid-19 is a respiratory tract disease which the human-to-human transmissible, can cause symptoms including fever, difficulty in breathing, dry cough. Many countries have taken several strict precautions such as limiting travel among cities, closing public area, quarantining cities to be able to control the contagious disease.

Covid-19 has become doubtlessly one of humanity's biggest epidemic problems. This epidemic has generated significant social, economic, and health problems. And many people died. Therefore, it has gained importance predicting how to progress of epidemic. Mathematical models which is based on a series of dynamic mathematical equation play an important role in epidemiology. The SIR model which is the well-known basic infectious disease model in the literature, was defined in 1927 by Kermack and McKendrick (Kermack and McKendrick 1927). Later, by adding new compartments to the SIR model, some infectious disease models such as SEIR, SEIRD which can simulate real life a little better were obtained (see, for instance, Li et al 1999, Weitz and Dushoff 2015 etc). Many study on the generalized version of the classical SEIR model known as SEIRDP model have been made after Covid-19 epidemic outbreak (see, for instance, Godio et al 2020, Afonso et al 2020, Peng et al 2020).

In this study, we have tried to apply to SEIRDP infectious disease model the spreading of Covid–19 in Turkey. For this, we have obtained real data that is publicly available at official web site of Turkey Government Ministry of Health by using web scraping program that we have written with Python programming language (Turkey Government Ministry of Health 2021, Python 2021). We have tried to analyze epidemic and to determine parameters of that model using nonlinear least squares method. We have aimed to be able to evaluate the daily number of infectious cases, recovered cases and deaths, the consequences of the precautions taken.

Key Words: Covid-19, epidemic models, SIR, generalized SEIR, python.

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Decay Estimate for the Klein-Gordon Equation in Anti de Sitter Spacetime

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ABSTRACT

The Klein-Gordon equation arises in relativistic physics such as cosmology, general relativity and quantum field theory. In this study, we consider the following initial value problem for the Klein-Gordon equation in the Anti-de Sitter space time,

$$\Phi_{tt} + nH\Phi_t - e^{2Ht}\Phi + m^2\Phi = 0, \qquad (x,t) \in \mathbb{R}^n \times [0,\infty),$$
$$\Phi(x,0) = \varphi_0(x), \qquad \Phi_t(x,0) = \varphi_1(x), \qquad x \in \mathbb{R}^n,$$

where $m > 0$ is the physical mass and H is denoted by Hubble constant. Here, the sign of H is negative which describes the contraction spatially for the model of the universe. If the sign of H is positive, then the so-called de Sitter space time model describes the expansion spatially. When H is zero, the model is called Minkowski space time. The fundamental solution of the initial value problem is shown in [1]. Then, $L^p - L^q$ decay estimate for the solution of the problem is obtained in [2]. Moreover, in [3], L^∞ decay estimate for the solution of the zero initial value problem with source term is obtained. Decay estimates play an important role to prove the global existence of the solutions for the nonlinear partial differential equations. Therefore, we prove the L^∞ decay estimate for the solution of the initial value problem nonzero initial data.

Key Words: Anti-de Sitter spacetime, Klein-Gordon equation, L^∞ estimate.

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Dynamical Behavior of Rational Difference Equation

$$x_{n+1} = \frac{x_{n-11}}{\pm 1 \pm x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}}$$

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ABSTRACT

The study and solution of nonlinear rational recursive sequence of high order is quite challenging and rewarding. In the recent times, nonlinear difference equations have a critical role in the fields of pyhsics, economy, ecology and computational science engineering, etc. Many researchers have investigated the behavior of the solution of nonlinear difference equations. So, recently there has been an increasing interest in the study of qualitative analysis of rational difference equations.

Our aim in this paper is to investigate the behavior of the solution of the following nonlinear difference equations

$$x_{n+1} = \frac{x_{n-11}}{\pm 1 \pm x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}}, \; n \in \mathbb{N}_0$$

where the initial conditions are arbitrary nonzero positive real numbers. Also, we get explicit forms of the solutions.

Key Words: Difference equations, local stability, recursive sequences.

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Existence Of Solutions For A Third-Order Boundary Value Problem With Integral Boundary Conditions On Infinite Intervals

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ABSTRACT

This paper deal with the existence of bounded or unbounded solutions for the following third order three-point integral conditional boundary value problem on a half-line:

$$x'''(t) + r(t)g\big(t, x(t), x'(t), x''(t)\big) = 0, \qquad t \in (0,\infty),$$
$$x(0) = \int_0^{\eta} x(s)ds, \qquad x'(0) = A, \qquad x'(\infty) = B,$$

where $\eta \in (0,\infty)$, $r: (0,\infty) \to (0,\infty)$ and $g: (0,\infty) \times \mathbb{R}^3 \to \mathbb{R}$ satisfies Nagumo's condition which plays an important role in the nonlinear term depend on the first-order derivative explicitly. This subject "third-order boundary value problem with integral boundary conditions on infinite intervals" is important and also complicated. Our main aim is to show that there is at least one solution to the considering problem. First we create the Green function for our problem. Moreover we define the lower and upper solutions and Nagumo's condition. By applying Schäuder's fixed point theorem and the lower and upper solutions method, we obtain the expected result. Then, we indicate the existence of at least one solution of the problem by determining suitable and sufficient conditions in our problem.

Key Words: Three-point integral conditional boundary value problem, lower and upper solutions, half-line, Nagumo's condition, Schäuder's fixed point theorem.

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Fitted Numerical Method for a Volterra Delay Integro-Differential Equation

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ABSTRACT

In this work, the authors propose a finite difference technique for approximating solutions of first-order Volterra delay integro-differential equation. The presented numerical method acquires a second-order convergence in discrete maximum norm. The derived results are numerically validated in test problems to support the theoretical analysis.

Key Words: Voltera integro- differantial equation, delay, maximum norm.

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Fuzzy Similarity Of Subalpine, Steppe And Segetal Vegetation For Black Sea Region In Turkey

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ABSTRACT

In this study, the results obtained in subalpine, steppe and segetal vegetation in the research project on plant sociology conducted in Black Sea region (Turkey) has been evaluated with the fuzzy similarity measures which is an important approach in applied mathematics. The similarity relations between the sample areas included in each plant community were calculated and interpreted. The issue of fuzzy similarity of sets and elements in sets is used. This study brings a different perspective to new researches in plant sociology. According to this point of view, the fuzzy similarity among plant associations and relevés came to the fore.

Fuzzy similarity is a highly preferred method in all applied sciences for a better understanding of the subject, with its practicality, time saving and contribution to the economy.

For the sake of mathematics point of view, we give the formulas for the fuzzy similarity among plant communities below:

Fuzzy Similarity of Two Fuzzy Sets:

$$FS\ (\ AS_i, AS_j\ ) = \max\ \{\ \min\ \{\ \mu AS_i\ (r_k), \mu AS_j\ (r_k)\}\ \};\ _k = 1(1)62,\ _{i,\ j} = 1(1)5.\ (1)$$

$$r_k \in AS_i \subset X$$

Fuzzy Similarity of Two Elements:

$$FS_e\ (r_k, r_m) = \max\ \{\min\ \{\ \mu_{ASi}\ (r_k), \mu_{ASi}\ (r_m)\ \}\};\ _{k,\ m} = 1(1)30,\ i = 1(1)3\ (2).$$

Key Words: Fuzzy Similarity, Plant Community, Phytosociology, Subalpine, Steppe, Segetal vegetation, Ecology.

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Global Existence and Uniqueness of The Inviscid Velocity-Vorticity Model of The g-Navier-Stokes Equations

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ABSTRACT

In this talk we consider a velocity-vorticity formulation for the g-Navier-Stokes equations. The system is performed by entegrating the velocity-pressure system which is involved by using the rotational formulation of the nonlinearity and the vorticity equation for the g-Navier-Stokes equations. In this study we particularly interest the following velocity-vorticity system of the g-Navier-Stokes equations over the two dimensional periodic box $\Omega = [0,1]^2 \subset R^2$:

$$\frac{\partial u}{\partial t} - \nu\Delta_g u + \nu\frac{1}{g}(\nabla g.\nabla)u + w \times u + \nabla \mathrm{P} = f,$$

$$\frac{\partial w}{\partial t} - \nu\Delta_g w + \nu\frac{1}{g}(\nabla g.\nabla)w + (u.\nabla)\mathrm{w} = \nabla \times \mathrm{f} + \mathrm{w}\left(\frac{\nabla g}{g}.u\right),$$

$$\nabla.(gu) = 0 \qquad \nabla.(gw) = 0,$$

$$u(x,0) = u_0 \qquad w(x,0) = w_0$$

where $P = p + \frac{1}{2}|u|^2$, p denotes pressure, u represent velocity, w which play the role of vorticity, f is the body (external) forcing on the fluid and $\nu > 0$ kinematic viscosity. We consider this problem under the periodic boundary conditions. We assume u, p and w and the first derivative of u, w to be spatially periodic, i.e.,

$$u(x_1 + 1, x_2) = u(x_1, x_2) = u(x_1, x_2 + 1) \qquad (x_1, x_2) \in R^2.$$

We investigate global existence and uniqueness of the inviscid velocity-vorticity model of the g-Navier-Stokes equations using the classical Picard iteration method. The inviscid velocity-vorticity model of the g-Navier-Stokes is equivalent to the functional differential equation

$$\frac{du}{dt} + P_g(w \times u) = P_g f,$$

$$\frac{dw}{dt} + B_g(u, w) = P_g(\nabla \times \mathrm{f}) + P_g\left(\mathrm{w}\left(\frac{\nabla g}{g}.u\right)\right),$$

$$\nabla.(gu) = 0 \qquad \nabla.(gw) = 0,$$
$$u(x,0) = u_0 \qquad w(x,0) = w_0$$

where, for simplicity, we assume f to be time independent. We rewrite $B_g = P_g\big((u.\nabla)\mathrm{w}\big)$ and $P_g: L^2_{per}(\Omega, g) \longrightarrow H_g(\Omega)$ Helmholtz-Leray orthogonal projection.

