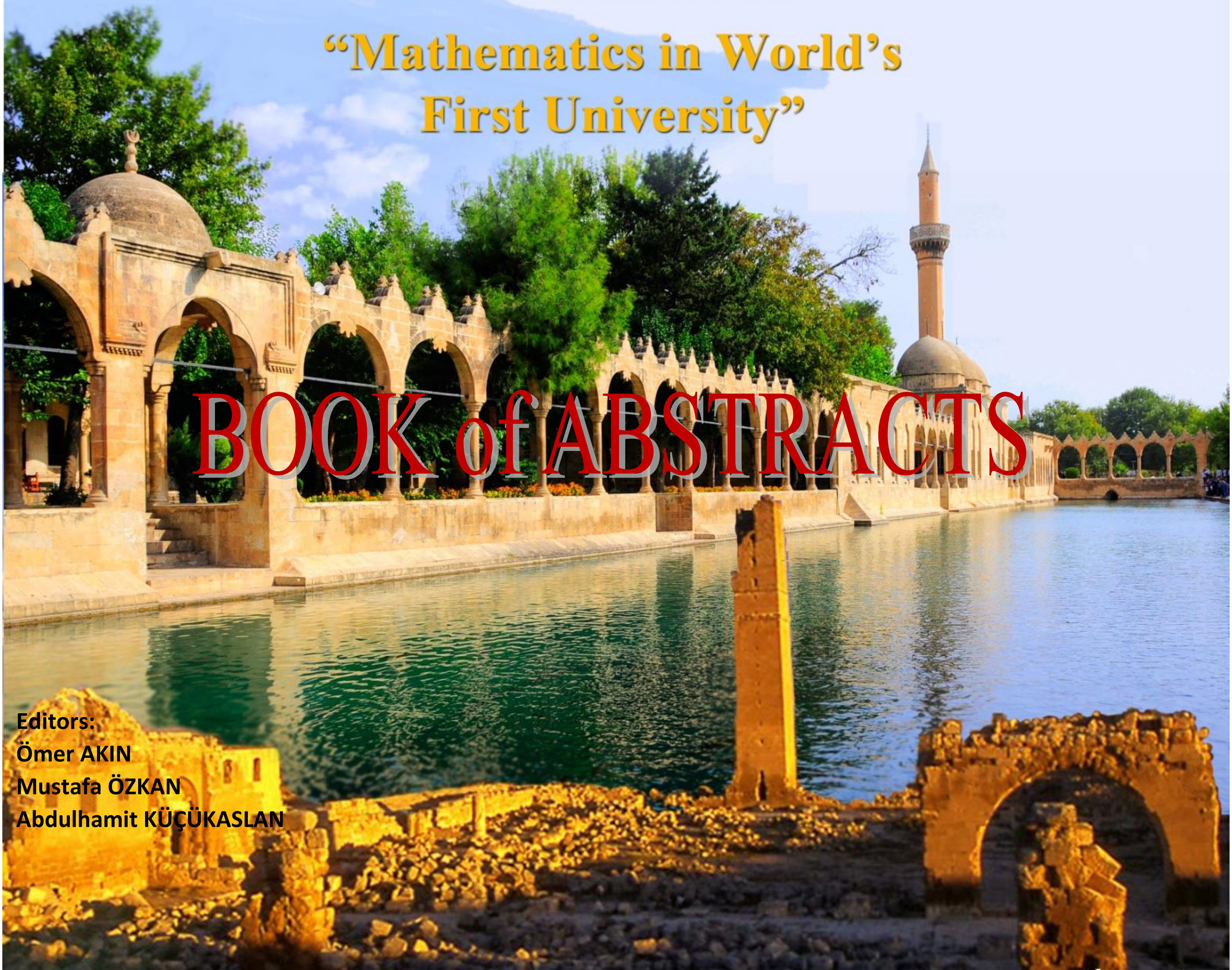
MATEMATİKÇİLER DERNEĞİ

INTERNATIONAL CONFERENCE ON MATHEMATICS AND MATHEMATICS EDUCATION (ICMME-2017)

HARRAN UNIVERSITY

11 - 13 MAY 2017

"Mathematics in World's First University"



BOOK of ABSTRACTS

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International Conference on Mathematics and Mathematics Education
(ICMME - 2017)

Harran University, Şanlıurfa, 11-13 May 2017

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PREFACE

The International Conference on Mathematics and Mathematics Education "Mathematics in World's First University" (ICMME-2017) will be held on May 11-13, 2017 in Şanlıurfa, Turkey.

MATDER- Association of Mathematicians is an association founded in 1995 by mathematicians in Turkey. Up to now, 14 national and 2 international mathematics conferences were organized by MATDER.

These meetings had traditionally been the main general national conferences in all areas of mathematics and mathematics education and had been well attended by mathematicians from academia and ministry of education. The last five conferences have been held in Elazığ ICMME-2016, Niğde (2015), Karabük (2014), Ankara (2013) and Samsun (2012). This year ICMME-2017 has been held at Harran University, Şanlıurfa, Turkey on 11-13 May 2017 as an international conference. Şanlıurfa is a city in South Eastern Anatolia, Turkey which is near to the historical and touristic city Harran. It also involves the world's first temple Potbelly Hill (Göbekli Tepe).

As in the previous meetings, the objective of this conference was to provide us to meet and discuss current research topics in the fields of mathematics and mathematics education.

This conference has been organized by MATDER-Association of Mathematicians and hosted by Harran University.

The topics of interest included;

Mathematics

- Algebra
- Algebraic Geometry
- Category Theory
- Complex Analysis
- Computer Sciences
- Control Theory and Optimization
- Differential Equations
- Differential Geometry
- Discrete Mathematics
- Dynamical Systems and Ergodic Theory
- Functional Analysis
- Fuzzy Logics and Its Applications
- Geometry
- Mathematical Logic and Foundations
- Mathematical Physics
- Number Theory

- Numerical Analysis
- Operator Theory
- Probability Theory and Statistics
- Real Analysis
- Topology

Mathematics Education

- Activities and Programs for Students with Special Needs
- Evaluation and Assessment in Mathematics Education
- Improving the Curriculum
- Popularization of Mathematics
- Problem Solving and Modelling
- Reasoning, Proof and Proving in Mathematics Education
- School Organization and Classroom Practices
- Teacher Preparation and Ongoing in-Service Work
- The Use of Mathematics in The Sciences, Informatics and in The Real World
- Using Technology in Mathematics Education

The main aim of this conference was to contribute to the development of mathematical sciences, mathematical education and their applications and to bring together the members of the mathematics community, interdisciplinary researchers, educators, mathematicians and statisticians from all over the world. In the conference new results and future challenges has been presented in series of invited and short talks, poster presentations, workshops and exhibitions. The presentations could be done in English. Moreover, selected and peer review articles will be published in the following journals:

- Turkish Journal of Mathematics & Computer Science (TJMCS)
- MATDER Mathematics Education Journal

Also, the 5th Mathematics competitions among high schools and chess tournament have been organized.

On Behalf of the Organizing Committee
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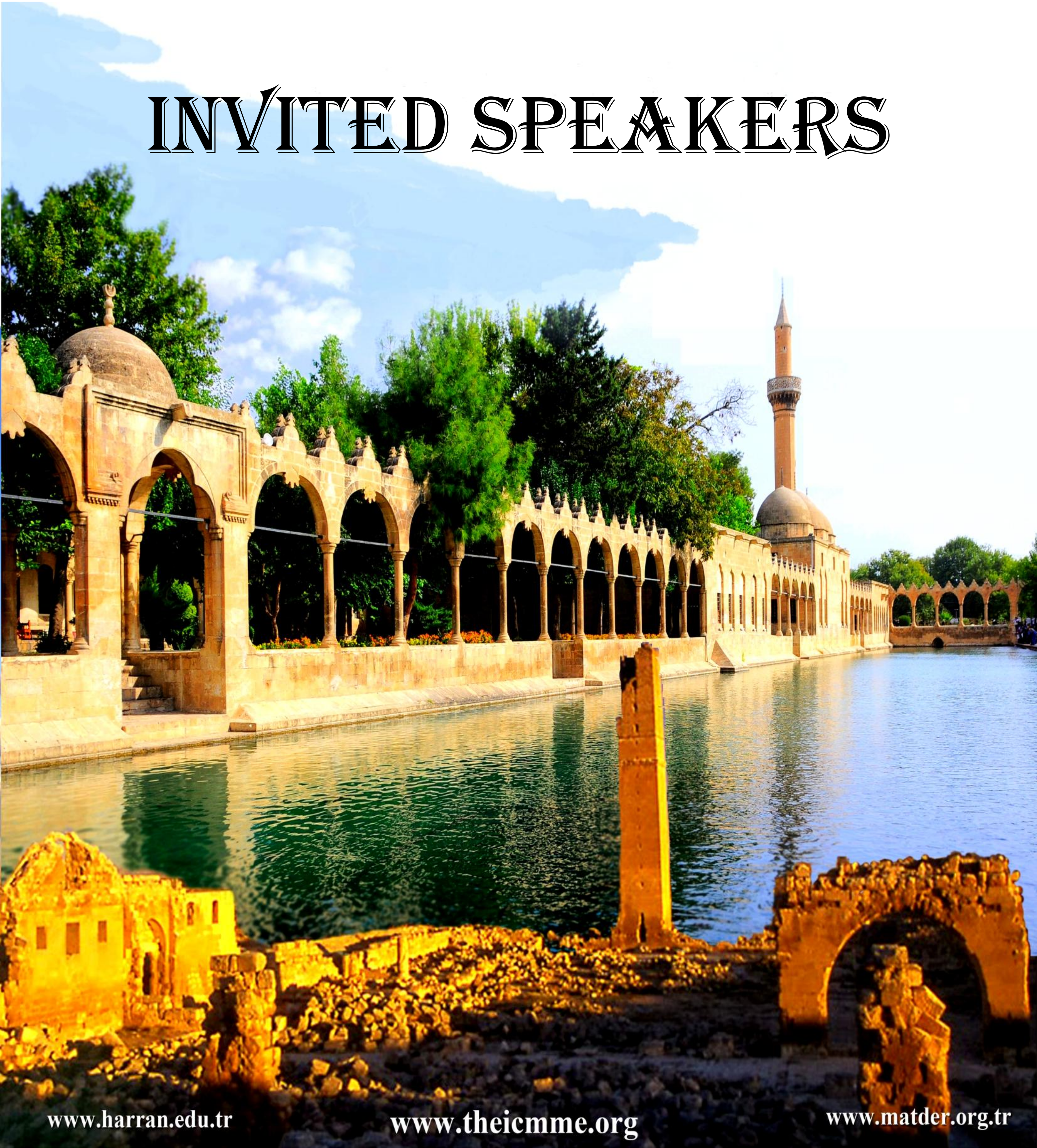

The Philosophy of Point

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ABSTRACT

In this talk, it will be explained how the concept of "point", the first definition of the first book of Elements of Euclid, emerged from the dialectical tension between the concepts of "existence and being". As a natural consequence of this philosophical process, it will be explained how the “atomic” is modeled on the geometric construction, and that the "point", the most basic concept built on geometric systems, is compatible with the “atomic” concept, the most basic concept used in the explanations of the nature theories for the construction of the physical world.

As a result, mathematical systems have been built with philosophical concepts. There are such philosophical concepts at the root of the theorems and systems we have worked on today. These philosophical concepts are also a historical root and there is a continuing story even today. Understanding these conceptual roots make mathematics more understandable and meaningful.

Key Words: Philosophy, point, atomic.

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Some New Results on Fuzzy Differential and Difference Equations

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ABSTRACT

Fuzzy differential and differences equations appear as natural ways to model the uncertainty in dynamical environments. When a real world problem is transformed into a deterministic ordinary differential equation, or a difference equation, we cannot usually be sure that the model is perfect. For example, the initial values may not be known exactly and the functions may contain uncertain parameters. Therefore, it is natural to consider differential equations and difference equations in the fuzzy concept. For the initiation of this aspect of fuzzy theory, the necessary calculus of fuzzy functions such as fuzzy derivatives and fuzzy integrals has also been studied. Consequently, the study of the theory of fuzzy differential and difference equations has recently been growing rapidly as an independent discipline [2,3]. In the recent years, the concept of Fuzzy Initial Value Problems (FIVPs), Fuzzy Partial Differential Equations (FPDEs), Fuzzy Fractional Differential Equations (FFDEs) and Fuzzy Difference Equations [1] has been proposed. Some new approaches and new derivatives are introduced to study new properties of fuzzy differential equations. In this talk, we will discuss some new results in the areas of fuzzy differential equations and fuzzy difference equations.

Key Words: Fuzzy arithmetic, Hukuhara derivative, generalized derivative, fuzzy valued functions, fuzzy difference equations, fuzzy difference equations.

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Sumudu Transform Powers: Theory and Applications

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ABSTRACT

In this presentation we attempt to briefly shed lights on the deeper propertiesof the Sumudu Operator , and present the various advantages it has over other transforms such as Laplace or Fourier while agreeing with them in the latter's temporal applications. We will show pragmatic application to the fact. Consequently, from an educationally pedagogical point of view, we should we be able to afford it and share its practice. The Sumudu can be easily recognized first as the antecedent of other transforms, through temporal dualities. Furthermore it ought to be presented to students and researchers alike, as a natural robust mathematical tool.

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School of Mathematics from Ancient Period: Kyzikos

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ABSTRACT

Throughout the course of the ancient period, it seems likely that Greek mathematicians generally worked alone. But there are some exceptions to this. During the Classical period in Athens were small groups of mathematicians who worked together, or at least on the same set of problems. Eudoxus of Cnidus is one of these. Athens the main center of mathematical activity throughout this period. Mathematicians in Athens then returned to their homes along the eastern Mediterranean and founded schools of mathematical and philosophical instruction. Among these schools, the school of Matematics at Kyzikos was emerges as a striking example [6].

Kyzikos city located on the south of Propontis (Sea Marmara) in Asia Minor. Kyzikos was fertile the waters rich in tuna fishes, the harbours excellent. Kyzikos was a city merchants and sailors. Kyzikos coins was one of the three dominant currencies of the Mediterranean. Another known feture is its role as a centre of science, especially in mathematics and astronomy, during the later fourth and early third centuries [4].

Eudoxus of Cnidus who student of Plato was a Greek astronemer, mathematician, physician, geografer, legislator and scholar [1, 5, 7]. Eudoxus who was the third great mathematician of the Athenian school and is also reckoned as the

founder of the school at Kyzikos. He was born in Cnidus in 408 B.C. [1,5,8]. He went to Tarentum and studied under Archytas the then head of the Pythagoreans. Eudoxus travelled with Plato to Egypt, and then settled at Kyzikos. There he established a school which proved very popular and he had many followers. The mathematicians Menaechmus, Dinostratus, and Athenaeus belonged to school at Kyzikos. Another member was the astronomer Helicon of Kyzikos [1,2,8]. Polyaenus of Kyzikos was among member of the Kyzikos school of mathematicians [3].

