Statistical Convergence, Statistical Boundedness and Their Generalizations for Sequences in Metric Spaces

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ABSTRACT

The main topic of this study is to give some generalizations of the statistical convergence, the statistical boundedness and strong Cesàro summability for the sequences in metric spaces. We will see that this concepts are related to each other under some conditions and we will construct these important relations. The statistical convergence was introduced and developed for number sequences during the period of 1935 - 1960 by Zygmund, Steinhaus, Fast and Schoenberg. In the last decades and under different names the subject was discussed in many different theories such as in the theory of Fourier analysis, number theory, ergodic theory, measure theory, trigonometric series and Banach spaces. It was further investigated from the sequence spaces and summability theory point of view and via summability theory by many mathematicians.

The order of statistical convergence of a sequence of positive linear operators was introduced by Gadjiev and Orhan in 2002 and then the statistical convergence of order α ($0<\alpha\leq 1$) and strong p-Cesàro summability of order α were introduced and studied by Çolak in 2010 for number sequences, using the notion α-density of a subset of the set $\mathbb{N}$ of positive integers. After then the subject have been studied by many mathematicians in last few years.

In this study we introduce and give d-statistical convergence of order α, d-statistical boundedness of order α and d-strong p-Cesàro summability of order α for a sequence in a metric space. Furthermore we investigate the relations between the sets of d-statistically convergent sequences of order α, between the sets of d-statistically bounded sequences of order α and between the sets of d-strongly p-Cesàro summable sequences of order α for various values of α's. Also we establish some relations between these concepts.

Key Words: Statistical convergence, statistical convergence of order α, statistical boundedness of order α

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