Inviscid forms of the models in computational fluid dynamics have been attracted and extensively studied by many researchers.

Key Words: Existence and uniqueness, g-Navier-Stokes equations, inviscid velocity-vorticity model.

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Hermite-Hadamard -Fejér type inequalities for higher order convex functions and quadrature formulae

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ABSTRACT

The Hermite-Hadamard-Fejér inequalities state:

$$f\left(\frac{a+b}{2}\right)\int_a^b p(x)\,dx \le \int_a^b p(x)\,f(x)\,dx \le \frac{f(a)+f(b)}{2}\int_a^b p(x)\,dx,$$

where $p\colon [a,b]\to\mathbb{R}$ is nonnegative, integrable and symmetric about $\frac{a+b}{2}$ and $f\colon [a,b]\to\mathbb{R}$ is a convex function. In a special case, for $p(x)\equiv 1$ above inequalities reduce to the Hermite-Hadamard inequalities.

Over the last decades, Hermite-Hadamard type inequalities and Hermite-Hadamard-Fejér type inequalities have been extensively investigated and applied in approximation theory, special means theory, optimisation theory, information theory and numerical analysis.

The main purpose of this talk is to present some new Hermite-Hadamard-Fejér type inequalities for higher order convex functions and the general weighted integral formula involving w-harmonic sequences of functions. Since, a special case of the obtained integral formula is the general weighted Euler integral identity, we also derive Hermite-Hadamard-Fejér type inequalities for the general three, four and five point quadrature formulae of Euler type. Further, we establish Hermite-Hadamard-Fejér type inequalities for the corrected three point formula, the corrected four point formula and the corrected five point quadrature formula of Euler type. These formulae include not only the values of the function in the chosen points, but also the values of the first derivative at the end points of the interval, so have a degree of exactness higher than the adjoint original formulae.

Finally, certain special cases will be presented such as Hermite-Hadamard-Fejér type estimates for Simpson, Maclaurin, corrected Simpson and corrected Maclaurin quadrature rules.

Key Words: Hermite-Hadamard-Fejér inequalities, higher order convex functions, quadrature formulae.

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IMPLICIT EXPONENTIAL FINITE DIFFERENCE METHOD FORNUMERICAL SOLUTIONS OF MODIFIED BURGERS EQUATION

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ABSTRACT

In this paper, implicit exponential finite difference scheme has been used to acquire the numerical solutions of modified Burgers equation. A model problem is used to test the performance of the method. Comparisons of the exact solutions and the numerical solutions acquired by several other methods are present with tables. L_2 and L_∞ error norms are used to demonstrate the correctness of the results. The method is analyzed by von Neumann stability analysis method and it is displayed that the method is unconditionally stable.

Key Words: Modified Burgers equation, implicit exponential finite difference method, von Neumann stability analysis.

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Modified Finite Difference Method for solution of two-interval boundary value problems with transition conditions

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ABSTRACT

A lot of mechanical and physical systems are modeled by ordinary or partial differential equations, for which exact solutions are impossible to find. Many researchers have tried to do this in various semi-analytical, numerical and approximate methods, such as the Finite Element Methods [1], the Adomian Decomposition Methods, the Differential Transform Method [2], the Runge-Kutta Method, the Explicit Euler Method, the Taylor's Expansion Method [3] etc. One of them is the Finite Difference Method [4], which is the simplest scheme and plays an important role in mathematical physics and engineering in all those cases where the differential equation can't be solved by analytical methods.

In this study, we have proposed a new modification of classical Finite Difference Method (FDM) for the solution of boundary value problems which are defined on two disjoint intervals and involved additional transition conditions at an common end of these intervals. The proposed modification of FDM differs from the classical FDM in calculating the iterative terms of numerical solutions. To illustrate the efficiency and reliability of the proposed modification of FDM some examples are solved. The obtained results are compared with those obtained by the standart FDM and by the analytical method. Corresponding graphical illustration are also presented.

Key Words: Finite Difference Method, transition conditions, boundary value problems.

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New Fractional Operators Including Wright Function in Their Kernels

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ABSTRACT

Fractional Calculus is a field of mathematics which deals with the definitions, properties and various applications of integrals and derivatives of arbitrary order. It started with G. W. Leibniz and has been developed up to nowadays by many mathematicians including P. S. Laplace, J. B. J. Fourier, N. H. Abel, J. Liouville, B. Riemann, A. K. Grünwald, A. V. Letnikov, J. Hadamard, A. Erdelyi and H. Kober. Fractional Calculus has been applied in various areas of applied mathematics, physics, engineering and finance.

Very recently, different types of functions have been used by many researchers in the kernels of integral operators for obtaining new definitions of fractional operators. Some of these operators are known as Caputo-Fabrizio, Losada-Nieto, Atangana-Baleanu and Gomez Aguilar-Atangana.

Motivated by these studies, we first defined two new fractional operators with a Wright function in their kernels. The first fractional operator is like Riemann-Liouville type, and the second one is like Caputo type. Then we obtained their Laplace and inverse Laplace transforms. We also determined fractional derivatives of some elementary functions and examined the relationships of new fractional operators with the other known fractional operators in the literature. Finally, as an example, we solved two differential equations with fractional derivatives obtained using both new fractional derivative operators.

Key Words: Gamma function, Wright function, Laplace transform, Inverse Laplace transform, Riemann-Liouville fractional operator, Caputo fractional operator.

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Nonexistence of Global Solutions for a Hyperbolic-Type Wave Equation With Delay

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ABSTRACT

In this paper, we consider a hyperbolic-type wave equation with delay. Under suitable conditions, the nonexistence of global solutions have been discussed. Generally, time delay effects arise in many applications and practical problems such as physical, chemical, biological, thermal and economic phenomena. Also, delay effects can be a source of instability. It is well known that delay effects may destroy the stabilizing properties of a well-behaved system. There are several examples that illustrate how time delays destabilize some internal or boundary control system.

Key Words: Nonexistence of solutions, hyperbolic-type, wave equation, delay.

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Numerical Analysis of Singularly Perturbed Problems with Integral Boundary Conditions

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ABSTRACT

In this research, exponential difference scheme is investigated to solve the singular perturbation problem with integral boundary condition. Some properties of the exact solution and its first-order derivative are obtained. A finite difference scheme is created on this mesh. The stability and convergence analysis of the method are shown as first-order convergent at the discrete maximum norm, independent of the perturbation parameter ε. Numerical datas are revealed by solving an example. These datas are demonstrated on table and figure.

The singularly perturbed problem is a problem involving a small parameter ε. So, there are thin transition layers where the solutions varies very rapidly for small values of ε, while away from layers it behaves regularly and varies slowly. Hence, the presence of small parameter in singularly perturbed problems gives severe difficulties that have to be addressed to ensure accurate numerical solutions [6-10]. To solve these type of problems, primarly there are some approaches namely, finite difference methods, finite element methods such as exponential elements, and fitted mesh methods which use or special piecewise uniform grids which condense in the boundary layers.

The solutions of like this problems include layers which occur in narrow layer regions of the domain. Singular perturbation problems emerge very frequently in fluid mechanics, fluid dynamics, quantum mechanics, elasticity, aerodynamics, meteorology, plasma dynamics, magnetohydrodynamics, rarefied gas dynamics, oceanography and other domains of the great world of fluid motion [1-5].

Key Words: Singular perturbation equation, finite difference scheme, uniform mesh, uniform convergence, integral conditions.

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Numerical Integration Of System Of Differentıal Equations By Iterative Decreasing Dimension Algorithm

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ABSTRACT:

Studying on solution of the system of linear algebraic equations

$$Ax = f \tag{1}$$

is a classical problem which is important not only in linear algebra but also in other branches of science, engineering, economics. In (Wang and Jiang, 2000), a decreasing dimension method (DDM) has been proposed to solve the system (1) where A is N× N - regular matrix, x and f are N-vectors. In (Zhang, 2002), it has been said that the proposed DDM in (Wang and Jiang, 2000) is same as the well known domain decomposition technique based on a Schur complement type method. So in (Keskin and Aydın, 2007), the authors improved DDM and gave iterative decreasing dimension method (IDDM) which decreases the dimension of the linear systems, one order in every step without any pre-process.

On the other hand; we consider a system of ordinary differential equations

$$\frac{dx}{dt} = \kappa(t,x), \quad t \in (t^0, \mathrm{T}], \tag{2}$$

where $\kappa(\cdot,\cdot)$ is continuous with respect to the first argument and Lipschitz continuous with respect to the second argument. The system (2) were converted to type (1) system by Euler's method and Newton's method under suitable conditions (initial value or boundary value conditions), respectively. Thus, the numerical integration of the type (2) system was converted to the type (1) system and investigated by Schur's complement method (Guibert and Tromeur-Dervout, 2008; Burden and Faires, 2010; Falgout et al., 2014; Badia and Olm, 2017).