The students of Kyzikos school have made important contributions to the history of science. Menaechmus of Kyzikos acquired great reputation as a teacher of geometry, and was appointed one of the tutors of Alexander the Great [8]. When Eudoxus was at Kyzikos with his school, he made astronomical observations [2]. The connection between Kyzikos school and that of Athens was very close, and it is now impossible to disentangle their histories [8]. Eudoxus of Cnidus and his followers in Kyzikos about 368 B.C. went Athens and joined Plato [5,7,8].

In this study, the Kyzikos School, which was established in the ancient city of Kyzikos, will focus on the studies carried out there and the famous mathematicians of Kyzikos.

Key Words: Cnidus of Eudoxus, Cyzicus, Propontis.

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Recent Results on Weighted Hardy and Rellich Inequalities on Manifolds

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ABSTRACT

The classical Hardy inequality in $\mathbb{R}^n$, $n \geq 3$, states that for any $\phi \in C_0^\infty(\mathbb{R}^n)$ the following inequality holds:

$$\int_{\mathbb{R}^n} |\nabla\phi(x)|^2 dx \geq \left(\frac{n-2}{2}\right)^2 \int_{\mathbb{R}^n} \frac{|\phi(x)|^2}{|x|^2} dx.$$

The constant $\left(\frac{n-2}{2}\right)^2$ is sharp, in the sense that

$$\left(\frac{n-2}{2}\right)^2 = inf_{0 \neq \phi \in C_0^\infty(\mathbb{R}^n)} \frac{\int_{\mathbb{R}^n} |\nabla\phi(x)|^2 dx}{\int_{\mathbb{R}^n} \frac{|\phi(x)|^2}{|x|^2} dx}.$$

Another famous inequality is the Rellich inequality

$$\int_{\mathbb{R}^n} |\Delta\phi(x)|^2 dx \geq \frac{n^2(n-4)^2}{16} \int_{\mathbb{R}^n} \frac{|\phi(x)|^2}{|x|^4} dx,$$

where $\phi \in C_0^\infty(\mathbb{R}^n)$, $n \geq 5$ and the constant $\frac{n^2(n-4)^2}{16}$ is again sharp. There are also versions for lower dimensions under additional hypotheses.

These classical inequalities play import roles in many questions from spectral theory, harmonic analysis, partial differential equations and continue to be the subject of extensive study and research.

In this talk, I shall present our recent results on weighted Hardy and Rellich type inequalities on manifolds. This is a joint work with Jerome Goldstein and Abdullah Yener.

Some Results on Statistical and Uniform Statistical Convergence and Limit Points

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ABSTRACT

Convergence of sequences has undergone numerous generalizations providing interesting results in summability theory. Among these are statistical convergence, almost convergence and uniform statistical convergence. Many authors have provided new insights into the field, including Fridy, Orhan, Miller, Balaz , Kostyrko, Mačaj, Šalat , Strauch, Yurdakadim and Miller-Van Wieren.

Here we will focus on the notions of statistical and uniform statistical convergence, as well as on (uniform) statistical cluster points and limit points of a sequence. We present some recent results in this immediate area. Namely, we present some theorems about the (uniform) statistical convergence of subsequences of a (uniformly) statistically convergent sequence and also some theorems about the sets of (uniform) statistical cluster points and limit points of a sequence.

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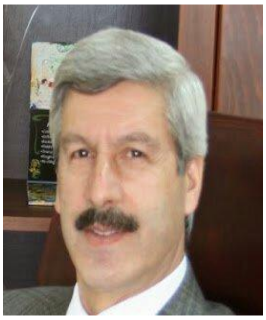
Statistical Convergence, Statistical Boundedness and Their Generalizations for Sequences in Metric Spaces



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ABSTRACT

The main topic of this study is to give some generalizations of the statistical convergence, the statistical boundedness and strong Cesàro summability for the sequences in metric spaces. We will see that this concepts are related to each other under some conditions and we will construct these important relations. The statistical convergence was introduced and developed for number sequences during the period of 1935 - 1960 by Zygmund, Steinhaus, Fast and Schoenberg. In the last decades and under different names the subject was discussed in many different theories such as in the theory of Fourier analysis, number theory, ergodic theory, measure theory, trigonometric series and Banach spaces. It was further investigated from the sequence spaces and summability theory point of view and via summability theory by many mathematicians.

The order of statistical convergence of a sequence of positive linear operators was introduced by Gadjiev and Orhan in 2002 and then the statistical convergence of order α $(0<\alpha\leq1)$ and strong p-Cesàro summability of order α were introduced and studied by Çolak in 2010 for number sequences, using the notion α-density of a subset of the set $\mathbb{N}$ of positive integers. After then the subject have been studied by many mathematicians in last few years.

In this study we introduce and give d-statistical convergence of order α, d-statistical boundedness of order α and d-strong p-Cesàro summability of order α for a sequence in a metric space. Furthermore we investigate the relations between the sets of d-statistically convergent sequences of order α, between the sets of d-statistically bounded sequences of order α and between the sets of d-strongly p-Cesàro summable sequences of order α for various values of α's. Also we establish some relations between these concepts.

Key Words: Statistical convergence, statistical convergence of order α, statistical boundedness of order α

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Education For The 21st Century Developing Mathematics Education

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ABSTRACT

In today's knowledge-based, globally-competitive economy, the types of skills for students need to succeed are far different when compared to past. Any educational system needs to develop a new set of 21st century skills for students. Without new efforts to help students to gain the competencies that prepare them to meet the demands of democracy, competitiveness and life; schools are increasingly irrelevant. These competencies include critical thinking, collaboration, communication and creativity skills. Other important skills include life skills, capacity for lifelong learning, technological and financial literacy, global awareness, and skills for effective civic engagement.

Every student in the 21st century needs to gain certain required skills such as critical thinking, problem solution, collaboration, communication, being innovative and technical literacy. Therefore, schools are supposed to set up these skills and capacities to allow them to figure things out on their own.

Creating an aligned 21st century education system that prepares Turkey to thrive is the central competitive challenge of the next decade. Addressing the challenge requires forceful and forward thinking leaderships from government policymakers.

Learning mathematics is a key fundamental in every education system that aims to prepare its citizens for a productive life 21st century. As a nation, the development of a highly-skilled and well- educated manpower is critical to support an innovation and technological-driven economy. A strong grounding in mathematics and a talent pool in mathematics are essential to support the wide range of value-added economic activities and innovations.

High competencies in Mathematics, Science and Technology (MST) are prerequisites in order to meet the great challenges of today and tomorrow. We need sufficient numbers of people with insight in MST in order to understand our challenges and to act in an appropriate way. Many exciting opportunities exist; ready to be discovered by curious scientists.

In 21st century national mathematics curriculum reform brought major changes in philosophy of instruction, teaching styles, assessment, teacher and student roles, and curriculum organization based on a constructivist approach in terms of instructional studies that began in Turkey in 2004s. The standards for school mathematics describe the mathematical understanding, knowledge, and skills through grades 1-5, grades 6-8 and grades 9-12 in Ministry of Education; 2004, 2005, 2005a, 2011, 2013.

The standards for K-12 in school mathematics curriculum reform are based on five categories: standard for intellectual development, standard for content, standard for pedagogy, learning, teaching and assessment, problem solving: concepts, skills, processes, metacognition and attitudes.

The aim of this study is to give a brief overview of Turkish educational system while analyzing the school mathematics curriculum integration related to 21st century skills into the classroom application, connections and participation of mathematics curriculum reform programs in Turkey.

The analysis of the research is based on five critical components:

1. Mathematics curriculum programs,
2. Teaching and learning process,
3. Assessment,
4. Connection of cross curricular and courses,
5. Content, skills, readiness, anxiety and weaknesses.

An Overview of Projective Geometry

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ABSTRACT

Three different geometries have been formed by reaching the fact that different axioms can be accepted in the 19th century from the controversies over two thousand years on the parallel postulate of the Euclid: Affine geometry, hyperbolic geometry and projective geometry.

Here, some informations about projective geometry will be given. A projective space can be constructed both axiomatically and analytically. In a projective space constructed analytically there exists some close relations between geometric properties of the space and algebraic properties of the system coordinated by the points.

It will mainly be focused on the projective planes and handled the concept of cross-ratio which is only a numerical value and an invariant under projective maps.

Besides, it will also talk about projective planes constructed on algebraic structures such as division ring, alternative ring, Cayley algebra, local ring, and module.

Finally, some applications of projective geometry will be mentioned. In this sense, it will be given examples in the main application areas such as physics, image analysis, cryptography.

Stochastic Processes And The Paradox Of Residual Waiting Time

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ABSTRACT

In this study, the limit distribution of residual waiting time is obtained and the paradox of the residual waiting time which is an important subject in probability theory is expressed. Then, various types of random walk and renewal reward processes such as with delaying barrier, reflecting barrier etc. are constructed mathematically. Next, weak convergence theorems under some conditions are proved for the ergodic distributions of these processes. As a result of the weak convergence theorems, the limit distributions of these ergodic processes are obtained and it is observed that each limit distribution is one of a kind of residual waiting time distribution.

Key Words: Residual waiting time, the paradox of residual waiting time, renewal reward process, random walk, reflecting and delaying barriers, weak convergence theorem.

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From Ernst Chladni to Kurt Gödel or on the Example of Realizing the Laws of the Unity and Conflict of Opposites and of the Negation of the Negation

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ABSTRACT

We will report a theory of construction of two-dim (with respect to spatial coordinates) mathematical models corresponding to thin walled thermodynamic but not only for elastic structures. The evolution on some sense is connected with works of Ernst Florens Friedrich Chladni and Kurt Friedrich Gödel and represents the example of realizing the laws of the unity and conflict of opposites and of the negation of the negation.

The key element for the effective development Project in the fields of Science-Education and Technology-Production is the Mathematics. The importance of Mathematics from the historical aspects will be explained in our presentation on the base of the works of Morris Kline and Albert Einstein. Then we consider in detail the problem how the theory of Clifford Truesdell, Walter Noll and other similar theories will be compared with our corresponding elaborations. The excellency of our methodology will be proved as a new convenient mathematical models, which is not only in theory with regard to natural generalization of widely penetrating initial-

boundary value problems but also in practice with regard to development and functioning of modern technologies (analysis and full design) for thin walled structures used in aircrafts, ships, long pipes and almost every structures characterized with boundary layer effects.

Key Words: The negation of the negation, two-dim mathematical models of thin walled structures, the importance of mathematics.

Applications Of High Performance Computing: Photoelectron Spectroscopy, Cultural Heritages And Drug Design

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ABSTRACT

The presentation is about the applications of the high performance computing such as photoelectron spectroscopy, cultural heritages and drug design. The high performance computing, which saves a considerable amount of time and energy for its users, is used in many different work (weather forecasts, defense industry, economic foresight, simulation of natural disasters, scientific works, etc.) today.

Clusters play an important role in understanding the transition from microscopic to macroscopic structures of matter [1]. Thus, by studying the properties of clusters as a function of size, the evolution of bulk properties can be revealed. In addition, atomic clusters have unique size specific properties that differ from their bulk systems. Clusters of atoms can adopt different atomic arrangements from bulk materials. They often exhibit novel structures and properties, which provide opportunities for new types of chemical bonding and stoichiometry. Consequently, materials synthesized by assembling clusters are technologically important [2, 3].

Throughout the history, Anatolia has hosted several civilizations which have different cultures and beliefs. Therefore, it is possible to see cultural heritages that were inherited from these civilizations at several places in the country. These

artifacts, which shed light from past to present, are destroyed or substantially damaged because of human activity and natural disasters. These precious artifacts are required to be documented as soon as possible for protection and reconstruction projects in order to transmit the artifact to future generations [4].

Drug design is the inventive process of finding new medications based on the knowledge of a biological target. The drug is most commonly an organic small molecule that activates or inhibits the function of a biomolecule such as a protein, which in turn results in a therapeutic benefit to the patient [5].

Key Words: High performance computing, photoelectron spectroscopy, cultural heritages, drug design

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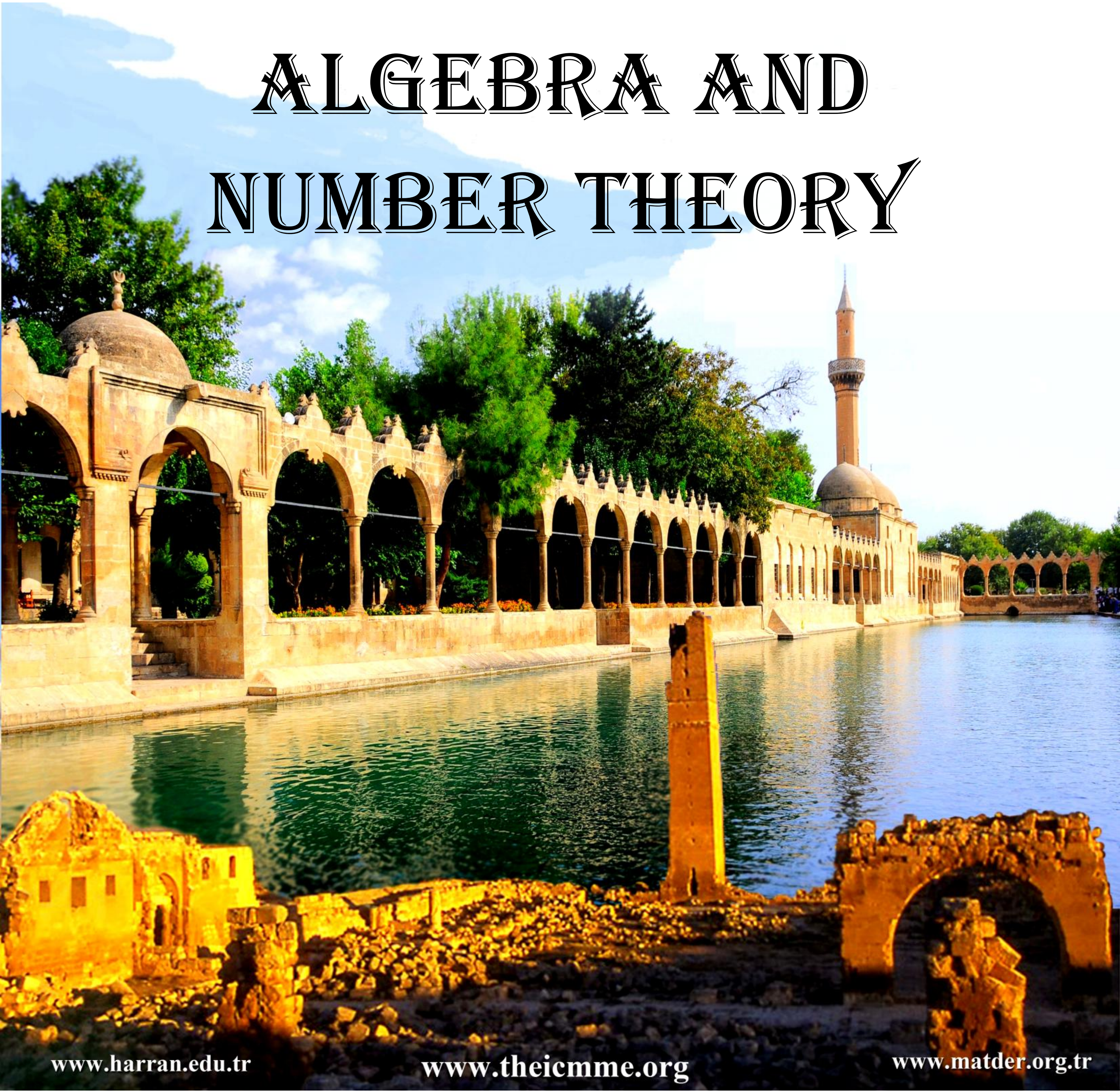
MATHEMATICS

ALGEBRA AND NUMBER THEORY



A New Generalization of Jacobsthal Lucas Sequence (Bi-Periodic Jacobsthal Lucas Sequence)

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ABSTRACT

Out of all the most common types of integer sequences such as Fibonacci, Lucas, Jacobsthal, Jacobsthal Lucas, Pell and the rest, the Fibonacci and Lucas sequences have been the most popular and have attracted mathematicians and researchers as well as engineers for many years now due to their numerous applications in almost every field of Science and Arts. For example, the ratio of two consecutive Fibonacci numbers converges to what is widely known as the Golden ratio whose applications appear in many research areas, particularly in Physics, Engineering, Architecture, Nature and Art [5].

The same can be said of Jacobsthal sequence. For instance, it is known that Microcontrollers and other computers change the flow of execution of a program using conditional instructions. Together with branch instructions, some microcontrollers use skip instructions which conditionally bypass the next instruction which boil down to being useful for one case out of the four possibilities on 2 bits, 3 cases on 3 bits, 5 cases on 4 bits, 11 on 5 bits, 21 on 6 bits, 43 on 7 bits, 85 on 8 bits and continue in that order, which are exactly the Jacobsthal numbers [8].

There are many generalization in literature on the above well-known integer sequences. For example, in [1,2], Edson and Yayenie defined the bi-periodic Fibonacci sequence also known as the generalized Fibonacci sequence as

$$q_n = \begin{cases} aq_{n-1}+q_{n-2} & if\ n\ is\ even \\ bq_{n-1}+q_{n-2} & if\ n\ is\ odd \end{cases} \qquad n \geq 2$$

With initail condition given as $q_0 = 0$ and $q_1 = 1$.

Again in [3], Bilgici also defined the bi-periodic Lucas sequence as

$$l_n = \begin{cases} al_{n-1} + l_{n-2} & \text{if } n \text{ is even} \\ bl_{n-1} + l_{n-2} & \text{if } n \text{ is odd} \end{cases} \quad n \geq 2$$

With initail condition given as $l_0 = 2$ and $l_1 = 1$.

In [4], we defined a new generalization for the Jacobsthal Sequence which we also called the bi-periodic Jacobsthal sequence as

$$J_n = \begin{cases} aJ_{n-1} + 2J_{n-2} & \text{if } n \text{ is even} \\ bJ_{n-1} + 2J_{n-2} & \text{if } n \text{ is odd} \end{cases} \quad n \geq 2$$

with initial conditions, $J_0 = 2,\ J_1 = 1$.

In all the above generalizations, useful properties of integer sequences such as the Binet formula, Generating function, Cassini identity, Catalan identity and many other relations were also obtained.

In this paper, we define a new generalization of Jacobsthal Lucas numbers which we shall call bi-periodic Jacobsthal Lucas sequences. We shall then proceed to find the Binet formula as well as its generating function. After examining the convergence properties of the consecutive terms of this sequence, the well known Cassini, Catalans and the D'ocagne's identities as well as some related binomial summation formulas are also given. We will then extablish a good number of relations between the bi-periodic Jacobsthal Jacobsthal Lucas sequences.

Key Words: Jacobsthal sequence, Jacobsthal Lucas sequence, Binet formula, generating function.

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A new generalization of split Fibonacci quaternions

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ABSTRACT

It is well known that the algebra of quaternions $H = \{\sum_{l=0}^{3} a_l e_l : a_l \in \mathbb{R}\} \cong \mathbb{C}^2$ is defined as a four-dimensional vector space over $\mathbb{R}$ having a basis $e_0 \cong 1$, $e_1 \cong i_1$, $e_2 \cong i_2$ and $e_3 \cong i_3$, which satisfies the following multiplication rules:

$$e_1^2 = e_2^2 = e_3^2 = -1,\quad e_1e_2 = -e_2e_1 = e_3,\ e_2e_3 = -e_3e_2 = e_1,\ e_3e_1 = -e_1e_3 = e_2.$$

Similar to the quaternion algebra, the algebra of split quaternions $S = \{\sum_{l=0}^{3} a_l e_l : a_l \in \mathbb{R}\}$ is defined as a four-dimensional associative algebra over $\mathbb{R}$, but the basis satisfies the multiplication rules:

$$e_1^2 = -1,\ e_2^2 = e_3^2 = 1,\quad e_1e_2 = -e_2e_1 = e_3,\ e_2e_3 = -e_3e_2 = -e_1,\ e_3e_1 = -e_1e_3 = e_2.$$

There are several studies on different types of sequences over quaternion algebra and split quaternion algebra [1,3,4,5,6]. In [3], Horadam defined the n-th Fibonacci quaternion Q_n as:

$$Q_n = F_n e_0 + F_{n+1} e_1 + F_{n+2} e_2 + F_{n+3} e_3$$

where F_n is the n-th Fibonacci number. By using split quaternion multiplication, Akyigit and et al. [1] introduced the n-th split Fibonacci quaternions.

In this study, we consider the sequence of the generalized bi-periodic Fibonacci quaternions $\{W_n\}$, which is defined in [6] as:

$$W_n = \left\{ \sum_{l=0}^{3} w_{n+l} e_l \ :\ a_l \in \mathbb{R} \right\}$$

where $\{w_n\}$ is the generalized bi-periodic Fibonacci sequence that is introduced in [2] as: $w_n = aw_{n-1} + w_{n-2}$ if n is even, $w_n = bw_{n-1} + w_{n-2}$ if n is odd with arbitrary initial conditions w_0, w_1 and nonzero numbers a, b. We present a new generalization of the split Fibonacci quaternions which give us a simple way to represent several number of split quaternion sequences in a unique sequence. By taking appropriate values for the generalized split Fibonacci quaternions, we can obtain several split quaternion sequences as a special case. Also, we give the generating function, the Binet formula, and some basic properties of these split quaternion sequences.

Key Words: bi-periodic Fibonacci sequences, split quaternions, bi-periodic Fibonacci quaternions.

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A New Radical Structures on the Soft Sets

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ABSTRACT

The algebraic structures have been transferred to soft set theory in two different types: The first one is the soft algebraic structures such as soft groups, soft rings, soft fields, soft vector spaces, soft near-rings, soft modules etc. The second one is the soft substructures of algebraic structures such as soft subgroups of a group, soft subring of a ring, soft ideal of a ring,soft submodule of a module etc. The notion of soft set was introduced in 1999 by Molodtsov[1] as a new mathematical tool for dealing with uncertainties. Since its inception, it has received much attention in the mean of algebraic structures such as groups, semirings, Soft set theory has continued to experience tremendous growth in the mean of algebraic structures since Aktas and Cagman [10] defined and studied soft groups, soft subgroups, normal soft subgroups, soft homomorphisms, adopting the definition of soft sets in [1]. Sezgin and Atagun [11] corrected some of the problematic cases in [10], introduced the concepts of normalistic soft group and normalistic soft group homomorphism. Atagun ve Sezgin [9] defined the concept of soft subrings and soft ideals of a ring and studied related properties with respect to soft set operations. Acar, Koyuncu and Tanay [13] introduced basic notions of soft rings, which are actually a parametrized family of subrings of a ring, over a ring R and the soft ideals of a soft ring. In [17], Shah and Medid, introduced several notions like soft prime ideals, soft maximal ideals, soft primary ideals, and soft radical for a soft ring over a given unitary commutative ring. In this study, we define a radical of an ideal in soft set theory by using different soft ideal concepts: a soft ideal of a ring [9] and a soft ideal of a soft ring [13]. We give some results and illustrate with several examples. Our paper differs from [17] as regards the basic definitions of soft radicals.

Key Words: Soft sets, ring structure, radicals, soft radicals, nil radicals.

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A note on Betti Series of the Universal Modules

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ABSTRACT

Let R be the coordinate ring of an affine irreducible curve serviced by $R=\frac{k[a_1,a_2,...,a_k]}{(g)}$ and ω is a maximal ideal of R. Assume that, R_ω the localization of R at ω is not a regular ring. Let $\Omega_n(R_\omega)$ be the universal module of n-th order derivations of R_ω. We show that $B(\Omega_n(R_\omega),t)$ the Betti series of $\Omega_n(R_\omega)$, is a rational function under some conditions. To conclude give examples related to $B(\Omega_n(R_\omega),t)$ for various rings R. In order to prove conclusions about algebraic sets and their coordinate rings, one of the methods is to study the universal module of di¤erential operators. This ideas of studying the universal module may decrease questions about algebras to questions of module theory. This notion of studying the universal module may reduce questions about algebras to questions of module theory. For example, if P is a point of an algebraic set U, then under suitable conditions it can be shown that P is a simple point of U if and only if $\Omega_1(R)$ is a free module. Here R is the local ring corresponding the point P of U and $\Omega_1(R)$ is the universal module of derivations of R. Throughout this study, unless the contrary is stated explicitly, by a k-algebra, we mean commutative ring which contains a field k of characteristic zero. All modules will be unitary. We use the following notation: The universal module of n-th order derivations of a ring R will be denoted $\Omega_n(R)$.

Key Words: Universal modules, Betti series, n-th order derivations.

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A Note on Soft Set Cyrptosystem

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ABSTRACT

The notion of soft set was introduced in 1999 by Molodtsov[1] as a new mathematical tool for dealing with uncertainties. Since its inception, it has received much attention in the mean of algebraic structures such as groups, semirings, rings, BCK/BCI-algebras, normalistic soft groups, BL-algebras, BCH-algebras and near-rings. Atagun and Sezgin defined the concepts of soft subrings and ideals of a ring, soft subfields of a field and soft submodules of a module and studied their related properties with respect to soft set operations also union soft substructues of near-rings and near-ring modules are studied in [5]. Cagman et al. defined two new type of group action on a soft set, called group SI-action and group SU-action, which are based on the inclusion relation and the intersection of sets and union of sets, respectively. Algebraic structures of soft sets have been studied by some authors. Maji et al. presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et al. introduced several operations of soft sets and Sezgin and Atagun studied on soft set operations as well. Soft set relations and functions and soft mappings were roposed and many related concepts were discussed, too. We intruduced two new operations on soft sets, called inverse production and chareacteristic production, by using Molodtsov's definition of soft sets[6]. We proved that the set of all soft sets over a universe U is an abelian group under the each operations and called "the inverse groups of soft sets" and "characteristic group of soft sets"[6]. In this work, the operations inverse and

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characteristic products of soft sets are redefined without using relation forms of soft sets. We defined soft cryptosystem which is a new cryptosystem method by using inverse and characteristic products of soft sets with symmetric groups and given some applications

Key Words: Soft sets, group structure, ring structure, inverse product, characteristic product

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A Note on the (p,q)- Beta Polynomials and Their Properties

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ABSTRACT

Polynomials are one of the fundamentals in the study of mathematics. A polynomial can be represented by an equation, a graph and a numerical table. Like functions, polynomials have mathematical operations such as addition, substraction, multiplication and division. Also, some polynomial families are very important in mathematics, physics, engineering, statistics and so on. For instance, the Genocchi polynomials have been studied in differential topology, quantum physics and Eistenien series. Recently, the Beta polynomials are studied by Bhandari and Vignat [3]. And then, Simsek studied the generating function of these polynomials and obtained some of their properties by using the generating function. Furthermore, Simsek expanded Beta polynomials to *q*-Beta polynomials and investigated their results that depend on *q*-integers ([1] and [2]). Nowadays, *(p,q)*-calculus which is generalization of *q*-calculus have been extensively studied in many branches of mathematics such as approximation theory, analytic number theory, special polynomials etc.

In this study, we give definition of Beta polynomials and obtain their generating function based on *(p,q)*-integers. Then, we show graphs of Beta polynomials and their generating function by using various variables of *p* and *q.* In addition, we derive some of their properties such as recurrence relations, relations with related to special polynomials and numbers and an identity in relation to *(p,q)*-Volkenborn integral.

Key Words: *(p,q)*-Calculus, Beta polynomials, *(p,q)*-Volkenborn integral.

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A Note on the Generalization q-Fibonacci Polynomials

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ABSTRACT

The Fibonacci sequences and their generalizations has an important place in field of number theory. One of these generalizations is the biperiodic Fibonacci sequence, denoted by

$$q_n = \begin{cases} aq_{n-1} + q_{n-2}\,; & if\ n\ is\ even \\ bq_{n-1} + q_{n-2}\,; & if\ n\ is\ odd \end{cases} \tag{1}$$

for $n \geq 2$ with initial values $q_0 = 0$ and $q_1 = 1$, where a and b are nonzero numbers in [1]. If we take $a = b = 1$ in (1), we obtain the Fibonacci sequence, if we take $a = b = 2$ in (1), we get the Pell sequence and if we take $a = b = k$ in (1), we get $k-$Fibonacci sequence. Some another generalizations are the incomplete Fibonacci and Lucas numbers are defined as

$$F_n(k) = \sum_{j=0}^{k} \binom{n-1-j}{i} \quad ; \qquad 0 \leq k \leq \left\lfloor \frac{n-1}{2} \right\rfloor$$

and

$$L_n(k) = \sum_{j=0}^{k} \frac{n}{n-j} \binom{n-j}{i} \quad ; \qquad 0 \leq k \leq \left\lfloor \frac{n}{2} \right\rfloor,$$

respectively in [4]. Note that for the case $k = \left\lfloor \frac{n-1}{2} \right\rfloor$ incomplete Fibonacci numbers are reduced to Fibonacci numbers and for the case $k = \left\lfloor \frac{n}{2} \right\rfloor$ incomplete Lucas numbers are reduced to Lucas numbers. Ramirez [2] defined the biperiodic incomplete Fibonacci sequence and obtain some properties of the biperiodic

incomplete Fibonacci sequence. In [8], Jia et al. defined the generalizations of $q-$Fibonacci polynomials as follows $S_{n+1}(t,q)=S_n(t,q)+tq^{n-2}S_{n-1}(t,q)$ for $n\geq 1$, where $S_0(t,q)=a$ and $S_1(t,q)=b$.

In this study, we defined the $q-$analogue of the biperiodic generalized Fibonacci and Lucas polynomials. We give the useful results and generating functions for the biperiodic generalized Fibonacci and Lucas polynomials. In addition, we introduces $q-$analogue of the biperiodic incomplete Fibonacci and Lucas polynomials. We obtain recurrence relations and some properties of the biperiodic incomplete Fibonacci and Lucas polynomials.