In this study, the numerical integration of the system type (2) was investigated with modified algorithms by IDDM based on Schur's complement method. Numerical examples are also given using these algorithms

Key Words: Differential equations system, Schur complement method, Iterative decreasing dimension method.

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On Controlling Chaos for Multi-Step Iteration Process and Its Special Iterations in Discrete Dynamical Systems

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ABSTRACT

Due to the close relationship between chaos and dynamical systems, it is a natural result to consider the iteration processes that are related to the dynamical systems of fixed point theory. From this natural relationship, the control of the chaos that occurs in fixed point iteration dynamics is the main problem of the article. To achieve this goal, analytical solutions have been given to control the chaos that occurs for unstable fixed points of multi-step iteration process. After, it has been shown that there is an effective regime for the parameters of multi-step iteration. It is well know that it is occur the chaos in the any system according to parameters in system. To illustrate this claim, well-known special cases of multi-step iteration process, Noor iteration, Ishikawa iteration, Mann iteration, Krasnoselskij iteration, Picard iteration processes have been introduced. In particular, among these iterations, the Noor iteration process has been studied in detail in terms of controlled chaos. The Lyapunov exponent was used to estimate the stability and unstability of fixed points and periods that cause chaos in iteration processes. Finally, with the help of MATLAB program, all these results are shown on logistic system and cubic equation with chaos properties.

This work was supported by the Yildiz Technical University Scientific Research Projects Coordination Unit. Project Number: FDK-2018-3348.

Key Words: Controlling chaos, multi-step iteration process, dynamical systems.

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On Some Applications of Korovkin Type Theorem for k-Positive Linear Operators

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ABSTRACT

During the last forty years, many researchers impressed with the Korovkin theorems and they applied these theorems to the setting of different function spaces like Banach algebras, Banach spaces and so on.

The study for the statistical convergence plays very important role as an expanding of the basic convergence for the sequence spaces. The concept of statistical convergence was introduced by Fast [1] and Steinhaus [2], later reintroduced by Schoenberg [3].

The first work for the convergence of analytic functions of complex variables by the idea of k-positive linear operators was given by Gadjiev [4]. He offered "k-positive operators" and applied the Korovkin type theorem for these operators in the space of holomorphic functions on the different domains.

Let us

$$D_r^R := D(r;R) := \{z \in \mathbb{C}^n : r_j < |z_j| < R_j \; ; j = \overline{1,n}\}$$

be polyradius in $\mathbb{C}^n$ and $\mathcal{H}(D(r;R))$ be holomorphic functions space over $D(r;R)$ with the topologically compact convergence. In this study, the concept of convergence will have the same meaning as uniform convergence on compact subsets of the polyradius $D(r;R)$. We will take into account the semi-norms for $r < r' < R' < R$, $\|f\|_{\mathcal{H}(D(r;R)),r',R'} := \|f\|_{\mathcal{R}(D(r;R))} := \max_{r'<|z|<R'} |f(z)|$

that convert $\mathcal{H}(D(r;R))$ into a Frechet-type space.

In this study we will be interested in the sequence of k-positive linear operators that stands for the Laurent coefficient. The aim of this research is to attain Korovkin-type theorems for k-positive linear operators on $\mathcal{H}(D(r;R))$.

Key Words: Korovkin type theorem, k-positive linear operators, statistical convergence, holomorphic functions in space of several complex variables.

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On stability of the bacterial infection model through incommensurate fractional-order system

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ABSTRACT

In this study, the time-dependent changes in antibiotic concentrations and the bacterial population causing the infection in a host receiving multiple antibiotic therapy in case of a bacterial infection were investigated by mathematically modeling with incommensurate fractional-order system (IFOS).

Let us consider that B represent the bacterial concentration in an individual at time t, and C_i for $i = 1,2,\dots,n$ represent the i-th antibiotic concentration at time t in an individual receiving multiple antibiotic therapy against the infectious bacteria. The model for $t > t_0$ is

$$\begin{aligned} D^{\alpha_1}B(t) &= \beta B\left(1-\frac{B}{\delta}\right) - B\left[\sum_{i=1}^{n} q_i C_i\right] - \eta B \\ D^{\alpha_2}C_1(t) &= \Lambda_1 - \mu_1 C_1 \\ &\vdots \\ D^{\alpha_{n+1}}C_n(t) &= \Lambda_n - \mu_n C_n \end{aligned} \tag{1}$$

where initial conditions are $B(0) = B_0$, $C_1(0) = C_{1_0}$,…,$C_n(0) = C_{n_0}$. In addition, for the numbers $\alpha_1, \alpha_2, \dots, \alpha_{n+1}$, which are derivative orders in the model, it is satisfied the followings:

$$0 < \alpha_1, \alpha_2, \dots, \alpha_{n+1} \leq 1 \tag{2}$$

such that

$$k_1, k_2, \dots, k_n, m_1, m_2, \dots, m_n \in \mathbb{Z}^+ \tag{3}$$

and

$$\alpha_1 = \frac{k_1}{m_1}, \alpha_2 = \frac{k_2}{m_2}, \dots, \alpha_n = \frac{k_n}{m_n} \tag{4}$$

where the number m is the smallest of the common multiples of $m_1, m_2, \dots, m_n$. The parameters used in the model can be summarized as follows. The bacteria causing the infection increase according to the logistic rules, where β is the growth rate of the bacteria and δ is the carrying capacity of the bacteria, In addition, η indicates the natural death rate of the bacterium and q_i for $i = 1,2,\dots,n$ indicates the death rate of the bacteria due to the i-th antibiotic. For $i = 1,2,\dots,n$, the i-th antibiotic is taken in a fixed amount of Λ_i and is excreted from the body at a rate of μ_i. Also, we have the following inequalities.

$$\beta, \delta, \eta, q_i, \Lambda_i, \mu_i > 0 \tag{5}$$

In addition, the results, which are compatible with real situations, have obtained in numerical studies.

Key Words: Incommensurate Fractional-Order Differential Equation; Bacterial Infection Modeling; Stability Analysis.

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Parametric Identification in ODE Models as a Tool for Solving Real-world Problems

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ABSTRACT

One of the best ways to understand the deeper connections between the different aspects of mathematics to start working on interesting real-world problem, which has a great number of applications. Engaging students in such a way helps them to understand the material on a deeper level and apply it to solving real-world problems. In the current talk, I shall present two real-world problems which could be solved using parametric identification (a method to estimate the unknown parameters of a model). The problems are suitable for students at the University level.

Compute the transmission rate of an infectious decease. There is a given data set, which consists of data for the number of susceptible and infectious individuals to a certain decease. The aim is to find transmission rate of the decease. Firstly, students shall get familiar with the SIR model, which splits the population to three groups – susceptible, infectious, and removed [1]. The model consists of three ordinary differential equations for the three different groups and appropriate initial conditions are applied. It has two unknown parameters – the recovery rate and the transmission rate. Secondly, the problem shall be solved with fixed parameters. Then the unknown parameters are found in such a way that the model fits the data best.

Axisymmetric drop shape analysis. The problem can be formulated as follows. Using an image of a pendant drop of unknown liquid, find the surface tension of a liquid. This problem has a great number of applications – in food industry, oil industry, etc. Firstly, the students shall get familiar with the Young-Laplace equation of capillarity, which gives a relationship between the surface tension and the mean curvature of the drop and the pressure, applied to it.

Computing the curvature of the drop and using some geometrical considerations, they shall arrive at mathematical model, consisting of three ODEs with two unknown parameters [2]. In order to find the unknown parameters, the profile of the drop shall be extracted and the unknown parameters shall be computed, using parametric identification. Detailed description of all experiments from student point of view could be found in [3].

Key Words: SIR model, Young-Laplace equation of capillarity, mathematics education.

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SINGULARLY PERTURBED FUZZY INITIAL VALUE AND FUZZY BOUNDARY VALUE PROBLEMS

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ABSTRACT

In this work, we firstly introduced singularly perturbed initial and two-point fuzzy boundary value problems and then we have given algorithms for obtaining the solutions of them. We gave some results on the behaviour of the α-cuts of the solutions. We fuzzified some examples and then we successfully applied the algorithms of solutions of them.

Key Words: Fuzzy Differential Equation, Initial Boundary Value Problem, Two Point Boundary Value Problem, Singularly Perturbed, Zadeh's Extension Principle.

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P.S. The subject of this study have been sent to journals to be evaluated as articles.

Some Further Properties of BLUPs in SUR Models

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ABSTRACT

Seemingly unrelated regression (SUR) models, proposed by [1], are a class of multiple regression models. These models may seem unrelated but each of them can be linked to their correlated error terms across the models. In general, instead of considering the group of linear regression models individually, merging them into a combined model by making use of block matrices is a common procedure to gain more efficiency in the prediction of the unknown vectors.

Characterization of algebraic and statistical properties of predictors of the unknown vectors in regression models is one of the main subjects in statistical analysis. Furthermore, establishing the results on the connection among predictors in different models is one of the primary problems encountered in the theory of regression analysis. Therefore, it is natural to consider certain links among predictors under SUR models and their combined models. In this study, the best linear unbiased predictors (BLUPs) are considered in the context of SUR models. Some properties of BLUPs in SUR models are investigated. We present analytical expressions of the BLUPs and establish some properties of BLUPs by using block matrices' rank and inertia formulas. Our main purpose is to give a variety of equalities and inequalities of BLUPs and their covariance matrices for comparison of these predictors in considered models.

There are extensive works on SUR models, we can refer to [2,3] among others. SUR models are a special class of a system of linear regression models. We may refer to [4,5] for a system of linear regression models. Some results related to inertias, ranks, and Löwner partial ordering of symmetric matrices can be found in, e.g., [6]. Furthermore, we may refer to, e.g., [7,8] for rank and inertia formulas for covariance matrices of predictors/estimators.

Key Words: BLUP, inertia, rank, seemingly unrelated regression model.

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Some Remarks on Predictors in Reduced Linear Models

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ABSTRACT

Establishing a relationship between two or more different linear models in the theory of regression analysis is one of the classical research problems. One of these problems is to examine the relations between original linear model and its transformed models. Sometimes, linear models need to be transformed to meet the requirements of the analysis in statistical inferences of linear regression models. Using several linear transformations of the observable random vector, we can face different transformed models of the original model. One of these transformed models corresponds the reduced linear models of the original model.

In this study, we consider the best linear unbiased predictors (BLUPs) in the context of reduced linear models. Some properties of the BLUPs and their analytical expressions are investigated in reduced linear models. The results are established by making use of formulas of inertias and ranks of block matrices which are effective algebraic tools in matrix theory. Covariance matrices of BLUPs are usually used as comparison criteria to determine optimal predictors among other types of unbiased predictors because of their minimum covariance requirement in the Löwner partial ordering. The main purpose of the study is to derive some results on the comparison of covariance matrices of BLUPs under the original linear model and its reduced linear models. The subjects related to the results obtained in this study can also be found in [1-8].

Key Words: BLUP, covariance matrix, inertia, rank, reduced linear model.

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Sub Pico-Second Optical Solitons in Birefringent Fibers with Kaup-Newell Equation by Two Strategic Integration Technologies

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ABSTRACT

This paper studies sub pico-second optical solitons in birefringent fibers. The term optical soliton is used to refer to any optical field that does not change during propagation because of a delicate balance between group velocity dispersion and linear effects in the medium. The coupled vector form of Kaup-Newell equation without four-wave mixing (4WM) is considered. The governing model is a case of derivative nonlinear Schrödinger's equation and describes sub pico-second optical solitons in birefringent fibers. These optical solitons are electromagnetic pulses which are commonly referred to as ultrafast events. Optical bright, dark, singular and combo singular solitons are recovered by the Riccati function and sine-Gordon function methods. These solitons depend on certain parameter restrictions which are given in this paper. The Riccati function method gives rise to dark and singular solitons while the sine-Gordon function method causes to bright, dark, singular and combo singular solitons. The steps of the Riccati function and sine-Gordon function methods are introduced. These methods are based on the principle of homogeneous balance between the highest order derivatives and the nonlinear terms of nonlinear evolution equations in abundant scientific fields such as in fluid dynamics, nonlinear optical fibers, nuclear physics, electromagnetism, medicine, finance, mathematical biosciences, etc.

Key Words: Kaup--Newell equation, Riccati function method, sine-Gordon function method, optical solitons.

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THE ASYMPTOTIC BEHAVIOR FOR GENERALIZED λ- BERNSTEIN-STANCU TYPE OPERATORS

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ABSTRACT

The Bernstein operators, which are positive linear operators, are of great importance for the theory of approximation. In [1], Bernstein operators were introduced by Bernstein to prove the Weierstrass approximation theorem. In [2], Cai et al. introduced the Bernstein operators with shape parameter $\lambda\epsilon[-1,1]$ as follows:

$$B_m^{\lambda}(g,x) = \sum_{j=0}^{m} g\left(\frac{j}{m}\right)\tilde{p}_{m,j}(\lambda,x), \quad m\epsilon\mathbb{N},\ x\epsilon[0,1], \tag{1}$$

where $\tilde{p}_{m,j}(\lambda,x)$ are Bézier basis functions with shape parameter λ defined by

$$\tilde{p}_{m,0}(\lambda,x) := p_{m,0}(x) - \frac{\lambda}{m+1}p_{m+1,1}(x)$$

$$\tilde{p}_{m,j}(\lambda,x) := p_{m,j}(x) - \lambda\left[\frac{m-2j+1}{m^2-1}p_{m+1,j}(x) - \frac{m-2j-1}{m^2-1}p_{m+1,j+1}(x)\right]; j = 1,2,\dots,m-1 \tag{2}$$

$$\tilde{p}_{m,m}(\lambda,x) := p_{m,m}(x) - \frac{\lambda}{m+1}p_{m+1,m}(x),$$

where the Bernstein basis functions $p_{m,j}(x)$ are defined as

$$p_{m,j}(x) := \binom{m}{j}x^j(1-x)^{m-j};\ \ j = 0,1,\dots,m..$$

In [3], Srivastava et al. constructed λ-Bernstein-Stancu operators defined by

$$B_{m,\alpha,\beta}^{\lambda}(g,x) = \sum_{j=0}^{m} g\left(\frac{j+\alpha}{m+\beta}\right)\tilde{p}_{m,j}(\lambda,x);\ \ m\epsilon\mathbb{N},\ x\epsilon[0,1], \tag{3}$$

where α and β are positive real numbers satisfying $0 \leq \alpha \leq \beta$, and Bézier basis functions $\tilde{p}_{m,j}(\lambda, x)$ with shape parameter λ are defined in (2).

In the present work, we introduce the following generalized λ-Bernstein-Stancu operators with shifted knots for $g \epsilon C[0,1]$,

$$G_{m,\lambda}^{\alpha,\beta}(g,x) = \left(\frac{m+\beta_2}{m}\right) \sum_{j=0}^{m} g\left(\frac{j+\alpha_1}{m+\beta_1}\right) \tilde{p}_{m,j}(\lambda, x); \quad m \epsilon \mathbb{N}, \tag{4}$$

where α_i and β_i are positive real numbers satisfying $0 \leq \alpha_i \leq \beta_i$, for $i = 1,2$ and $x \epsilon \left[\frac{\alpha_2}{m+\beta_2}, \frac{m+\alpha_2}{m+\beta_2}\right]$, and Bézier basis functions $\tilde{p}_{m,j}(\lambda, x)$ with shape parameter $\lambda \epsilon [-1,1]$ are defined in (2) such that

$$p_{m,j}(x) := \binom{m}{j} \left(x - \frac{\alpha_2}{m+\beta_2}\right)^j \left(\frac{m+\alpha_2}{m+\beta_2} - x\right)^{m-j}; \quad j = 0,1, \dots, m.$$

We give a Korovkin type approximation theorem and obtaine the rate of convergence of these type operators for Lipschitz type functions. Then, we give a Voronovskaja type theorem for the asymptotic behavior for these operators. Finally, we give numerical examples and their graphs to demonstrate the convergence of $G_{m,\lambda}^{\alpha,\beta}(f,x)$ to $f(x)$ with respect to m values.

Key Words: λ-Bernstein-Stancu type operators, Bézier bases functions, Voronovskaja type theorem.

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The Marshall–Olkin transmuted Rayleigh distribution. Statistical properties and different methods of estimation

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ABSTRACT

In this paper we have introduced a new four parameter Rayleigh distribution called the Marshall Olkin transmuted Rayleigh distribution obtained by new family of continuous distributions called the Marshall Olkin transmuted G family of univariate distribution given by Afify et al [1]. Some of its mathematical properties including the quantile function, limit behavior, hazard rate function, ordinary and incomplete moments, characteristic function, mean residual lifetime, mean past lifetime, mean deviation about mean and median, order statistics, probability weighted moments, Shannon and Rényi entropies, mean deviations, Bonferroni and Lorenz curves and stress-strength parameter are derived.

The estimation of the model parameters is performed by different methods like the maximum likelihood method, moments estimators, least squares estimators, weighted least squares, maximum product spacing estimates, methods of minimum distances, method of Cramer- von-Misses, methods of Anderson-Darling and right-tail Anderson-Darling and compare them using extensive simulations studies. We investigate the mean estimates, biases, and root mean square errors using Monte Carlo simulations and we showed that the methods of Cramer- von-Misses has the smallest mean square errors and mean relative estimates, for parameters, proving to be the most efficient method compared to other methods.

Two real data set are using to illustrate its applicability, and it is shown that Marshall Olkin transmuted Rayleigh distribution fits much better than some other distributions.

Key Words: Marshal-Olkin transmuted-G; maximum likelihood estimation; Order Statistics.

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The Poisson Topp Leone generalized exponential distribution

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ABSTRACT

Recently many authors constructed several classes of distributions by extending common families of continuous distributions. In 2020 Merovci, Yousouf and Hamedani proposed a new family of distributions called Poisson Topp Leone-G family of distributions. By using this generator, in this paper we have proposed a new extension of the generalized exponential distribution called Poisson Topp Leone generalized exponential distribution. We study the main properties of this new distribution including the quantile function, limit behavior, hazard rate function, ordinary and incomplete moments, characteristic function, order statistics, Shannon and Rényi entropies, mean deviations and Bonferroni and Lorenz curves.

Several methods are used for the estimation of the unknown parameters: the maximum likelihood method, moments estimators, least squares estimators, weighted least squares, maximum product spacing estimates, methods of minimum distances, method of Cramer- von-Misses and methods of Anderson-Darling. We investigate the mean estimates, biases, and root mean square errors using Monte Carlo simulations and we showed that the methods of Anderson-Darling has the smallest mean square errors and mean relative estimates, for parameters, proving to be the most efficient method compared to other methods.