Key Words: $q-$Fibonacci polynomials, $q-$biperiodic Fibonacci polynomials, $q-$biperiodic Lucas polynomials.

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A Relationship Between Orders Obtained by Uninorms and T-Norms (T-Conorms)

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ABSTRACT

Uninorms, t-norms and t-conorms are important aggregation operators. An operation U is called a uninorm on bounded lattice L, if it is commutative, associative, increasing with respect to the both variables and has a neutral element on L. Uninorms are t-norms if its neutral element e=1 and uninorms are t-conorms if its neutral element e=0. These basic properties give that uninorms are generalization of t-norms and t-conorms because its neutral element can be any element on the lattice.

Uninorms were defined on unit interval [0,1] by Yager and Rybalov [6]. Researchers have studied uninorms on more general structures such as partially ordered sets, bounded lattices [3,4]. Ordering obtained from logical operators are hot topic to study for researchers. These studies started with defining the partially ordered relation from t-norms [5].

In this study, relationships between ordering based on uninorms and ordering based on underlying t-norms and t-conorms are deeply studied. It is showed that if underlying t-norms and t-conorms are divisible, the order obtained uninorms coincides the natural order on bounded lattice. Moreover, it is posed that if e in $L\backslash\{0, 1\}$ is comparable with all elements of L, the partially ordered set L equipped with the order obtained by uninorm is lattice iff [0,e] with order obtained from underlying t-norm and [e,1] with order obtained from underlying t-conorm are lattices. An example is given to show that this argue may not be true in general.

Key Words: Uninorm, t-norm, t-conorm, bounded lattice.

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A Study on Continuous Z- Numbers

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ABSTRACT

In this paper, we study Z numbers and continuous Z numbers. The concept of a Z number relates to the issue of reliability of information. The concept of a Z number is based on the concept of a fuzzy granule. It should be noted that the concept of a Z number is much more general than the concept of confidence interval in probability theory. There are some links between the concept of a Z number, the concept of a fuzzy random number and the concept of a fuzzy random variable. An alternative interpretation of the concept of a Z number may be based on the concept of a fuzzy set valued random variable.

Firstly, we give definition of the Z numbers. A Z number, Z=(A,B) has two components, The first component A is a probability measure and the second component B is restriction on the probability measure of A.

Secondly, we give continuous Z numbers. A continuous Z number is an ordered pair Z=(A,B) where A is a continuous fuzzy number playing a role of a fuzzy constraint on values that a random variable X may take: X is A and B is a continuous fuzzy number with a membership function $\mu_B\colon [0,1] \rightarrow [0,1]$ playing a role of a fuzzy constraint on the probability measure of A: P(A) is B.

Key Words: Z-numbers, fuzzy numbers, probability measure, membership function.

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A Study on Z-Numbers

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ABSTRACT

In this talk, we discuss algebraic properties of Z numbers under the four arithmetic operations, namely addition, subtraction, multiplication and division which are defined by the extension principle. The concept of a Z number relates to the issue of reliability of information.

Firstly, we give definition of the Z numbers. A Z number, Z=(A,B) has two components, The first component A is a restriction (constraint) on the values which a real valued uncertain variable, X, is allowed to take. The second component B is a measure of reliability (certainty) of the first component. The concept of a Z number has a potential for many applications, especially decision analysis, risk assessment, and economics. The concept of a restriction has greater generality than the concept of a constraint. A probability distribution is a restriction but is not a constraint. A restriction may be viewed as a generalized constraint. In this note the terms constraint and restriction are used interchangeably.

The restriction R(X): X is A -> A; is referred to as a possibility restriction (constraint), with A playing the role of the possibility distribution of X. More specifically, R(X): X is A -> Poss (X=u) = $\mu_A(u)$ where μ_A is the membership function of A and u is the generic value of X.

Secondly, we give generalization level of uncertain numbers and operations on random variables.

Key Words: Z numbers, fuzzy numbers, uncertain numbers.

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An Application of Ecological Models in Fuzzy Initial Value Problems

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ABSTRACT

Using differential equations is widespread method to model dynamical systems. Moreover, analyzing the solutions and their behavior of ecological models play an important role to understand the phenomenon. Obtaining more realistic models is a crucial issue that scientist work on. The biological parameters which are used in differential equations are not always fixed. In real world, parameters vary due to the changing environmental conditions such as natural disasters, climate and financial changings. Therefore, it is natural to determine the impreciseness of the biological parameters in the behavior of biological models. To add the impreciseness, stochasticity is added in the models. However, the problem of choosing appropriate probability distribution for imprecise parameters arises. So, this makes the problem more complicated. Fuzzy set theory, which is developed by Zadeh [1], is used to overcome to this impreciseness.

There are few literature exists in the field of mathematical biology. Bassanazi et al. [2] used fuzzy differential equations to study the stability of fuzzy dynamical system by considering the variables and parameters are uncertain in nature. Barros et al. [3] considered environmental fuzziness of a life expectancy model by taking the parameters is fuzzy in nature. Peixoto et al. [4] presented predator-prey model under fuzziness. Mizukoshi et al. [5] studied the fuzzy initial value problem with parameters and/or initial conditions under fuzziness. Guo et al. [6] studied logistic model and Gompertz model under fuzziness. Pal et al. [7] presented proportional harvesting model with fuzzy intrinsic growth rate and fuzzy harvesting quantity.

In this work, we study on some fuzzy ecological models by using results which are given by Barros et al. Moreover, we introduce interval arithmetic and its application on fuzzy dynamical systems.

Key Words: Fuzzy initial value problem, Fuzzy ecological models.

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An Outranking Approach For Multi-Criteria Decision-Making Problems With Interval-Valued Bipolar Neutrosophic Sets

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ABSTRACT

The multi-attribute decision making (MADM) problems and methods are the important in real decision making deal with incomplete, indeterminate, and inconsistent data. The MADM problems can be modelling with fuzzy set theory [1] and intuitionistic fuzzy set theory [2]. Because of membership functions, these theories have some disadvantages and cannot modelling MADM problems. Based on the theories, Smarandache [4] developed the neutrosophic set theory which overcomes the disadvantage of fuzzy set theory and intuitionistic fuzzy set theory which independently has a truth-membership degree, an indeterminacy-membership degree and a falsity-membership degree. In this paper, a novel outranking approach for multi-criteria decision-making (MCDM) problems is proposed to address situations where there is a set of numbers in the real unit interval and not just a specific number with a bipolar neutrosophic set. Firstly, the operations of interval bipolar neutrosophic sets and their related properties are introduced. Then some outranking relations for interval bipolar neutrosophic numbers (INNs) are defined based on ELECTRE, and the properties of the outranking relations are further discussed in detail. Additionally, based on the outranking relations of INNs, a ranking approach is developed in order to solve MCDM problems.

Key Words: Neutrosophic sets, bipolar neutrosophic sets, Interval-valued bipolar neutrosophic sets, Multicriteria decision-making, Outranking method.

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Application of Intuitionistic Fuzzy Set Via Normalized Hamming Distance Method in Education

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ABSTRACT

The notion of fuzzy logic was defined firstly by L.A.Zadeh in 1965 [1]. The membership function of an element to a fuzzy set is a value between zero and one the non-membership function of an element to a fuzzy set is equal to 1 minus the membership degree in fuzzy set theory. Then, Intuitionistic fuzzy sets (shortly IFS) were defined by K.Atanassov in 1986 [2]. Intuitionistic fuzzy sets form a generalization of the notion of fuzzy sets. In intuitionistic fuzzy set theory, sum of the membership function and the non-membership function is a value between zero and one. The hesitation degree is defined as 1 minus the sum of membership and non-membership degrees respectively in intuitionistic fuzzy sets. In fuzzy set theory, the hesitation degree is zero because sum of the membership function and the non-membership function is 1. The intuitionistic fuzzy set theory is useful in many application areas, such as algebraic structures, education, robotics, control systems, agriculture areas, computer, irrigation, economy and many engineering fields. Many applications of intuitionistic fuzzy set are carried out using distance measures approach. Distance measure in intuitionistic fuzzy sets is an important notion because of its wide applications in real world, such as decision making, medical diagnosis, career determination [6],[7],[8].

In this paper, we proposed an application of intuitionistic fuzzy set in high school determination using normalized hamming distance method. Intuitionistic fuzzy set is lucrative model to detail uncertainty involved in decision making. Normalized hamming distance method was utilized to measure the distance between each student and each high school. School which settled each student determined using normalized hamming distance method according to examination transition to high school education. Solution is stated by measuring the smallest distance between each student and each school.

Key Words: intuitionistic fuzzy sets, high school determination, normalized hamming distance method, distance measure.

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Approximately Homomorphisms in Proximal Relator Spaces

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ABSTRACT

Let X be a nonempty set. A relator is family of relations $\mathcal{R}$ on a nonempty set X and the pair $(X, \mathcal{R})$ (or $X(\mathcal{R})$) is a relator space. If we consider a family of proximity relations on X, we have a proximal relator space $(X, \mathcal{R}_\delta)$ ($X(\mathcal{R}_\delta)$).

The concept of ordinary algebraic structures are consist of a nonempty set of abstract points with one or more binary operations which are required to satisfy certain axioms such as a groupoid is an algebraic structure $(A,\circ)$ consist of a nonempty set A and a binary operation "$\circ$" defined on A. And binary operation "$\circ$" must be closed in A whereas in proximal relator spaces, the sets are composed of non-abstract points instead of abstract points and these points are describable with feature vectors. Descriptively upper approximation of a nonempty set is obtained by using the set of points composed by the proximal relator space together with matching features of points and these are the basic tools for defining algebraic structures on proximal relator spaces and binary operations on any groupoid A in proximal relator space must be closed in descriptively upper approximation of A.

In this work, approximately group homomorphisms in proximal relator spaces have been given. Moreover, some properties of descriptively approximations using object descriptive homomorphism have been obtained.

Key Words: Proximity spaces, Relator spaces, Approximately group, Homomorphisms.

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Binomial Transforms of the k-Jacobsthal Sequences

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ABSTRACT

The most common types of integer sequences are Fibonacci, Lucas, Jacobsthal, Pell etc. There have been many studies in literature about these special number sequences because of their numerous applications in almost every field of science and arts. For instance, the ratio of two consecutive elements of Fibonacci sequence is always golden ratio, is very important number almost every area of science and art. It is well known that computers use conditional directives to change the flow of execution of a program. In addition to branch instructions, some microcontrollers use skip instructions which conditionally bypass the next instruction.This brings out being useful for one case out of the four possibilities on 2 bits, 3 cases on 3 bits, 5 cases on 4 bits, 11 cases on 5 bits, 21 cases on 6 bits,..., which are exactly the Jacobsthal numbers. For n≥2 Jacobsthal sequence is given by the recurrence relations

$$j_n = j_{n-1} + 2j_{n-2}, \qquad j_0 = 0, \ j_1 = 1 .$$

For any positive integer k, the k-Jacobsthal sequence $\{j_{k,n}\}_{n\in N}$ is defined recurrently by

$$j_{k,n} = kj_{k,n-1} + 2j_{k,n-2}, \qquad j_{k,0} = 0, \ j_{k,1} = 1 .$$

The binomial transform of the k-Jacobsthal sequence $\{b_{k,n}\}_{n\in N}$ is defined as the following formula

$$b_{k,n} = \sum_{i=0}^{n} \binom{n}{i} j_{k,i}.$$

S. Falcon and A. Plaza have obtained the binomial transform of the k- Fibonacci sequence in 2009. P. Bhadouria has established the binomial transform of the k- Lucas sequence in 2014.

In this paper firstly we define the binomial transform of the k- Jacobsthal sequence. And also we obtain new binomial transforms which are called k- binomial, rising, falling binomial transforms of the k- Jacobsthal sequence. Many formulas relating these transforms of sequences are established. And by using these formulas we establish Binet formulas and generating functions and also we get the Jacobsthal Pascal triangles. By using Jacobsthal Pascal triangles, we can easily obtain the values of the binomial, k binomial , rising, falling transforms of the k- Jacobsthal sequence.

Key Words: Jacobsthal sequence, binomial transform, Binet formulas, generating functions.

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Bipartite Graphs Associated with Circulant Matrices

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ABSTRACT

A bipartite graph G is a graph whose vertex set V can be partitioned into two subsets V_1 and V_2 such that every edge of G joins a vertex in V_1 and a vertex in V_2. A perfect matching (or 1-factor) of a graph with $2n$ vertices is a spanning subgraph of G in which every vertex has degree 1. The enumeration or actual construction of perfect matching of a bipartite graph has many applications, for example, in maximal flow problems and in assignment and scheduling problems [1-2].

A circulant matrix is a special kind of Toeplitz matrix where each row vector is rotated one element to the right relative to the preceding row vector. Circulant matrices have a wide range of applications; for examples, in graph theory, signal processing, coding theory and image processing, etc. Numerical solutions of certain types of elliptic and parabolic partial differential equations with periodic boundary conditions often involve linear systems associated with circulant matrices [3].

The famous integer sequences (e.g. Fibonacci, Pell and Jacobsthal) provide invaluable opportunities for exploration, and contribute handsomely to the beauty of mathematics, especially number theory [4].

In this study, we consider an $n \times n$ $(0,1)$-circulant matrix. Then we show that the numbers of perfect matchings of bipartite graphs associated this matrix generate the Fibonacci numbers.

Key Words: Bipartite graph, permanent, circulant matrix, Fibonacci number.

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Bivariate Jacobsthal and Bivariate Jacobsthal-Lucas Matrix Polynomial Sequences

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ABSTRACT

The most common types of integer sequences are Fibonacci, Lucas, Jacobsthal, Pell etc. There have been many studies in literature about these special number sequences because of their numerous applications in almost every field of science and arts. For instance, the ratio of two consecutive elements of Fibonacci sequence is always golden ratio, is very important number almost every area of science and art. It is well known that computers use conditional directives to change the flow of execution of a program. In addition to branch instructions, some microcontrollers use skip instructions which conditionally bypass the next instruction.This brings out being useful for one case out of the four possibilities on 2 bits, 3 cases on 3 bits, 5 cases on 4 bits, 11 cases on 5 bits, 21 cases on 6 bits,..., which are exactly the Jacobsthal numbers [3]. The Jacobsthal and Jacobsthal Lucas sequences are defined recurrently by

$$j_n = j_{n-1} + 2j_{n-2}, \qquad (j_0 = 0,\ j_1 = 1)$$

$$c_n = c_{n-1} + 2c_{n-2}, \qquad (c_0 = 2,\ c_1 = 1)$$

where $n \geq 1$ any integer.