We present two applications with real data set to illustrate its applicability, and it is shown that Poisson Topp Leone generalized exponential distribution fits much better than its parent distribution.

Key Words: exponential distribution; maximum likelihood method; Shannon and Rényi entropies.

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GEOMETRY

A STUDY ON ZERMELO NAVIGATION PROBLEM AND THE CURVATURE PROPERTIES ON CONIC KROPINA MANIFOLDS

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ABSTRACT

In this paper, mathematical formulation of Zermelo Navigation problem using Riemannian & Finsler geometry is discussed. Special situations with complete mathematical solutions of the problem are mentioned, namely Finsler space with constant flag curvature and other. We establish the relationships between some curvature properties of F and the corresponding properties of new metric$\tilde{F}$, which involve S curvature, flag curvature and Ricci curvature. Modelling of the aircraft motion is introduced.

Key Words: Finsler Space, Riemannian Geometry, Geodesics

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An Application of Second Order Picone type Identity for the p-Biharmonic Operator on Riemannian Manifolds

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ABSTRACT

The study of various kinds of weighted integral inequalities for differential operators have been a target of intensive research for a long time. This interest is due to the importance of such inequalities in applications to problems in analysis, mathematical physics, spectral theory, geometry and quantum mechanics.

In this paper, we introduce suitable characterizations for a pair of functions $a(x)$ and $b(x)$ in order to have general weighted Rellich inequality on a complete noncompact Riemannian manifold *M. We then use it to derive several refined geometric Rellich type inequalities.* Our approach relies on the second order Picone type identity which is an extension of the method of Allegretto-Huang presented in [1]. We should point out here that the method we use is quite practical and constructive to obtain both known and new weighted Rellich type inequalities. We shall demonstrate these cases by giving many explicit examples of Rellich type inequalities including radial, logarithmic and particularly non-radial weights.

On the other hand, we also establish weighted L^p Rellich type inequality that connects first to second order derivatives.

Key Words: Riemannian manifold, Rellich inequality, Picone identity.

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An Extension of Smarandache Ruled Surfaces According to Frenet Frame in E^3

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ABSTRACT

In this paper, we address the new notion namely Smarandache ruled surfaces according to Frenet frame and as an extension we introduce new ruled surfaces where all possible linear combinations of the vector elements of Frenet frame are taken into account as a base curve and the generator. Next, by referring the coefficients of first and second fundamental forms, we calculate the mean and Gaussian curvatures so that we could provide some characteristics for these ruled surfaces. In accordance with the calculations, we give the necessary conditions for any surface defined to be developable and minimal, as well. Finally, we present examples for each created ruled surfaces and picture those by using Maple 17 program.

Key Words: Smarandache ruled surfaces, first and second fundamental forms, mean and Gaussian curvatures, developable surfaces, minimal surfaces.

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Bertrand Curves and B-Lift Curves in Lorentzian 3-Space

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ABSTRACT

The theory of curves has an important place in differential geometry. One of the known curves is the natural lift curve. The definition of the natural lift was introduced in Thorpe's "Elementary Topics in Differential Geometry" book. According to the definition, the natural lift curve was formed by combining the end points of the tangent vector of the main curve. In this study, based on Thorpe's definition, we define a new curve in Minkowski 3-space which called B-lift curve and we examine the Frenet operators of the B-lift curve. Besides, we investigate the correpondence of the Frenet vectors between the Bertrand and the B-Lift curve. Finally, we give some examples on these results.

Key Words: B-Lift curve, Bertrand curve, Frenet vectors.

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Generating Circle-Foliated Transition Surfaces in Lorentz Space

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ABSTRACT

Cyclic surfaces can be defined as a one-parametric family of circles. Canal surfaces, Dupin cyclides, quadrics, the surface of revolution and transition surfaces are considered as well-known examples of cyclic surfaces. This study deals with transition surfaces foliated by the system of generating Lorentzian circles. The system of generating Lorentzian circles are constructed by a set of centers and radii functions with two parameters respect to the carrier Lorentzian planes. We give the structural properties of these surfaces by the fundamental forms and curvatures. Finally, we obtain the curvature lines and the singularity conditions so make some classifications for them.

Key Words: Cyclic surface, transition surface, foliation.

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On Some Closed Sets and Circles of the Added Sierpinski Triangle

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ABSTRACT

Fractals are interesting and fascinating shapes. One of the most important features of the fractals is self-similarity. There are basic fractal models that are solutions to many mathematical problems such as Cantor set, Sierpinski triangle and carpet, Vicsek fractal, Koch curve and snowflake, Menger sponge, Mandelbrot set and Julia sets. Many properties of these self-similar sets have been investigated from every aspect for years. Especially the Sierpinski triangle, S, has been considered as a fundamental model in various studies. In recent years, studies in which the intrinsic metrics are formulated on this fractal come to the fore. To define the intrinsic metrics by using the code representations of the points on classical fractals make possible to investigate different geometrical and topological properties of these sets. In different studies, the intrinsic metrics are formulated on the classical Sierpinski triangles, added and mod-3 Sierpinski triangle and also Vicsek fractal by this method.

In this study, we consider the added Sierpinski triangle and its intrinsic metric formula and then we investigate some circles and closed sets of this fractal. Finally, we express them as the code sets with the help of the code representations of the points by using the intrinsic metric and give figures of some of them.

Key Words: Sierpinski gasket, fractal, intrinsic metric.

Acknowledgement: This study was supported by TÜBİTAK-1002 under contract 118F356.

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Several Weighted Hardy type Inequalities with Robin Boundary Conditions

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ABSTRACT

The classical Hardy inequality on $\mathbb{R}^n$ says, for $1< p<\infty$,

$$\int_{\mathbb{R}^n} |\nabla \phi(x)|^p dx \geq \int_{\mathbb{R}^n} \frac{|\phi(x)|^p}{|x|^p} dx;$$

the constant is best possible and there are no nonzero extremals. Even when $p = n$; the inequality is interesting because of the optimality of the constant. For $p < n$ one assumes $\phi \in C_0^\infty(\mathbb{R}^n)$, while $p > n$ one assumes $\phi \in C_0^\infty(\mathbb{R}^n \setminus \{0\})$. Here the subscript zero signifies compact support, ∇ is the usual Euclidean gradient, and $|x| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$.

Hardy-type inequalities have evoked the interest of many mathematicians because of their applications to elliptic and parabolic PDE's motivated by physics and geometry, and the literature concerning different versions of Hardy inequalities and their applications is extensive on the Euclidean setting.

On the other hand, very little work has been done on Hardy type inequalities with
Robin boundary conditions. However, there has been some initiation in this area of interest. In an interesting paper, H. Kovarik and A. Laptev [3] proved, among other results, a Hardy inequality for Laplace operators with Robin boundary conditions. For convex domains they also showed how the corresponding Hardy weight depends on the coefficient of the Robin boundary conditions. Later, T. Ekholm, H. Kovarik and A. Laptev [2] studied the best constant in a Hardy inequality for the p-Laplace operator on convex domains with Robin boundary conditions.

The main objective of this article is to study the general weighted Hardy type inequalities for the p−Laplace operators with Robin boundary conditions. We should emphasize that our unifying method is quite practical and constructive to obtain several weighted Hardy, Maz'ya and Heisenberg-Pauli-Weyl type inequalities with boundary terms.

Key Words: Hardy inequality, Boundary term, Robin boundary condition.

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Some Notes on N(k)-contact Metric Manifolds

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ABSTRACT

Differential geometry, which has a wide application area in many branches of science, is one of the most active fields of study in modern mathematics. The theory of manifolds is one of the most comprehensive and important fields of study of differential geometry since manifolds explain spaces in terms of simpler and easily understandable structures. There are many special and different classes of manifolds in the literature. One of them is N(k)-contact metric manifolds that are almost contact metric manifolds with nullity distribution. The first study on such manifolds were given by Tanno. Since then, such manifolds have been studied and investigated widely by several mathematicians.

On the other hand, in differential geometry, mathematicians have used many different methods to characterize the geometric properties and class of a given manifold. One of them is special vector fields. The existence of the vector fields defined on the manifolds plays a significant role in determining and characterizing the most geometric properties of the related object of the manifolds. This situation make vector fields useful tools in the study of differential geometry of manifolds. As a result of this, Therefore, in recent years, the manifolds which admit special vector fields have been investigated comprehensively by many geometers in many context.

Motivated by the above circumstances, in this work we deal with some special vector fields on N(k)-contact metric manifolds, which prove to be rich in geometrical structures. Also, we obtain some important characterizations as regards both such vector fields and such manifolds.

Key Words: N(k)-contact metric manifold, Sasakian Manifold, Torse-forming vector field.

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Some Special Smarandache Ruled Surfaces According to Flc Frame in E^3

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ABSTRACT

In this study, we introduce some special ruled surfaces according to the Flc frame of a given polynomial curve. We name these ruled surfaces as T-D_2, T-D_1 ve D_1-D_2 Smarandache ruled surfaces and provide their characteristics such as Gauss and mean curvatures in order to specify their developability and minimality conditions. Moreover, we examine the conditions if the parametric curves of the surfaces are asymptotic, geodesic or curvature line. Such conditions are also argued in terms of the developability and minimality conditions. Finally, we give an example and picture the corresponding graphs of ruled surfaces by using Maple 17.

Key Words: Smarandache ruled surfaces, Mean Curvature, Gaussian Curvature, Flc Frame, Polynomial curves.