In this study we consider sequences named bivariate Jacobsthal, bivariate Jacobsthal Lucas polynomial sequences. After that, by using these sequences, we define bivariate Jacobsthal and bivariate Jacobsthal Lucas matrix polynomial sequences. . And, Binet formulas for nth bivariate Jacobsthal polynomial and nth bivariate Jacobsthal Lucas polynomial are given. And also we give the generating function of these sequences. We also establish some properties of bivariate

Jacobsthal and bivariate Jacobsthal Lucas polynomial sequences by using matrix sequences. Finally we investigate some properties of these sequences, present some important relationship between bivariate Jacobsthal matrix polynomial sequence and bivariate Jacobsthal Lucas matrix polynomial sequences. We get some sum formulas bivariate Jacobsthal and bivariate Jacobsthal Lucas matrix polynomial sequences. We use elementary matrix algebra and Binet formulas for getting these formulas.

Key Words:

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Convolved Lucas matrices

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ABSTRACT

The generalized Lucas polynomial or (p,q)-Fibonacci polynomial is a generalized version of several polynomial sequences defined by recurrence relations of order two. Some examples of (p,q)-Fibonacci polynomials are the Fibonacci numbers and polynomials, Lucas numbers and polynomials, Fermat polynomials, Chebyshev polynomials, Jacobsthal polynomials, Pell polynomials, etc. This sequence was first studied by Lucas in [5]. Then, Hoggatt and Long [3] studied on this sequence. Some convolved Fibonacci and Lucas numbers have been studied in several papers, one of them is convoluted generalized Fibonacci and Lucas numbers studied in [4].

In [2] Şahin and Ramirez defined the convolved generalized Lucas polynomials as an extension of the classical convolved Fibonacci numbers. Then they gave some determinantal and permanental representations of this new integer sequence. Şahin [1] studied on q-analogue of Fibonacci and Lucas matrices and using a method for obtaining inverse matrices of these matrices. This method based on determinants of some Hessenberg matrices which obtained from a part of q-analogue of Fibonacci and Lucas matrices.

In this study, we defined convolved Lucas matrices using convolved generalized Lucas polynomials. We study some algebraic properties of these matrices. We also obtain the inverse of Convolved Lucas matrices by using the method in [1].

Key Words: Convolved Lucas matrices, convolved Fibonacci numbers, (p,q)-Fibonacci polynomial.

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Derivatives in Fuzzy Logic and Some Applications

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ABSTRACT

Being a mathematical discipline, fuzzy logic is based on fuzzy set theory. Fuzzy logic is close to human reasoning; it works according to some mediate values. It likewise provides more reliable results. In 1965, fuzzy set theory was introduced by Zadeh as an extension of crisp sets. Existing crisp set theory utilizes the characteristic functions, as defined by two valued logic, it can only take values of zero and one. In fuzzy set theory, membership function (a function $\mu: X \to [0,1]$) can take any values in closed unit interval. Furthermore, the study of fuzzy differential equations (FDE) establishes a suitable setting for mathematical modelling of a real world event in which there is uncertainty and vagueness. The first approach to the concept of fuzzy derivative for FDE is given by Chang and Zadeh. It is followed up by Dubois and Prade, the mathematicians who employed the extention principle in their approach. Other methods have been discussed by Puri and Ralescu and by Goetschel and Voxman.

In this study, we aim to define fuzzy derivatives and related theorems. Subsequently, we will apply and illustrate some forms of the FDE's. Differential equations have an important role in the embodiment of numerous biological events and analysis of their circumstances. Consequently, we will be examining fuzzy initial value SIR epidemic model that models the way an epidemic which emerges in a population spreads.

Key Words: Fuzzy Number, Fuzzy Derivative, Fuzzy Differential Equations

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Dice Vector Similarity Measure Based on Multi-Criteria Decision Making with Trapezoidal Fuzzy Multi-Numbers

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ABSTRACT

In 1965, Zadeh [1] proposed fuzzy sets to handle imperfect, vague, uncertain and imprecise information as a fuzzy subset of the classical universe set A. Soon after the definition of fuzzy set, the set has been successfully applied in engineering, game theory, multi-agent systems, and control systems, decision-making and so on. In the fuzzy sets, an element in a universe has a membership value in [0, 1]; however, the membership value is inadequate for providing complete information in some problems as there are situations where each element has different membership values. For this reason, a different generalization of fuzzy sets, namely multi-fuzzy sets, has been introduced. An element of a multi-fuzzy set may possess more-than-one membership value in [0, 1] (or there may be repeated occurrences of an element). Ulucay et al. [5] first proposed fuzzy multi-number as a generalization of fuzzy numbers. The main aim of this study is to present a novel method based on multi-criteria decision making for trapezoidal fuzzy multi-numbers. Therefore, Dice vector similarity and weighted Dice vector similarity measure is defined to develop the trapezoidal fuzzy multi-numbers decision making method. In addition, the method is applied to a numerical example in order to confirm the practicality and accuracy of the proposed method.

Key Words: Fuzzy set, Interval fuzzy multi-sets, Trapezoidal fuzzy multi-numbers

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Evaluating the Number of k-normal Elements over Finite Fields

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ABSTRACT

In this study we obtain an enumeration of k-normal elements over Finite Fields. In Theorem 1 their number is given implicitly. We will give explicit number and furthermore in some cases we will give their number as a sum whose values will depend on certain linear diophantine equations. Our results depend on the factorization of cyclotomic polynomials. k-normal elements were introduced in [Huz2013]. They are generalization of normal elements.

Let $\mathbb{F}_q$ and $\mathbb{F}_{q^n}$ be finite fields of order q and q^n respectively. Let $\alpha \in \mathbb{F}_{q^n}$ and define a set $B = \{\alpha, \alpha^q, \alpha^{q^2}, \dots, \alpha^{q^{n-1}}\}$. We say that B is a normal basis if B forms a basis of $\mathbb{F}_{q^n}$ over $\mathbb{F}_q$. Theory of normal bases is an interesting mathematical subject. Beside this normal bases play an important role in coding and cryptography. Because they are building blocks in an arithmetic of a finite field, especially multiplication and exponentiation.

It is known that [2] an element $\alpha \in \mathbb{F}_{q^n}$ is normal over $\mathbb{F}_q$ if and only if $\gcd(x^n - 1, g_\alpha(x)) = 1$, where $g_\alpha(x) = \alpha x^{n-1} + \alpha^q x^{n-2} + \alpha^{q^2} x^{n-3} + \dots + \alpha^{q^{n-1}}$. Generalizing this fact, and hence generalizing the notion of normal elements, k-normal elements were introduced in [1].

Definition 1: Let $\alpha \in \mathbb{F}_{q^n}$ and let $g_\alpha(x) = \alpha x^{n-1} + \alpha^q x^{n-2} + \alpha^{q^2} x^{n-3} + \dots + \alpha^{q^{n-1}}$. Assume that the degree of $\gcd(x^n - 1, g_\alpha(x))$ is k. Then we say that α is k-normal element over $\mathbb{F}_q$.

It is known that the number of k-normal elements is given implicitly.

Theorem 1: [1] The number of k-normal elements of $\mathbb{F}_{q^n}$ over $\mathbb{F}_q$ is given by

$$\sum_{\substack{h|x^n-1\\ \deg h=n-k}} \Phi_q(h)$$

where h is monic and $\Phi_q(h)$ is Euler Phi function for polynomials over $\mathbb{F}_q$, that gives the number of nonzero polynomials of degree less than $\deg h$ and relatively prime to h.

This theorem tells us that if we know the factorization of x^n-1 then we can evaluate explicitly the number of k-normal elements. Recall the well-known fact that the polynomial x^n-1 is factorized into cyclotomic polynomials.

Theorem 2: [2] Let $Q_d(x)$ be the cyclotomic polynomial of degree d. Then we have

$$x^n-1=\prod_{d|n} Q_d(x).$$

Now, one needs the factorization of cyclotomic polynomials in order to get the factorization of x^n-1. One of the earliest results on factorization of cyclotomic polynomials is given in [2].

Theorem 3: [2] Let $q\equiv 1(mod\,4)$ and let $q=2^A u+1$ where u is an odd integer. Also let m be a positive integer. Then the cyclotomic polynomial $Q_{2^m}(x)$ factorizes over $\mathbb{F}_q$ as follows

i) If $m\leq A$ then

$$Q_{2^m}(x)=\prod_{\substack{\rho-primitive\\ 2^m th\ roots}}(x-\rho)$$

ii) If $m>A$ then

$$Q_{2^m}(x)=\prod_{\substack{\rho-primitive\\ 2^A th\ roots}}\left(x^{2^{m-A}}-\rho\right)$$

Key Words: k-normal element, normal element, normal base, cyclotomic polynomial.

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Extended Multivariable Fourth Type Horn Functions

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ABSTRACT

The hypergeometric functions play an important role in mathematics as all special functions stated in terms of these functions. Last two decades, most of the researchers studying on special functions theory have growing interest in extensions of classical gamma, beta and hypergeometric functions, which are including new extra parameter (see [1] and [2]). In this paper, we motivate from extensions of hypergeometric and multivariable hypergeometric functions. Some of them are extensions of the Gauss hypergeometric function, the confluent hypergeometric function, the first kind and the second kind Appell functions, the first kind and the fourth kind Lauricella functions that are containing two parameters (see [2]). Firstly, we define extensions of fourth kind Horn functions and multivariable fourth kind Horn functions by using similar method in [2]. Then, we obtain some generating functions for the extended multivariable fourth kind Horn functions by using series rearrangement technique. Furthermore, we get a class of bilateral generating functions for the extended multivariable fourth kind Horn functions and extended first kind Lauricella functions. At the end of this study, we have some theorems which give various families of multilinear and multilateral generating functions for the extended multivariable fourth kind Horn functions. Then we also discuss some applications of our results regarding these classes of generating functions.

Key Words: Hypergeometric function, generating function, Horn functions, Multivariable Horn functions, Appell functions, Lauricella functions.

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$\mathfrak{F}$-Perfect Groups With Some FC-Subgroups

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ABSTRACT

We deal with the groups having FC-subgroups and define the class Θ of groups. We provide two points of view and consider such a group as a subgroup of $M(\mathbb{Q}, GF(p))$ for some prime p, the McLain groups, or represent as a finitary permutation group on an infinite set.

Having certain FC-subgroups in groups may provide some opportunities to figure out the structure of the groups (see [1,Theorem 1.1(b)] for example). In this vork we give certain useful results. The following is a slightly generalized form of [4,Theorem 2.4] (see 4 for the definition of the notion “locally degree preserving").

In this study, we obtained the following results:

Let G be a perfect locally finite p-group for some prime p. If there exits $a \in G \backslash Z(G)$ such that $< a^G, g >$ is an FC-group for every $g \in G$ then there exists a locally degree-preserving embedding of an epimorphic image of G into $M(\mathbb{Q}, GF(p))$ for some prime p.

The above result provides a restriction our attention to McLain goups for some perfect groups (of course if such groups exist)

Define the class Θ of groups as follows:

The normal closure of every two generated subgroup of the group is an FC-group.

Let G be an infinitely generated locally finite $\mathfrak{F}$-perfect-p-group for some prime p with trivial center. If G is a Θ-group then G has an epimorphic image which can be represented as a finitary permutation group.

Key Words: FC-subgroups, $\mathfrak{F}$-perfect-p-group, McLain group, finitary permutation group, perfect group, Θ-group.

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Fuzzy Logic Approach to Friend Recommendation Systems for Social Networks

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ABSTRACT

Most of the real life problems contain uncertainties. Depending on their kind, the uncertainties can be modelled in different ways. An uncertainty of random nature can be modelled by means of probability theory and stochastic analysis. However, in most problems, the uncertainty is not random. Such uncertainties can be modelled by fuzzy sets, or intervals. If the uncertain variable, which is not of random nature, can take each value within a range with the same possibility, then it can be modelled by an interval. But, if the variable can take different values from the interval with different possibilities, then it will be proper to model the uncertainty by a fuzzy number.

In this study, we focus on the problem of recommending a friend in social media. Social media tools suggest the users to contact with new people under the title of "People you might know". The algorithm, that is currently used, is straightforward. It often recommends candidates, selected among friends of user's friends. Such recommendation sometimes is relevant. However, often the user is not familiar with the recommended person, or has no common interests with him. In the existing algorithms a graph model is used, where users are represented by vertexes, and relationships between them are represented by edges. This means that graphs not including uncertainty are used. However, usually the relationship between two people is uncertain, and can be classified as "Very Strong", "Strong", "Medium", "Weak" and "Very Weak". Since these concepts are subjective, we model the problem by means of fuzzy logic. The degree of the relationship between two people is determined by studying in social media the contents which they share, and the things they like or do not like. In such a model, the user will be offered new people

from among friends having “Very Strong”, “Strong” or “Medium” relationship with the user’s friends, who in turn have “Very Strong”, “Strong” or “Medium” relationship with the user.

Key Words: Recommender system, Social media, Fuzzy logic.

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Generalized Hukuhara Differentiability in Intuitionistic Fuzzy Environment

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ABSTRACT

The fuzzy set theory was firstly introduced by L. Zadeh in 1965 [1]. Since for many reasons, it is known that vagueness or uncertainty is inevitable in most practices he reminded us that things are not always black or white. To get more realistic mathematical models for such situations he introduced a novel set concept having elements with a function $\mu: X \to [0,1]$ (called membership function). Later fuzzy set theory has been applied to the various fields, and been shown its powerful practicality to handle uncertainty in data [2, 3]. However the shape of the membership function may have uncertainty in some applications owing to the subjectivity of the expert knowledge and also the imprecision of the models. That's why, it was seen that fuzzy set theory does not give any information for this hesitation degree. To avoid this disadvantage of the fuzzy set theory, some extensions for it were introduced. One of these extensions is Atanassov's intuitionistic fuzzy set theory [4]. In 1986, Atanassov [4] introduced the concept of an intuitionistic fuzzy set. This new set concept is characterized by membership and non-membership function such that the sum of both functions is not always equal to 1.

In fuzzy set theory and Interval Analysis inverse element of a set is a problematic issue as the Minkowski difference does not always give the identity element. In [5] Hukuhara difference (H-difference) was introduced as an attempt to partially handle this problem. Later based on H-difference, The Hukuhara derivative (H-derivative) of a fuzzy number valued functions was introduced in [3] and it was studied in several papers. In [6] the H-derivative of a fuzzy function and fuzzy initial value problem was studied. However in applications it was seen that H-difference

and H-derivative concept have very restrictive limitations. Hence Stefanini et al. [7] proposed and studied some generalizations for Hukuhara difference and derivative concept for compact and convex sets and for fuzzy numbers.

In this study we introduce and study the generalized Hukuhara difference and differentiability concept for Atanassov's intuitionistic fuzzy sets. We introduce the notion of generalized Hukuhara difference and give related properties in intuitionistic fuzzy environment. Hence this enables us to introduce the concepts of differentiability and solve differential equations by using generalized Hukuhara concept.

Key Words: Intuitionistic fuzzy number, Generalized Hukuhara difference, Generalized Hukuhara derivative.

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Gröbner-Shirshov Basis and Normal Forms for the Affine Weyl GrouP $\tilde{C}_n$

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ABSTRACT

If a group G is defined by generators S and relations R, then each element $g \in G$ can be written as a product of generators

$g = s_1 s_2 \cdots s_k$

where $s_i \in S$. If k is minimal among all such expressions for g, then k is called the length of g, written $l(g) = k$ and the product $s_1 s_2 \cdots s_k$ is called a reduced decomposition for g. In general, every group element corresponds to a very high number of reduced decompositions. Among this decompositions, we have to choose a specic one for each element of the group, called its normal form.