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The properties of projective, concircular and conharmonic curvature tensor fields on the complex Sasakian manifold

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ABSTRACT

Real Sasakian manifolds are well known in the literature, and there are many articles on this subject. But the concept of complex Sasakian manifold is a new field of study. Studies on complex Sasakian manifolds are scarce in the literature. Properties of many tensors and curvatures on real Sasakian manifolds have been studied, and there are many articles in the literature on this subject. Complex Sasakian manifolds are very difficult to work with, the calculations are complex and long. For this reason, obtaining the properties of tensor fields and curvature tensor fields on the complex Sasakian manifold requires a very laborious work. The best known of the curvature tensor fields are the projective, concircular and conharmonic curvature tensor fields. Therefore, in this article, the properties of projective, concircular and conharmonic curvature tensor fields on the complex Sasakian manifold are investigated.

Key Words: Projective, concircular, conharmonic, complex Sasakian manifolds

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MATHEMATICS EDUCATION

A Disease Modeling Approach to Simulate the Spread of Math Anxiety Between Students

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ABSTRACT

Tobias and Weissbrod (1980) define math anxiety as "the panic, helplessness, paralysis, and mental disorganization that arises among some people when they are required to solve a mathematical problem" that affects a large percentage of the population [1]. There is mounting evidence that math anxiety is contagious [2]. In the present study, we will simulate the spread of math anxiety between students using a disease modeling approach. We use a next generation matrix approach to calculate the basic reproduction number, R_0, associated with math anxiety. We prove that students remain anxiety free when R0 <1 whereas math anxiety becomes endemic when R0 > 1. In other words, when one anxious student transmits their math anxiety to more than one classmate, math anxiety will spread throughout the sample population. Otherwise, the class of students will eventually become anxiety-free. Using numerical simulations of our model, we show that the number of students with math anxiety can periodically increase or decrease during the semester. The merit of the present work is the innovative use of disease modeling to understand the dynamics of math anxiety in a population of students enrolled in a math course.

Key Words: Math Anxiety, Mathematical Modelling, Compartment Model, Numerical Simulation, Mathematical Epidemiology, Stability Analysis.

ACKNOWLEDGEMENT:

The authors would like to acknowledge the work and comments of Mr. Arash Arjmand and Deepak Sireeshan.

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An Application On Comparison of Student Achievements in Face-to-Face and Online Education Models in University Mathematics Education

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ABSTRACT

Today, online education is an education model that is used not only in the education sector, but also in different sectors, regardless of time and place. With the development of technology this education model has become one of the basic education models that provide lifelong learning opportunities.

Due to the Covid-19 coronavirus, which emerged in the last months of 2019 and affected the whole world as a global epidemic, universities in our country had to take a break from traditional face-to-face education in line with the decision taken by YÖK. In this period, which is called the coronavirus pandemic, universities continued their education via online, depending on their digital infrastructure. In this way, it is aimed to reduce the negative effect of the pandemic by participating in the classes in the virtual environment from where they are, without coming to the university campuses.

Mathematics education is fundamental in our country as in the world. The existence of a general prejudice about the difficulty of learning mathematics can affect the educational process. For this reason, the effectiveness of face-to-face and online education methods in mathematics education can be discussed.

In this study, students' achievements in face-to-face and online education methods of a basic theoretical course given in freshmen year in the Department of Mathematics were compared. For this, the grades of the students obtained by face-to-face education method in the 2019-2020 Fall semester and the grades obtained by the online education method in the 2020-2021 Fall semester were used. Whether

these two methods differ in terms of average success or their superiority to each other was examined using the t-statistic.

Key Words: Mathematics education, face-to-face education, online education, t-statistic.

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Assessment Of The Online Course Process During The Pandemic Period From The Perspective Of Instructor (Mathematics Teacher)

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ABSTRACT

Due to the Covid-19 virus epidemic, distance education has been started in our country since 23 March 2020. Based on the number of cases and the course of the pandemic, schools were opened and closed from time to time. With the pandemic, which has been alleviated by the measures and vaccination studies, it is planned to return to face-to-face training on September 6, 2021. However, even if schools are opened, there is a possibility that online classes will continue with the thought that they may close again. It is important for teachers to evaluate online courses that can be reapplied according to the course of the pandemic. In this context, the aim of the research is to evaluate the online course process in the pandemic period from the perspective of the instructor (mathematics teacher). The data of the research were collected at the beginning of the 2021-2022 academic year. The study group consists of a total of 4 mathematics teachers (1 female, 1 male, 2 secondary school mathematics teachers and 1 female, 1 male, 2 high school mathematics teachers) who are still working in the Ministry of National Education and giving online lessons during the pandemic process. While determining the study group of the research, easily accessible case sampling was used. Data were collected through semi-structured interviews with the study group. Content analysis was used in the analysis of the obtained data. According to the data obtained as a result of the research, we see that teachers have difficulties in online lessons at first and make an effort to adapt to technological tools. Teachers make easier figure drawing and question solving using tablets or graphic tablets, enriched their course processes with various Web 2.0 tools, they mostly teach the lessons with the help of Z books, material sharing has become easier in online lessons, class participation is higher at the grade levels preparing for the exam, the disciplinary problems experienced at first in

online courses decreased later, classroom privacy is not paid much attention in online classes and therefore the cameras are not opened by the teachers, that more teacher-centered teaching takes place in online courses, homework can be done more easily in online lessons, mathematics, which is a numerical course, is difficult and time-consuming for the instructor to teach in the online course, math teachers spends a lot of time preparing online course material, the curriculum is completed in the given time. It was stated that online courses made a positive contribution to teachers pedagogically. Teachers have developed their technological pedagogical content knowledge, in a way, by experiencing how to use technology effectively in the online course process.

Key Words: Distance education, mathematics teaching, teacher opinions, online course.

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Behavioural Analysis of a Predator-Prey Model

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ABSTRACT

Multiple mathematical models have been proposed in the studies on populations, since Lotka [1] and Volterra [2] suggested to model population dynamics of predator- prey model. Population dynamics is an important subject in the mathematical biology. Among the models mentioned above, predator-prey models have an important place in the biological field. In mathematical biology, the dynamics of the predator-prey models, which can show relationships between two predator-prey species, have been a great interest by mathematician [3-8] and the reference therein. Qualitative analyses of these studies were found many rich dynamics.

The recent articles have been shown that discrete-time models described by difference equations are appropriate than the continuous-time models governed by differential equations. So, in this study we considered a discrete-time model. We showed that the existence and topological classification of the fixed points of the considered model. Moreover, we discussed bifurcation analysis of a predator-prey model via bifurcation theory. Furthermore, our theoretical prediction is proved by numerical results that all calculations described with the aid of Mathematica. Finally, the analytical results obtained are interpreted biologically in support of numerical studies.

Key Words: Predator- Prey Model, Stability, Bifurcation.

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Development of Preservice Mathematics Teachers' TPACK through Digital Storytelling

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ABSTRACT

Mathematics education, that students are forced to learn, can become more enjoyable when taught by a method that attracts students. In this respect, the use of digital stories with visual and auditory elements that both narrators and listeners enjoy may be effective in mathematics education. Digital storytelling can be used as a beneficial tool enhancing learning and interaction among students (Hofer & Swan, 2005). In this respect, it may be important to provide preservice mathematics teachers opportunities to acquire the necessary knowledge and skills about creating and using digital stories effectively in classrooms. Hence, the study focused on the preservice mathematics teachers' technological pedagogical content knowledge (TPACK) (Mishra & Koehler, 2006) in the context of digital stories. The participants of the current study designed with one group pretest-posttest experimental design as a kind of experimental design was composed of 50 preservice mathematics teachers. Initially, they were asked to design lessons by using any technological tool. Then, they were instructed based on designed instructional sequence related to how to use the Powtoon, and to write stories for creating digital stories. Then, the participants were asked to design lessons using digital stories prepared by them. These lessons were analysed by a rubric prepared by the author. The findings of the study showed that most of the lessons designed and created using digital stories could show preservice teachers' technological pedagogical content knowledge effectively and help preservice mathematics teachers improve their TPACK thorough the instructional process enacted based on digital storytelling.

Key Words: Digital storytelling, mathematics, technological pedagogical content knowledge, preservice mathematics teachers.

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EXAMINING THE EFFECTS OF PRE-SCHOOL EDUCATION ON MATHEMATICS ACHIEVEMENT AT DIFFERENT SOCIOECONOMIC LEVELS: THE CASE OF TIMSS 2019 TURKEY

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ABSTRACT

Research shows that the skills acquired during the pre-school education process in early childhood are important for children to take the necessary foundation for their success in primary school and beyond. Because the experiences that children have from pre-school to the end of primary school form the basis of their future life skills (OECD, 2011a).

Studies show that children who receive pre-school education are in a better position in terms of social, emotional and physical development, language and self-care skills development, academic success in the first years of primary education compared to children who do not (Tantekin, Erden & Altun, 2014). Pre-school education prepares children for primary school in terms of all developmental areas and reduces the differences arising from socioeconomic level (Yazar, 2013). A qualified pre-school education strengthens social equality by compensating for the developmental differences of children.

Studies reveal that socioeconomic differences are also effective on students' academic success (Yayan & Berberoğlu, 2004; Sandoval-Hernandez & Bialowolski, 2016). Studies show that children from families with low and middle socioeconomic status are more disadvantaged than others. The fact that children in this disadvantaged group are included in the formal education process at an early age and receive a good education further increases the importance of pre-school education.