Knowing Gröbner-Shirshov basis of a group defined by generators and relations enable us to find normal forms via the famous Composition-Diamond lemma. Because of these and some other reasons, they are several paper in literature dealing with computing Gröbner-Shirshov bases of groups. Gröbner-Shirshov bases and normal forms of the elements were already found for the Coxeter groups of type A_n, B_n and D_n in [1]. The Gröbner-Shirshov bases of the other finite Coxeter groups are given in [4] and [5]. The authors study Gröbner-Shirshov basis and normal forms of the affine Weyl group $\tilde{A}_n$ in [6]. This paper is another example of finding Gröbner-Shirshov basis for an affine Weyl group.

The most important difference of this work from others, we do not only find Gröbner-Shirshov basis and normal forms for the affine Weyl group $\tilde{C}_n$ but also give a method to convert a product of generators into its normal form. This allow us to describe multiplication in terms of normal forms, since the product of normal forms will not, in general, be the normal form.

Key Words: Weyl Groups, Gröbner-Shirshov Bases, Normal Forms, Permutation Groups

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Hessenberg Determinants via Generating Function Method

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ABSTRACT

Hessenberg matrices are one of the most important matrices in many area of mathematics. The authors of [1-7] studied algebraic properties of some classes of Hessenberg matrices such as inverses, determinants, permanents etc. For example, Cahill et al. [1] gave a recurrence relation for the determinant of the a general Hessenberg matrices*.* However this relation is not convenient to compute determinant directly. Moreover, authors of [2,4-6] gave the determinantal and permenantal representations of some recursive sequences by using Hessenberg matrices. Getu [3] computed determinants of a class of Hessenberg matrices by using *generating function* method. We refer to [8] for more detail about generating functions. Recently, by using generating function method, Mircea [7] showed that determinant of an *n* by *n* Toeplitz-Hessenberg matrix is expressed as a sum over the integer partitions of *n*.

In this work, we use generating function method to determine the relationships between determinants of three classes of Hessenberg matrices whose entries are of terms of certain sequences and generating functions of these sequences. So determinants of these Hessenberg matrices could be easily found by these relations. Some of our results will generalize the results of [3]. Finally, we give an elegant method to compute determinants of Hessenberg matrices whose entries consist of terms of recurrence sequences: our approach is to find an adjacency-factor matrix.

Key Words: Hessenberg matrix, determinant, generating function, recursive sequence.

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Impulsive Diffusion Equation on Time Scales

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ABSTRACT

Time scale theory was first considered by Stefan Hilger [1] in 1988 in his doctoral dissertation under supervision of Bernard Aulbach to unify the two approaches of dynamic modelling: difference and differential equations. However, similar ideas have been used before and go back at least to the introduction of the Rieamann-Stieltjes integral which unifies sums and integrals. More specifically, T is an arbitrary, non-empty, closed subset of $\mathbb{R}$. Many results related to differential equations carry over quite easily to corresponding results for difference equations, while other results seem to be totally different in nature. Because of these reasons, the theory of dynamic equations is an active area of research. The time scale calculus can be applied to any fields in which dynamic processes are described by discrete or continuous time models. In recent years, several authors have obtained many important results about different topics on time scales. Application of boundary value problems (BVP's) on an arbitrary time scale T is a fairly new and important subject in mathematics. In this study, we deal with an eigenvalue problem for impulsive diffusion equation with boundary conditions on T. We generalize some noteworthy results about spectral theory of classical diffusion equation into T. Also, some eigenfunction estimates of the impulsive diffusion eigenvalue problem are established on T.

Key Words: Time Scales, Impulsive Diffusion Equation, Spectral Theory.

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Independent Sets in Some Graphs

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ABSTRACT

The Merrifield-Simmons (or σ-index) index was introduced by Richard Merrifield and Howard Simmons in a series of articles [1-5] and this theory was outlined in the book [6].

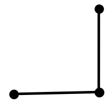
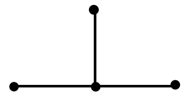
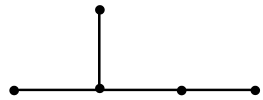
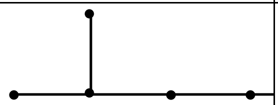
The name "Merrifield-Simmons index" for the graph invariant σ was first time used by Ivan Gutman [7]. Today, in mathematical chemistry this name is commonly accepted. For details of the theory of the Merrifield-Simmons index see the review [8].

Let $G = (V, E)$ be a simple connected graph whose vertex set V and the edge set E. A set X of the vertices of the graph G is called independent if no two distinct vertices of X are adjacent. A k-independent set of G is as set of k-mutually independent vertices. The number of k-independent sets of G is denoted by $\sigma(G, k)$. By definition $\sigma(G, 0) = 1$ for any graph G. Furthermore the Merrifield-Simmons index of a graph G, denoted by $\sigma(G)$, is defined as

$$\sigma(G) = \sum_{k=0}^{n} \sigma(G, k)$$

in other words, $\sigma(G)$ is equal to the total number of independent sets of G.

Table 1. Merrifield-Simmons indices of the first four 2-Methyl Alkanes

Numbers Of Points	Moleculer Graph	Name	σ-Index
3		propane	5
4		2-methyl propane	9
5		2-methyl butane	14
6		2-methyl pentane	23

Theorem 1. If G_n is a graph of a 2-methyl alkane, then

$$\sigma(G_n) = L_n + F_{n-1}\ (n \geq 3).$$

Proof. We calculate Merrifield-Simmons index of 2-methyl alkanes.

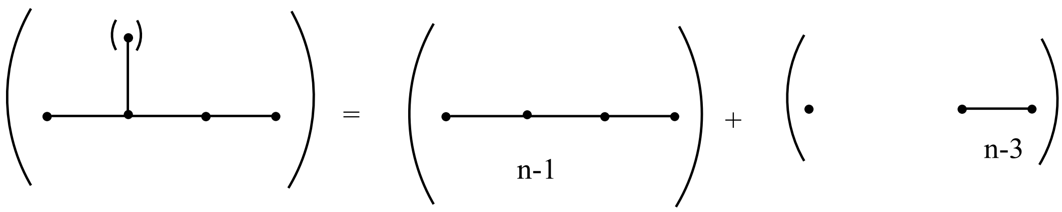


Figure 1. Molecular graph of a 2-methyl alkane which has n points needed for the calculation of the $\sigma(G_n)$.

We delete the point on the top of the graph which is showed in small brackets firstly.

$$\begin{aligned}\sigma(G_n) &= \sigma(P_{n-1}) + \sigma(P_1)\sigma(P_{n-3}) \\ &= F_{n+1} + F_3 F_{n-1} \\ &= F_{n+1} + 2F_{n-1} \\ &= F_{n+1} + F_{n-1} + F_{n-1} \\ &= L_n + F_{n-1}.\end{aligned}$$

Key Words: Independent sets, graphs, topological index.

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Kahler Differents and Torsion free Kahler Differentials

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ABSTRACT

In this study, firstly we introduce the notion of Kahler different (Jacobian ideal) of $S/R,$ which is an algebra for which the universally finite differential module $\Omega^1{}_{S/R}$ exists.

The i-th Fitting ideal of $\Omega^1{}_{S/R}$ is defined by $\partial^{(i)}(S/R) := F_i(\Omega^1{}_{S/R})$ $\quad$ $(\mathrm{i} \in N)$ [1].

$\partial^{(i)}(S/R)$ is called i-th Kahler different (Jacobian ideal) of S/R. For these ideals, the followings are true:

a) $\partial^{(0)}(S/R) \subset \partial^{(1)}(S/R) \subset \cdots \subset \partial^{(i)}(S/R) \subset \cdots$ and $\partial^{(i)}(S/R) = S$ for $\mathrm{i} \geq \mu(\Omega^1{}_{S/R})$.

b) Let I be an ideal of R and $\bar{S} := S/IS,$ $\bar{R} := R/I,$ then $\partial^{(i)}(\bar{S}/\bar{R}) = \overline{\partial^{(i)}(S/R)}$ where $\overline{\partial^{(i)}(S/R)}$ denotes the image of $\partial^{(i)}(S/R)$.

c) Let S_1/R and S_2/R be algebras. If $\Omega^1{}_{S\ /R}$ and $\Omega^1{}_{S_2/R}$ are finitely generated, then $\partial^{(i)}(S_1 \otimes_R S_2/R) = \sum_{p+q=i} \partial^{(p)}(S_1/R) \otimes_R \partial^{(q)}(S_2/R)$.

There is an important relation between rank of free module and Kahler differents. If $\Omega^1{}_{S/R}$ has rank r, then $\partial^{(i)}(S/R) = 0$ for $i < r;$ $\partial^{(i)}(S/R) = S$ for $i \geq r.$ Then we can conclude that $\Omega^1{}_{S/R}$ is projective with rank r if and only if $\partial^{(i)}(S/R) = 0$ for $i < r;$ $\partial^{(i)}(S/R) = S$ for $i \geq r.$ In literature, there are important studies about projective dimension of modules and their Fitting ideals [5], [6], [7].

Other important relation between regular ring and Kahler different is given by if S is a Noetherian integral domain and $\partial^{(n)}(S/R)$ is not an invertible ideal for some $n \in N$, then S is not a regular ring [4]. In this case, we can determine whether S is

regular or not examining in Fitting ideals of $\Omega^1_{S/R}$. Hence, we obtain an alternative solution to Jacobian Criteria in some cases.

In last time, we mention about whether Kahler module is torsion free or not, what we can say that its projective dimension [7]. For these cases, we will give some examples.

Key Words: Kahler Different, Kahler Modules, Fitting Ideals.

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k-Autocorrelation and Its Applications

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ABSTRACT

The standart autocorrelation is used to measure the similarities between a binary sequence and its any shifted form. It has applications in communication systems and cryptography. Let $a=(a_0,a_1,a_2,\dots,a_{n-1})$ be a binary sequence and its shifted forms be $a_\tau=(a_{-\tau},a_{1-\tau},a_{2-\tau},\dots,a_{n-1-\tau})$ for $\tau=1,2,\dots,n-1$,where indices of this sequence are in modulo n. The standart autocorrelation of the sequences a and a_τ is defined by

$$c_\tau(a):=\sum_{i=0}^{n-1}(-1)^{a_i+a_{i-\tau}} .$$

$\{c_\tau(a)\}_{\tau=0}^{n-1}$ sequence is called autocorrelation coefficients. In this study, we define k-autocorrelation of for a binary sequence and its $k-1$ shifted forms. The k-autocorrelation is a generalization of standart autocorrelation. If we take $k=2$, then we get the standart autocorrelation. Also, for given $\tau_1,\tau_2,\dots,\tau_{k-1}\in\mathbf{Z}$ such that $1\le\tau_1<\tau_2<\cdots<\tau_{k-1}\le n-1$, we call

$$s:=a_{\tau_1}+a_{\tau_2}+\cdots+a_{\tau_{k-1}}$$

total shift sequence for any binary sequence a. The k-autocorrelation measures the similarity between the sequence a and the total shift sequence s.

We give two application of the k-autocorrelation. In the first application, we would like to motivate our definition by providing an example related to design theory. In this specific example, we will explain the relation between k-autocorrelation coefficients of a binary sequence and corresponding lines in Fano plane. The second application is the circulant additive codes over F_4 in coding theory. We use the k-autocorrelation to determine the minimum distance of circulant additive codes over F_4.

Key Words: autocorrelation, Fano plane, circulant additive codes.

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k-Fibonacci and k-Lucas Sequences Taken to Modulo m

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ABSTRACT

Falcon and Plaza [3] studied for any positive real number k, the k-Fibonacci sequence say $\{F_{k,n}\}_{n\in\mathbb{N}}$ is defined recurrently by

$F_{k,n+1} = kF_{k,n} + F_{k,n-1}$ for n≥ 1 with initial conditions $F_{k,0} = 0$ and $F_{k,1} = 1$.

k-Fibonacci sequence was defined by studying the recursive application of the two geometrical transformations used in the well known for-triangle langest-edge(4TLE) partion in [4].

Besides, the sequence $\{L_{k,n}\}_{n\in\mathbb{N}}$ as in the form

$L_{k,0} = 2,\ L_{k,1} = k,$

$L_{k,n+1} = kL_{k,n} + L_{k,n-1},$

for k,n≥ 1 is called the k −Lucas sequence [7].

The period of the Fibonacci sequence mod m was first studied by Wall [1]. The recurrence part in the sequence creates a new sequence and gives the length of the periods of these sequences. The periods are calculated according to the values of modulo m. Campbell and Rogers [5] studied the periodicity of the Fibonacci and Lucas sequences according to the prime mode using the roots of the characteristic equation. Falcon and Plaza [6] studied the period length of the k-Fibonacci sequence modulo m. The period of the the k-Fibonacci sequence modulo m is called the Pisano period and the period length is shown as $\pi_k(m)$. $\{F_{k,n} \bmod m\}_{n\in\mathbb{N}}$ is defined as the sequence residues of k-Fibonacci sequence modulo m.

In this study, periods of k-Fibonacci and k-Lucas sequences taken as modulo prime and prime power were obtained using the roots of the characteristic equation.

Key Words: k-Fibonacci numbers, k-Lucas numbers , Period.

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Laplacian Eigenvalues of Vertex-Weighted Graphs

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ABSTRACT

A graph G as a pair (V, E), where V denotes its set of vertices(or nodes) and E denotes the set of edges. Each edge in E is an unordered pair of vertices, with the edge connecting distinct vertices a and b, written as $e = (a, b)$. A weighted graph is a triple $G = (V, E, w)$, where (V, E) is the associated graph and w is a function from E to real numbers, assigns a weight to each edge $w(e)$,which is called the weight of edge e. In many applications, the edge weights are usually represented by nonnegative integers or square matrices. The Laplacian $L(G) = (L_{ij})$ of a graph G of order n is the $n \times n$ matrix L, where $L_{ij} = d_i$ (degree of the vertex i) if $i = j$; $L_{ij} = -1$ if $(i, j) \in E$ *and* $L_{ij} = 0$ otherwise.

In this paper, finite simple graphs namely, graphs which have no loops or parallel edges are considered. The weights are taken from edges and assigned to the vertices, so the new composed graph is called vertex-weighted graph. Moreover, the Laplacian matrix of the new defined graph is arranged and the vertex weights are considered as positive real numbers or positive definite matrices of the same order. Further, some upper bounds are obtained for the eigenvalues of the Laplacian matrix of the vertex-weighted graphs.

Key Words: Vertex-weighted graph, weighted graph, upper bound.

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Lexicographic and Cartesian Product of Zero-Divisor Graphs on Finite Free Semilattices

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ABSTRACT

In recent time we have researched zero-divisor graphs on finite free semilattices, in that study the diameter, radius, girth, degree of any vertex, domination number, independence number, clique number, chromatic number and chromatic index of zero-divisor graph on a finite free semilattice have been established and we have determined when this graph is perfect graph and when its core is Hamiltonian graph [5]. In this presentation our aim is extend of this study to over lexicographic product of those graphs and cartesian product of those graphs.