When we look at the results of the international exam practices (TIMSS, PISA), which Turkey also participates in, two points related to pre-school education point to important findings on the academic success of students. First of all, whether students receive pre-school education or not is an important factor that causes differences between students' achievement scores. However, the results show that there is a strong relationship between the duration of preschool education and achievement scores (OECD, 2014; Mullis, I.V.S.vd, 2020).

Purpose of the study

The aim of the study is to determine whether the pre-school education status and duration of education, and depending on these variables, the literacy and numerical skills and socioeconomic levels of the students at the time they started school predict the mathematics achievement of the students.

Research Group

The research was conducted on the data of 3675 fourth grade students who participated in the TIMSS 2019 application from Turkey. Mathematics scores and questionnaire data of fourth grade students were used in the study.

Analysis of Data and Results

Multivariate linear regression analysis was used to determine the variables that predicted students' success. Considering the findings, it was determined that 1094 students, approximately one third of the group, did not receive pre-school education. Also, it was determined that as the duration of pre-school education increased, mathematics achievement also increased. When the results of the regression analysis are examined, it is seen that *socioeconomic level, early numeracy activities beginning school, early literacy activities beginning school*, are significant predictors of students' mathematics achievement. This variable gives a moderate and significant relationship with students' math scores ($R=0.584$, $R^2=0.341$, $p<0.05$).

Key Words: Mathematics achievement, pre-school education, socioeconomic level, TIMSS.

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On Stability Analysis and Flip Bifurcation of a Discrete-Time Predator-Prey System with Allee Effect and Prey Immigration

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ABSTRACT

In this study the dynamics of a discrete-time predator prey model including both Allee effect and immigration factor on prey population is studied. Predator-prey model is an important subject in mathematical biology. In 1930's, famous ecologist Warder Clyde Allee observed that individual fitness declines at lower critical density often occurs in many natural populations. By Allee effect, a positive correlation is described between any measure of species fitness and population numbers. Biologically, the Allee effect term can be explained as the factors that genetic inbreeding and loss of hetereozygosity lead to decreased fitness and that there will be shortage of cooperation interactions among conspecific at low densities.

Allee effect and the immigration parameter have an important role in increasing the realism of the population models, besides they help to gain a more accurate description of the model. We have concentrated on the existence and stability analysis of the equilibrium points of the model. Also, the topological classification of the equilibrium points has been made. Furthermore, dealing with the bifurcation analyses of the model the existence of Flip bifurcation is shown. The dynamical analysis of Flip bifurcation is determined by using the center manifold theorem. Some numerical simulations including stability and bifurcation diagrams are presented.

Key Words: Predator-prey model, stability analysis, Flip bifurcation.

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ON THE DEVELOPMENT OF PLANNING ABILITY IN PROBLEM SOLVING FROM GRADES 5 THROUGH 7

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ABSTRACT

Planning ability is considered an important feature for successful problem-solving in mathematics which should develop over the years at school. It involves many different processes and cannot be measured with imprecise or over-simplified operationalizations (Boghi et al, 2006). Most studies have focused on problem-solving longitudinally within planning ability (Siegler,1983; Gauvain and Rogoff, 1989). In this study, with the help of the Tower of London (TOL) test, which has been developed to measure planning ability, the effectiveness of executive functions for problem-solving in different degrees of complexity is investigated with N=223 students at secondary school in grades five, six and seven. In the five and six move problem sections the planning skills increased; in fact, a significant difference depending on the youngest and the oldest age is showing ($p < .05$). However, no statistically significant difference is found between the planning with respect to gender ($p > .05$).

Key Words: Tower of London Test, planning ability, problem-solving.

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Pre-service and In-service Teachers' Proof Schemes and Their Opinions on Mathematical Proof

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ABSTRACT

Making sense of mathematical proof is equivalent to making sense of mathematics as a discipline (Pair & Calva, 2020). At this point, questions such as how to teach mathematics, how to learn it and how to understand how it is learned arise (Harel, 2008). How and in what ways to teach mathematics; The thought patterns that are desired to be gained in the student and many more are the focus of proof-based teaching, and educators have important tasks at this point. In this study, the views and proof schemes of primary school teachers and prospective teachers who will be teachers of the future on mathematical proof were examined. When the literature is examined, the skills in probability and statistics at the secondary school level do not develop at the expected level (Bakırcı, 2014; Dereli, 2009; Güven & Özmen, 2014; Memnun, Altun & Yılmaz, 2010; Mills & Holloway, 2013), and the majority of secondary school students (Aydoğdu İskenderoğlu, 2003; Ören, 2007) cannot think analytically during the proof process. These results show that these subjects and areas should be supported for lower level students such as primary school and even pre-school. In this study, perspectives of primary school teachers and pre-service primary teachers about mathematical proof and proof schemes were examined. Case study design and maximum diversity sampling were used. Participants' views on mathematical proof were collected with a questionnaire consisting of open-ended questions. In order to examine the proof schemes, the problems created by the researchers that require graphing about data processing were used. The analysis of the obtained data was analyzed using the content analysis method, which is the most commonly used method among qualitative data analysis methods. According to the findings obtained in the research, the classroom teachers and candidates emphasized the meanings of proving, proving the truth and

revealing with concrete evidence; It has been determined that they associate the purpose of proving with increasing confidence in information and confirming meaningfulness. It has been observed that being understandable, consistent, objective, and free from contradictions are the most frequently stated expressions among the features of the proof. According to the findings regarding proof schemes, it was seen that teachers and pre-service teachers mostly had experimental proof schemes. In the light of these results, researchers are recommended to conduct research involving pupils, primary teachers and teacher candidates for the development of primary school level mathematics education.

Key Words: Pre-service and in-service teachers, proof based research, problem solving, data process

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Preservice High School Mathematics Teachers' Experiences on their Fieldwork During the COVID-19 Pandemic

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ABSTRACT

The COVID-19 Pandemic impacted many fields including the field of education (Bakker & Wagner, 2020; Erduran, 2020; Onyema, Eucheria, Obafemic, Sen, Atonye, Sharma, 2020). Most schools had to shift from face-to-face learning to distance learning in a short period of time (Ng, P. T., 2021). Students, teachers, and preservice teachers had different learning and teaching experiences during the pandemic (Baber, 2020). The purpose of this study was to explore three high school preservice teachers' field experience during the COVID-19 pandemic. Their performances on the Teaching Practice course that they took in their eighth semester were analysed. The researcher of this study was the instructor of the course. One preservice teacher observed one high school teacher's classroom, and two preservice teachers observed another high school teacher's classroom online. They shared their experiences on the virtual fieldwork in the Teaching Practice course. All course sessions were recorded. In addition to attending course meetings online, as part of the course, the preservice teachers reported their experience over the semester. The reports consist of 12 week online observations of the high school classroom. The data consists of the preservice teachers' videotaped sessions as well as their written work. The findings and conclusions will be shared.

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Key Words: Pandemic, preservice teacher education, field experience.

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THE EFFECTS OF ACTIVITIES CARRIED OUT WITH WEB 2.0 TOOLS ON SOME FIELD COMPETENCIES OF MATHEMATICS TEACHER CANDIDATES

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ABSTRACT

The fact that educational environments are affected by rapid changes in information and communication technologies necessitate the use of technology in these environments (Akkoyunlu, 1995). This situation has revealed the importance of developing the competencies of teachers who will integrate technology to education (Dağ, 2016). Web 2.0 tools, one of the best technologies that can provide integration into education nowadays, offer innovative web-based alternative digital applications that can provide professional development of teachers without time and place restrictions (Dalgarno ve Lee, 2010). Therefore, the aim of this study was to examine the effects of design experiences of mathematics teaching course activities organized with Web 2.0 tools on teacher candidates' technological pedagogical content knowledge, their attitude towards instructional technologies, and their self-efficacy for practical content development with Web 2.0 tools.

The study was structured in a single-group pretest-posttest design, one of the quantitative methods. The universe of the study consists of 33 teachers candidates studying in a primary school mathematics teaching program in a state university in the 2020-2021 academic year. "Technological Pedagogical Content Knowledge", "Web 2.0 Practical Content Development Self-Efficacy Belief" and "Attitudes Towards Instructional Technologies" scales were used as data collection tools. The study was carried out within the scope of the "Teaching Numbers" course. Twelve different Web 2.0 tools, and how to create examples of activities that can be used in mathematics teaching was presented at the end of the course through the videos prepared by the researcher for 10 weeks. The content of the presented activity examples was consist

of the topics covered in the course. At the end of the process, teacher candidates were asked to prepare and present a lesson plan integrated with Web 2.0 tools.

According to the results obtained from the study, it is concluded that, design experiences of mathematics teaching course activities of the teacher candidates had significant effects on “Web 2.0 Practical Content Development Self-Efficacy Belief” level and the “belief regarding usage of instructional technology in lesson”, “appreciation to usage of instructional technology in lesson” and “unappreciated using instructional technology” sub-dimensions of the “Attitude Towards Instructional Technologies” scale; however there were no significant effect on the “Technological Pedagogical Content Knowledge” scale and it’s sub-dimensions.