Let S_1 be finite free semilattice on a non-empty finite set X_1 and let S_2 be finite free semilattice on a non-empty finite set X_2. Let Γ_1 be zero-divisor graph on S_1, let Γ_2 be a zero-divisor graph on S_2. We use notation $\Gamma_1[\Gamma_2]$ for lexicographic product of Γ_1 and Γ_2, we use notation $\Gamma_1 \square \Gamma_2$ for cartesian product of Γ_1 and Γ_2. For lexicographic product of those graphs, we have researched girth, maximum degree, minimum degree and degree of any vertices in graph, diameter, radius, domination number, chromatic number and we have resulted when this graph is perfect graph. For cartesian product of those graphs, we have researched girth, degree of any vertices, radius, clique number, we have determined when this graph is perfect graph and we have resulted diameter, chromatic number for this graph.

Key Words: Zero-divisor graphs, finite free semilattices, product of graphs.

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Lie Ideals of Semiprime Rings with Generalized Derivations

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ABSTRACT

Let R be a $2-$torsion free semiprime ring. For any $x, y \in R,$ the symbol $[x, y]$ stands for the commutator $xy - yx$ and the symbol $x \circ y$ denotes the anti-commutator $xy + yx.$ An additive subgroup U of R is said to be a Lie ideal of R if $[u, r] \in U,$ for all $u \in U,$ $r \in R.$ U is called a square closed Lie ideal of R if U is a Lie ideal and $u^2 \in U$ for all $u \in U$. Let S be a nonempty subset of R. A mapping F from R to R is called centralizing on S if $[f(x), x] \in Z$, for all $x \in S$ and is called commuting on S if $[f(x), x] = 0,$ for all $x \in S.$ A mapping $f: R \to R$ is called skew-centralizing on R if $f(x)x + xf(x) \in Z(R)$ holds for all $x \in R$; in particular, if $f(x)x + xf(x) = 0$ holds for all $x \in R,$ then it is called skew-commuting on R. An additive mapping $d: R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R.$ An additive map $F: R \to R$ is called a generalized derivation if there exists a derivation $d: R \to R$ such that $F(xy) = F(x)y + xd(y),$ for all $x, y \in R.$

In the present study, we shall prove that h is commuting map on U if any one of the following holds: i) $F(u)u = \pm uG(u),$ ii) $[F(u), v] = \pm[u, G(v)],$ iii) $F(u) \circ v = \pm u \circ G(v),$ iv) $[F(u), v] = \pm u \circ G(v),$ v)$F([u, v]) = [F(u), v] + [d(v), u]$ for all $u, v \in U,$ where $G: R \to R$ is a generalized derivation associated with the derivation $h: R \to R.$

Key Words: Semiprime ring, Lie ideal, generalized derivation.

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Matrix Representation of Bi-Periodic Jacobsthal Sequence

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ABSTRACT

The most common types of integer sequences are Fibonacci, Lucas, Jacobsthal, Pell etc. There have been many studies in literature about these special number sequences because of their numerous applications in almost every field of science and arts. For instance, the ratio of two consecutive elements of Fibonacci sequence is always golden ratio, is very important number almost every area of science and art. It is well known that computers use conditional directives to change the flow of execution of a program. In addition to branch instructions, some microcontrollers use skip instructions which conditionally bypass the next instruction.This brings out being useful for one case out of the four possibilities on 2 bits, 3 cases on 3 bits, 5 cases on 4 bits, 11 cases on 5 bits, 21 cases on 6 bits,..., which are exactly the Jacobsthal numbers. Jacobsthal and Jacobsthal Lucas numbers are given by the recurrence relations Jn=Jn-1+2Jn-2, $J_0=0$, $J_1=1$ and Cn=Cn-1+2Cn-2, $C_0=2$, $C_1=1$ for n≥2, respectively. In this paper, we bring into light the matrix representation of bi-periodic Jacobsthal sequence, which we shall call the bi-periodic Jacobsthal Matrix sequence depending on two parameters a and b. In the literatüre bi-periodic Fibonacci sequence was fırst introduced into literature in 2009 by Edson and Yayenie [1] after which the bi-periodic Lucas sequence was defined in a similar fashion in 2004 by Bilgici [2]. We introduced a new generalization of the Jacobsthal numbers which we shall call bi-periodic Jacobsthal sequences similar to the bi-periodic Fibonacci and Lucas sequences. We use a for even terms ans b for odd terms. We find a 2x2 matrix for defining matrix representation of bi-periodic Jacobsthal sequence. We obtain the nth general term of this new matrix sequence by using elements of bi-periodic Jacobsthal sequence . By studying the properties of this new matrix sequence, the well-known Cassini or Simpson's formula was obtained. We gave a general formula for determanants of all elements of bi-periodic Jacobsthal matrix sequence. We then proceed to find its generating function as well as the Binet formula. Some new properties and two summation formulas for this new generalized matrix sequence were also given.

Key Words: Jacobsthal numbers, matrix sequences, Binet formula, generating function.

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Neuro-Fuzzy Approach to Adapt Vehicle Calibration Maps

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ABSTRACT

The world regulations on spark and compression ignition engine emissions are getting pretty tight and this results additional costs on design and manufacturing the vehicles [1]. The main pollutants for compression ignition (diesel) engines are Nitrogen oxides (NO_X), Hydrocarbons (HC), Carbon dioxide (CO_2), and Carbon monoxide (CO) emissions. Calibration maps are mostly used in automotive applications in order calculate true emission rates, soot mass, and soot mass flow under different circumstances, such as cold and hot engine, and various weather conditions. This paper presents a neuro-fuzzy model fo train and obtain emission maps for Engine Control Unit. In the vehicle applications, look-up maps and curves are used for operating point variations. Kalman and Extended Kalman filters are also used for table adaptation of these maps [1]. In this study, we first did 6 driving tests in different weather conditions and engine temperature, then we used our adaptation algorithm to better calculate soot mass flow with the optimized emission maps.

Fuzzy Adaptation model has a nonlinear structure and based on membership functions. It needs to an expert opinion, so all the membership functions and rules are examined by the expert opinion. Additionally, neural network is also used to train membership functions. Then, fuzzy inference model is provided in the critical choice. The prominent fuzzy inference systems are Sugeno, Mamdani, Tsukamoto, Şen [2]. Mamdani is very similar to human behaviour and so it can model the nonlinear systems like human behaviour better than Sugeno [3,4]. However, Sugeno method gives exact results and needs less mathematical load. As a result, we first used neural network approach to train membership functions to calculate the calibration maps. Next, Sugeno fuzzy inference system is used for calculating soot mass flow for different driving conditions. Finally, we are able to calculate the rates of soot mass flow with maximum %13 error in comparison with the sensor measurements.

Key Words: Emission Maps, Neuro-Fuzzy Adaptation System, Sugeno.Inference System

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Neutrosophic Modules

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ABSTRACT

The objective of this presentation is to study Neutrosophic Modules. The concept of neutrosophic algebraic structures was introduced by F. Smarandache and W.B. Vasantha Kandasamy in 2006. Neutrosophic logic is an extension of the fuzzy logic in which indeterminancy is included. In the neutrosophic logic, each proposition is characterized by the degree of truth in the set (T), the degree of falsehood in the set (F) and the degree of indeterminancy in the set (I) where T,F,I are subsets of] ,0,1,+ [. Neutrosophic logic has wide applications in science, engineering, IT, law, politics, economics, finance, etc.

Neutrosophic set is defined by the following way:

Let U be a universe of discourse and let M be a subset of U. M is called a neutrosophic set if an element x= x(T, I, F) ∈ U belongs to M in the following way:

(1) x is t% true in M,

(2) x is i% indeterminate in M, and

(3) x is f% false in M,

where t∈ T , i∈ I and f ∈F.

It is possible to have t+i+f=1 as in the case of classical and fuzzy logics and probability. Also, it is possible to have t+i+f<1 as in the case of intuitionistic logic and as in the case of paraconsistent logic, it is possible to have t+i+f>1.

In this talk some basic definitions and properties of the Modules are generalized. It is shown that every Neutrosophic modules over a Neutrosophic Ring is a module.

Key Words: Left Neutrosophic R-Module M, Right Neutrosophic R-Module M , Neutrosophic Ring , Neutrosophic Subring .

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Neutrosophic Rings

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ABSTRACT

This talking is about the Neutrosophic Rings. In this talk we give some basic definitions and properties of the Neutrosophic Rings. Just as complex rings or complex fields include in them the notion of imaginary element i with I^2 = –1 or the complex number i, in the neutrosophic rings we include the indeterminate element I where I^2 = I. In most of the real world problems or in situations we see in general we cannot always predict the occurrence or nonoccurrence of an event, the occurrence or the nonoccurrence may be indeterminate so we introduce the neutrosophic rings which can very easily cater to such situations. In many cases we felt the concept of indeterminacy was more concrete than the notion of 'imaginary' so we have ventured to define neutrosophic algebraic concepts.

This presentation has two sections. In the first section we give the notion of neutrosophic rings and their substructures like ideals and subrings. Section two introduces several types of neutrosophic rings.

Throughout this section Z+ will denote the set of positive integers, Z- the set of negative integers, Z the set of positive and negative integers with zero. Z+ ∪ {0} will show we have adjoined 0 with Z+. Like wise the rationals Q, the reals R, the complex number C can have one part of it denoted by Q+, Q–, R+, R– and so on. Also Zn will denote the set of modulo integers i.e. {0, 1, 2, …, n – 1} i.e. n ≡ 0 (mod n).

Key Words: Neutrosophic set, Neutrosophic Ring , Neutrosophic Subring .

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Neutrosophic Triplet Metric Spaces and Neutrosophic Triplert Normed Spaces

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ABSTRACT

Neutrosophy is a new branch of philosophy which studies the nature, origin and scope of neutralities as well as their interaction with ideational spectra. Florentin Smarandache [1] in 1999, first introduced the concept of neutrosophic logic and neutrosophic set where each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic especially to intuitionistic fuzzy logic. In fact neutrosophic set is the generalization of classical sets [7], fuzzy set [3] and intuitionistic fuzzy set [5] etc. This mathematical tool is used to handle problems consisting uncertainty, imprecision, indeterminacy, inconsistency, incompleteness and falsity. By utilizing the idea of neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache studied neutrosophic algebraic structures in [6] by inserting an indeterminate element ‘‘I’’ in the algebraic structure and then combine ‘‘I’’ with each element of the structure with respect to corresponding binary operation *. Smarandache and Ali [4] utilized these neutrosophic triplets to introduce the innovative notion of neutrosophic triplet group which is completely different from the classical group in the structural properties. In this presentation an idea of neutrosophic triplet metric spaces and neutrosophic triplert normed spaces are given and some of their properties are studied. We also show that a non-connection between the classical field and the classical vector space is between the neutrosophic triplet field and the neutrosophic triplet vector space.

Key Words: Neutrosophic triplet sets, Neutrosophic triplet metric, Neutrosophic triplet vector space, Neutrosophic triplet normed space

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New Degree Concepts in Graph Theory

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ABSTRACT

Topological indices have important place in theoretical chemistry. Many topological indices were defined by using vertex degree concept. The Zagreb and Randić indices are the most used degree based topological indices so far in mathematical and chemical literature among the all topological indices. Very recently, Chellali, Haynes, Hedetniemi and Lewis have published a seminal study: On *ve*-degrees and *ev*-degrees in graphs [1]. The authors defined two novel degree concepts in graph theory; *ev*-degrees and *ve*-degrees and investigate some basic mathematical properties of both novel graph invariants with regard to graph regularity and irregularity [1]. After given the equality of the total *ev*-degree and total *ve*-degree for any graph, also the total *ev*-degree and the total *ve*-degree were stated as in relation to the first Zagreb index.

Definition 1.1 [1*] Let G be a connected graph and $v \in V(G)$. The ve-degree of the vertex v, $deg_{ve}(v)$, equals the number of different edges that incident to any vertex from the closed neighborhood of v. For convenience we prefer to show the ve-degree of the vertex v, by c_v.*

Definition 1.2 [1] *Let G be a connected graph and $e = uv \in E(G)$. The ev-degree of the edge e, $deg_{ev}(e)$, equals the number of vertices of the union of the closed neighborhoods of u and v. For convenience we prefer to show the ev-degree of the edge $e = uv$, by c_e or c_{uv}.*

Definition 1.3 [1] *Let G be a connected graph and $v \in V(G)$. The total ev-degree of the graph G is defined as;*

$$T_e = T_e(G) = \sum_{e \in E(G)} c_e$$

and the total ve-degree of the graph G is defined as;

$$T_v = T_v(G) = \sum_{v \in V(G)} c_v.$$

Observation 1.4 [1] *For any connected graph G,*

$$T_e(G) = T_v(G).$$

These degree concepts can applicate topological indices which are stated by degrees of vertices or edges. The valid topological indices can be modificated by these degree concepts.

Key Words: degree, graphs, topological index.

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Notes on prime near-rings with multiplicative derivation

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ABSTRACT

An additively written group $(N,+)$ equipped with a binary operation $.: N \to N, (x,y) \to xy$, such that $x(yz) = (xy)z$ and $x(y+z) = xy + xz$ for all $x,y,z \in N$ is called a left near-ring. A near-ring N is said to be 3 −prime if $xNy = \{0\}$ implies $x = 0$ or $y = 0$. An additive mapping d is called a homomorphism or an anti-homomorphism of N, if d satisfies $d(xy) = d(x)d(y)$ or $d(xy) = d(y)d(x)$ for all $x,y \in N$.

The definition of derivation on rings is given by E. C. Posner in 1957. There has been a great deal of work concerning commutativity of prime or semiprime rings and near-rings with derivations satisfying with certain differential identities. The notion of multiplicative derivation for rings was introduced by Daif in [4]. A map $d: N \to N$ is called a nonzero multiplicative derivation if $d(xy) = xd(y) + d(x)y$ holds for all $x,y \in N.$ These maps are not additive. In view of some results on rings and near-rings with derivation, it is natural to look for comparable results with multiplicative derivation of rings or near-rings.

Many authors have generalized that derivation d is contained the center of R, i.e. $d(R) \subseteq Z$ or d acts a homomorphism or an anti-homomorphism on R. In the present study, we shall extend above mentioned results for multiplicative derivations of a zero symmetric 3 −prime left near-ring N.

Key Words: Near ring, derivation, multiplicative derivation.

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On (p,q)-Analogue of Lupaş-Schurer-Stancu Operators

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ABSTRACT

Bernstein polynomial was first introduced by Bernstein [5]. Then, in 1987, Lupaş [6] pioneered the research of q-Bernstein operators. After that the applications of q-calculus have been studied by several researchers for thirty years. Very recently, Mursaleen, Ansari and Khan [3] initiated the research of (p,q)-analogue of Bernstein operators which reduce to q-Bernstein operators for p=1.