Key Words: Web 2.0 tools, Instructional technology, Teacher candidates

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The Effectiveness of the Subject Area Textbook Review Course Amid the COVID-19 Pandemic

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ABSTRACT

Even though courses had to be switched to online during the COVID-19, the effectiveness of this shift needs to be investigated (Daniel, 2020; Tartavuela, Albu, Albu, Dieaconescu, & Petre, 2020). Institutions including schools had to take precautions to survive or continue to function (Azhari & Fajri, 2021, Engelbrecht, Borba, Llinares, & Kaiser, 2020). Compared to the past, research on textbook analysis has also been increased (citation). Textbooks could have the potential to improve preservice teachers' knowledge. Thus, the purpose of this study is to explore the effectiveness of the Subject Area Textbook Review Course during the COVID-19 pandemic. The participants were 57 preservice mathematics teachers, and they took the course over the summer 2019 during the pandemic. The course was a required course that preservice mathematics teachers needed to take in their eighth semester in their program. They met 4,5 hours per week consisting of 3 days of 90 minutes per day. Mathematics textbooks from the website of the Educational Informatics Network (EBA) were reviewed over the course. Preservice teachers needed to present their review of the books as part of the course. The course sessions over the course were videotaped. The results of the analysis of the course materials will be discussed.

Key Words: COVID-19, pandemic, preservice teacher education, textbook analysis

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THE FIELD EXPERIENCES ON THE ONLINE APPLICATION OF THE CASE STUDY METHOD IN MATHEMATICS LESSON

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ABSTRACT

The case study method is one of the effective methods that can be used to provide learning [1], [2] and has areas of use in different disciplines of education [3]. The case study method which brings students together with real life problems [4], enables students to develop their ability to use the information they learn in lessons in practice and to develop their thinking and decision-making skills together with their friends [5]. Although the case study method is not a new teaching method, there are limited studies in this field in mathematics education [6], [7], [8]. For this reason, the applicability of the case study method in the mathematics lesson was discussed.

With this study it is aimed to examine the field experiences obtained in terms of teachers and students within the scope of the online application of the case study method in the mathematics lesson. The study was carried out with 7th grade students from two different branches who were studying at a public school in Sakarya during the fall semester of the 2020-2021 academic year and took distance education courses. The participants of the study consisted of 23 students who constantly followed the online courses during the pandemic process. The study was designed in a qualitative research design and the data collection tools of the study; The opinions of the researchers about the preparation of the case study lesson plans before the application, the teacher's in-class observations during the application of the lesson plans and the written opinions of the students about the application. The experiences gained during this process are presented separately from the perspective of the teacher and the student.

According to the findings; there have been difficulties experienced in the design of the case study, in which daily life problems can be questioned in an empathic, intellectual and moral dimension. It has been determined that the case studies designed according to the experiences gained from the teacher's point of view are

structured based on the pre-cognitive knowledge of the students, the difficulties experienced in the efforts to communicate the case studies within the scope of the mathematical context, the difficulties experienced by the students in the course follow-up due to the online learning environment. In terms of students, even if they are not at a sufficient level of mathematical knowledge, the ease of expressing their opinions within the scope of the case studies prepared, the ease of intellectual flexibility brought by the transfer of mathematical structures to a social context, the gains in awareness of their own personal rights, the recognition of the responsibilities required by living in a social environment, the respect for different views. As a result of this process, suggestions will be presented to researchers about how to design case studies on different subjects and the issues to be considered during the implementation.

Key Words: Mathematics education, case study method, online mathematics lesson, field experiences.

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The Investigation of Studies on Case Study Method in Education

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ABSTRACT

The case study method is a method that helps fill the gap between theory and practice in the teaching environment by bringing students face to face with real life experiences [1]. The case study method has application in different disciplines and students can learn problem solving, critical thinking, reflective thinking, decision making, multicultural awareness, etc. It is very effective in gaining skills [2].

The study aimed to examine the researches on the case study method in education through systematic review. Thus, a general framework from the descriptive features of the relevant researches from their aims to their results was drawn and their tendencies were evaluated. In the study, a total of 48 researches that consisted of 21 articles, 20 master's thesis and 7 doctoral dissertations published between 2010-2020 were included. The researches were examined within the scope of the education field that the case study method was used, purpose, method, sample, data collection tools and results by content analysis. The obtained data were presented on tables with frequencies.

As a result of the study there were determined that as the education area where the case study method was used, that the most researches were made in the fields of teacher education, chemistry education and Turkish education; that quantitative research methods were mostly preferred; that mostly working with students and preservice teachers; that it was determined that scale and achievement tests were used. When the researches examined were evaluated within the scope of their purposes, the effects of the use of the case study method on academic achievement were related to critical thinking, creative thinking, social problem solving, decision making, etc. in the field of education. It was observed that the effect on gaining skills was tried to be determined. Within the scope of the results obtained, the findings regarding the effects of the use of the case study method on knowledge, skills and affective areas were determined. In this direction, different studies can be designed and implemented with the participation of interdisciplinary studies on the use of the

case study method in the field of education and with the participation of student groups at all levels, starting from younger age groups, instructors, teachers and preservice teachers who are the implementers of the method and students' parents who have an important role in the education of students. The effect size can be increased by sharing the results.

Key Words: Education, case study method, systematic review.

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The Opinions of Secondary School Students' Parents about Mathematics Course in Distance Education

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ABSTRACT

This study aims to determine the opinions of secondary school students' parents related to the mathematics course in distant education. A case study method was employed in the research. The sample of the research consisted of 405 parents whose children were studying at the state secondary schools in the city centre of a medium-scale province in the Eastern Anatolia Region. An online form was prepared o determine the opinions of the parents and sent to them. The parents' opinions were asked with the questions related to distance education, mathematics course in distance education, their level of responsibility and their ability to help students. In addition, the demographic information of the students was obtained. The SPSS package program was applied in the data analysis and the chi-square test was done. According to the findings, it was determined that the opinions of the majority of parents on distance education were negative (56%); therewithal, it was not appropriate to the mathematics course (55%). Moreover, it was noticed that the parents, who claimed positive opinion for the distance education, stated, at the same time, that mathematics was appropriate to distance education; on the other hand, those, who expressed a negative opinion, thought that distance education was not appropriate to mathematics course. It was determined in the study that, there was not a significant correlation between the opinions of the parents related to mathematics in distance teaching and students' gender, number of siblings, grade reports variables; there was a significant relationship with class-level variables. Considering this, it can be claimed that the parents of 8th class students expressed more negative opinions compared with the parents of other classes. In addition, it was determined that the parents, who expressed positive opinions related to mathematics in distance education, generally expressed positive opinions; those, who expressed negative opinions related to mathematics in distance education, generally expressed a negative opinion. Besides, a significant correlation was found between the opinions of parents related to mathematics in distance education and the responsibility towards the student. Most of the parents, who claimed negative opinions in distance education, stated that they had more responsibility. It was determined that half of the parents thought that the students needed extra help in a mathematics course in distance education, and some of the parents stated that they helped their children. Consequently, it can be claimed that as the class level increases the opinions of students and parents on distance education are negative because of the anxiety related to the exam. In addition, it was noticed that the responsibility of parents increased in this process, the students needed help in a mathematics course. For this reason, it can be claimed that they stated negative

opinions about mathematics courses in distance education. The findings of the study can be supported by the findings of qualitative studies by Kuzu (2020) and Aslan, Arı and Kanat (2021). These results can be taken into consideration in the education policies.

Key Words: Distance education, mathematics, parent

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Translation of the Digital Game Critter Corral into Turkish and Its Evaluation in Terms of Mathematics Education Standards and Appropriateness for Children

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ABSTRACT

Tablet computers are significantly effective in helping preschool children gain mathematical skills (Papadakis, Kalogiannakis & Zaranis, 2016; Papadakis, Kalogiannakis & Zaranis 2018). The number of digital games on tablet computers is increasing day by day (Blair, Pfaffman, Cutumisu, Hallinen & Schwartz, 2015). Therefore, it is needed to have research-based educational digital games. The aim of this research is to translate the game Critter Corral (CC), which was developed for pre-school children, into Turkish and to evaluate it in terms of mathematics education standards, its appropriateness for pre-school children. CC which is developed at Stanford University (Blair, 2013), is a research-based educational digital game for pre-school children aiming to support basic math skills related to counting such as one-to-one matching, comparison, counting, addition and subtraction. In the translation process of this game, permission was obtained from the game developer (K. P. Blair). Blair sent the English audio recordings of CC, the written version of these recordings, and the written documents containing the English expressions on the game screen, all of which were translated into Turkish by an expert in the Department of English Language Education, together with the researchers. Then, expert opinion was taken to evaluate these translations in terms of their appropriateness for the age and development of children, and the translations were completed by making necessary corrections in line with expert opinions. The researcher voiced the Turkish translations of the audio recordings and sent them to the game developer. Blair prepared the Turkish version of the game by integrating the Turkish translation of CC into the game software and requested the Turkish version to be tested by the researchers before uploading it to the application market.

Researchers played the Turkish version of the game from beginning to end and checked whether there were any deficiencies in the Turkish translations. The researchers noticed several omissions and technical errors in the audio recordings and translations and reported them to Blair. After the necessary corrections, the game was uploaded to the application market. Thus, CC was translated into Turkish and took its place in the Apple Store under the name Critter Corral Turkish. Besides, in this study the "Educational Digital Game Evaluation Form for Preschool Mathematics Education" prepared by Genç Çopur, Dağlıoğlu and Dağlı (2020) was used for the evaluation of the game. In conclusion, it is observed that CC Turkish game includes mathematical content and process standards together, supports the cognitive and psychomotor development areas of the child, has an interesting story in the game, has visual and verbal instructions, is suitable for the child's independent use, progresses gradually from simple to difficult, does not have any negative content, and includes hints and directions.

Key Words: Preschool, Math education, Digital game, Number Concept.

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