The purpose of this paper is to introduce Lupaş-Schurer-Stancu operators based on (p,q)-integers. We consider for each $p>0$, $q>0$ and for any $m\in \mathbb{N}$, $0\le\alpha\le\beta$, $x\in[0,1]$ and $f\in C[0,l+1]$, fixed $l\in \mathbb{N}^{+}\cup\{0\}$. We construct the (p,q)-analogue of Lupaş-Schurer operators by

$$L_{m,l,p,q}^{\alpha,\beta}\left(f;x\right)=\sum_{k=0}^{m+l}\frac{f\left(\dfrac{p^{m+1-k}[k]_{p,q}+\alpha}{[m]_{p,q}+\beta}\right)\begin{bmatrix} m+l \\ k \end{bmatrix}_{p,q} p^{\frac{(m+l-k)(m+l-k-1)}{2}} q^{\frac{k(k-1)}{2}} x^{k}(1-x)^{m+l-k}}{\prod_{j=1}^{m+l}\{p^{j-1}(1-x)+q^{j-1}x\}}$$

.

Note that, if we take $\alpha=\beta=0$, (p,q)-Lupaş-Schurer-Stancu operators reduce to be Lupaş-Schurer operators. Also, if we take p=q=1 and $\alpha=\beta=0$, (p,q)-Lupaş-Schurer-Stancu operators reduce to be Schurer-Bernstein operators.

Then, we deal with the approximation properties for (p,q)-Lupaş-Schurer-Stancu operators based on Korovkin type approximation theorem. We prove some auxiliary results which will be needed to establish the main results. Moreover, we estimate the rate of convergence by using modulus of continuity, with the help of functions of

Lipschitz class and Peetre’s K-functionals. Furthermore, we illustrate the convergence of the operator to some function by using MATLAB.

Key Words: (p,q)-integers, Lupaş operators, modulus of continuity, functions of Lipschitz class, Peetre's K-functionals.

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On Aes Encryption and Soft Sets

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ABSTRACT

In year 2000, the Rijndael block cipher was adopted by NIST as the Advanced Encryption Standard (AES), the new standard encryption algorithm of the US government to replace DES. The algorithm is a member of the family of squaretype algorithms [2] designed by Vincent Rijmen and John Daemen. It is currently one of the most widely used and analyzed ciphers in the world. AES is a 128-bit block cipher and accepts key sizes of 128, 192 and 256 bits. These versions of AES are called AES-128, AES-192 and AES-256 and the number of rounds for these versions are 10, 12 and 14 respectively. The AES encryption algorithm organizes the plaintext as a 4×4 table of 1-byte entries, where the bytes are treated as elements of the finite field $GF(2^8)$. There are three main operations used in AES: the s-box substitution, shift row, and mix column operations. In 1999, Molodtsov[1] proposed soft set theory as a new mathematical tool to deal with uncertainties which are free from the difficulties affecting existing methods. There are has been a rapid growth of interest in soft set theory and its applications in recent year. This theory has been applied to many fields such as information systems, decision making problems, optimization theory, algebraic structure and basic mathematics analysis, etc. which contain uncertainties. Çağman and Enginoğlu [8] defined and studied soft matrix. Atagün and Sezgin [6] studied on soft set operations with many corresponding examples as well. Sezgin, Atagün and Aygün [7] studied soft near-rings and idealistic soft near-rings. In [4], the same authors introduced two new operations on soft sets, called inverse production and characteristic production. In this study, we redefine the operations inverse and characteristic products on the set of soft matrices and we define soft encryption.

Key Words: Aes encryption,soft sets, inverse product, characteristic product

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On Arf Closures of Some Numerical Semigroups Generated by Three Elements

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ABSTRACT

A numerical semigroup is a subset of the set of nonnegative integers (denoted here by $\mathbb{N}$) closed under addition, containing the zero element and with finite complement in $\mathbb{N}$. A numerical semigroup is a set of the form $S = \langle u_1, u_2, \ldots, u_p \rangle = u_1\mathbb{N} + u_2\mathbb{N} + \ldots + u_p\mathbb{N}$ where are positive integers, such that $\gcd(u_1, u_2, \ldots, u_p) = 1$. The condition is saying that S has finite complement in $\mathbb{N}$ (where short $\gcd$ is the greatest common divisor),(Barucci, Dobbs & Fontana, 1997; .Fröberg, Gottlieb & Haggkvist, 1987; . Rosales &Garcia-Sanchez, 2009).

If S is a numerical semigroup, then $F(S) = \max(\mathbb{Z} \backslash S)$ is called Frobenius number of S. Any numerical semigroup write this form $S = < a_1, a_2, ..., a_n > = \{0 = s_0, s_1, s_2, ..., s_n = F(S) + 1, \rightarrow ...\}$. Where “$\rightarrow$” means that every integer greater than $F(S) + 1$ belongs to the set.

A numerical semigroup *S* is called Arf if $x + y - z \in S$ for all $x, y, z \in S$, where $x \geq y \geq z$. The smallest Arf numerical semigroup containing a numerical semigroup S is called the Arf closure of S, and it is denoted by $Arf(S)$.

Arf numerical semigroup and their applications to algebraic error corerecting codes have been a special interest in recent times (Brass-Amaros, 2004; Campillo, Farran & Munuera, 2000). The families of Arf numerical semigroups are related with the problem solution of singularities in curve.

In this presentation, we will give some results between numerical semigroups and theirs Arf closure. Also, we will obtain some relation for Arf closure of these numerical semigroups.

Key Words: Arf numerical semigroup, Arf closures, Genus.

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On Automorphisms of Free Nilpotent Lie Algebras

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ABSTRACT

Let $L_{m,c}$ be the free nilpotent of class $c-1$ Lie algebra of finite rank $m,\ m\geq 2$ over a field K of characteristic zero. By $L'_{m,c}$ and $C\left(L_{m,c}\right)$, we denote the derived subalgebra and the center of $L_{m,c}$ respectively. If an automorphism of $L_{m,c}$ induces the identity mapping on $L_{m,c}/L'_{m,c}$, it is called an IA-automorphism. If θ is an IA-automorphism then for every $u\in L_{m,c}$, we have $\theta(u)-u\in L'_{m,c}$. The set of all IA automorphism of $L_{m,c}$ is denoted by $IA\left(L_{m,c}\right)$. An automorphism of $L_{m,c}$ is called a central automorphism if it induces the identity mapping on the algebra $L_{m,c}/C\left(L_{m,c}\right)$, that is α is a central automorphism then for all u in $L_{m,c}$ we have $\alpha(u)-u\in C\left(L_{m,c}\right)$ and also the central automorphism of $L_{m,c}$ commutes with every automorphism in $Inn\left(L_{m,c}\right)$, the group of inner automorphism of $L_{m,c}$. We shall denote the group of all central automorphisms of $L_{m,c}$ by $Aut_C\left(L_{m,c}\right)$. For σ be an automorphism of $L_{m,c}$ and $u\in L_{m,c}$, the element $\left(\sigma_k,u\right)$ is defined inductively by $\left(\sigma_1,u\right)=\sigma(u)-u$ and, when $k>0$, $\left(\sigma_{k+1},u\right)=\sigma\left(\sigma_k,u\right)-\left(\sigma_k,u\right)$. For every $u\in L_{m,c}$, if the integer k can be chosen independently of σ, then σ is said to be unipotent. In this study, we determine sufficient conditions for inner and IA automorphisms to be central automorphisms. We prove that $Aut_C\left(L_{m,c}\right)=Inn\left(L_{m,c}\right)$ if and only if $L_{m,c}$ is nilpotent of class 2 and rank 2. And we also show that central aurtomorphisms are unipotent automorphisms.

Key Words: Central automorphism, IA-automorphism, inner automorphism, unipotent automorphism, free nilpotent Lie algebra.

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On Certain $\mathfrak{F}$-Perfect Groups

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ABSTRACT

We consider certain groups which have no proper subgroup of finite index such as minimal non-soluble p-groups and give some descriptions of them.

A group which has no proper subgroup of finite index is called $\mathfrak{F}$-perfect. Let $\mathcal{X}$ be a class of groups. If a group G is not in $\mathcal{X}$ but every proper subgroup of G is in $\mathcal{X}$ then G is called a minimal non $\mathcal{X}$-group and denoted usually by $MN\mathcal{X}$. Clearly every perfect p-group for some prime p is $\mathfrak{F}$-perfect and the structure of many $MN\mathcal{X}$-groups is investigated in perfect p-case.

In the present article we mostly take $\mathfrak{S}$, the class of all soluble groups, as the class $\mathcal{X}$, i.e. we consider certain $MN\mathfrak{S}$-groups. It not known yet that if locally finite $MN\mathfrak{S}$-p-group which is not finitely generated exist. Such groups are mainly studied in [5], [4], [2], [3] (in some general form) and given certain descriptions.

Let G be a group and H be a subgroup of G. If $|G:H|$ is infinite, $Core_G H = \cap_{g\epsilon G} H^g = 1$ and for every proper subgroup K of G, $|K:K\cap H|$ is finite, then G is called a barely transitive group and H is called a point stabilizer. Though thr definition of barely transitive groups has permutation groups origin, we use above abstract definition of these groups (see [7]).

We give certain applications of Khukhro-Makarenko Theorem to $\mathfrak{F}$-groups. Also we provide a corollary to [8,Satz 6] which is used effectively in most of the cited articles and give certain applications of it. Finally we define Weak Fitting groups and a useful class Ξ of groups and give certain results related to these notions.

Key Words: $\mathcal{X}$-group, $MN\mathfrak{S}$-groups, $\mathfrak{F}$-Perfect groups, barely transitive group.

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On Generators Of Some Transformation Semigroups

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ABSTRACT

Let T_n be a full (total) transformations semigroup on a finite set X_n and S_n be the symmetric group on a finite set X_n. $K_{n,r}$ is the semigroup of full transformation $\alpha : X_n \to X_n$ such that $|im(\alpha)| \le r$ for $1 \le r \le n-1$. For $1 \le r \le n-1$, $K_{n,r}$ is the subsemigroup of T_n, in addition $K_{n,r}$ is the ideal of T_n. Firstly, I mention on some recent contributions which are related to our new results. J. M. Howie and R. B. McFadden proved in [6] that the rank of the subsemigroup $K_{n,r}$ of T_n is the Stirling number of the second kind for $2 \le r \le n-1$. Ayık et al. developed a notation for certain primitive elements of T_n, called path-cycle, and described an algorithm to decompose an arbitrary map in T_n into a product of path-cycles in [1]. In addition, they used these techniques to obtain some informations about generators of T_n. In [2] Ayık et al. found necessary and sufficient conditions for any set for transformations of defect ($|X_n \setminus im(\alpha)|$) 1 in singular transformations semigroup on X_n to be a (minimal) generating set for singular transformations semigroup on X_n. In [3] Ayık and Bugay found necessary and sufficient conditions for any set of full transformations of height r in the subsemigroup $K_{n,r}$ of T_n to be a (minimal) generating set of $K_{n,r}$ for $2 \le r \le n-1$. In this talk I focus on $K_{n,r}$, which together with S_n. We prove some properties which are useful for our main result. Then, for each $1 \le r \le n-1$, we prove that the necessary and sufficient conditions for any subset of $K_{n,r}$ to be a generating set of $K_{n,r}$, which together with S_n.

This is a joint work with Gonca Ayık and Hayrullah Ayık.

Key Words: Full transformation semigroup, symmetric group, generating set, relative rank.

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On one type of ladder graphs and the Pell numbers

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ABSTRACT

In this study, we consider ladder-type graphs, which can be obtained as the Cartesian product of two path graphs. Since graphs are visual objects, to analyse them, computer support often requires. More precisely, to find out graph properties, matrix representation is used. The adjacency matrix of a graph whose entries are defined as (for $V = \{x_1, x_2, \dots, x_n\}$), $\mathrm{A(G)} = [a_{ij}]_1^n$, where

$$a_{ij} = \begin{cases} 1, & if\ x_i \sim x_j \\ 0, & otherwise. \end{cases}$$

Another way Another way of matrix representation of a graph is the Laplacian matrix which is denoted by *L(G)*, defined as *L(G)=D(G)-A(G)* where *D(G)* is *nxn* degree matrix and *A(G)* is the adjacency matrix. In this content, we obtain a general form of the adjacency and Laplacian matrices of this kind of graphs. As it is well-known, the energy of a graph is the sum of the absolute values of the eigenvalues of the adjacency matrix. As it is known, the degree matrix is a diagonal matrix which contains information about the degree of each vertex that is the number of edges attached to each vertex. Taking into account this fact, we acquire the energy of this graph and an upper bound for Laplacian energy. In graph theory, a perfect matching is a matching which matches all vertices of the graph. This idea brought us that the perfect matching numbers of the graph are equal to famous Pell numbers.

Key Words: ladder graph, energy, Laplacian energy, perfect matching number.

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On *r*-Circulant Matrices

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ABSTRACT

A circulant-type matrix is a special kind of Toeplitz matrix where each row vector is rotated one element to the right relative to the preceding row vector. Circulant-type matrices and their applications are a fundamental key in many areas of pure and applied science. They have a wide range of application in signal processing, image processing, digital image disposal, linear forecast, error correcting code theory.

r-circulant matrix is also a circulant-type matrix defined by $C_n := \mathrm{circ}_r(c_0, c_1, \ldots, c_{n-1})$, associated with the numbers $c_0, c_1, \ldots, c_{n-1}$, is denoted as

$$C_n = \begin{pmatrix} c_0 & c_1 & \cdots & c_{n-2} & c_{n-1} \\ rc_{n-1} & c_0 & \cdots & c_{n-3} & c_{n-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ rc_2 & rc_3 & \cdots & c_0 & c_1 \\ rc_1 & rc_2 & \cdots & rc_{n-1} & c_0 \end{pmatrix} \tag{1}$$

Recently, some scholars gave the explicit determinant and inverse of the special matrices involving famous numbers. For example, in [1], Shen and Cen obtained upper and lower bounds for the spectral norms of *r*-circulant matrices involving Fibonacci and Lucas numbers. Bozkurt and Tam [2] defined *r*-circulant matrices with general second order number sequences.

In this study, we also consider third order linear recurrence, which has the form:

$$W_n = pW_{n-1} + qW_{n-2} + tW_{n-3}$$

with initial conditions $W_0 = 0$, $W_1 = a$ and $W_2 = b$, and $n > 2$. By substituting W_i into c_i at the *r*-circulant matrix given by (1), we get some spectacular formulas for the general form of circulant matrices.

Key Words: *r*-circulant matrix, third order linear recurrence

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On Second Order Exterior Derivation of the Universal Modules

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ABSTRACT

This study is interested in special case of the exterior and symmetric derivation on universal modules. In this study, we investigated some interesting homological results about second order exterior derivation of universal modules and we gave some examples related to universal modules of second order exterior derivations.

In order to prove conclusions about algebraic sets and their coordinate rings, one of the methods is to study the universal module of di¤erential operators. This ideas of studying the universal module may decrease questions about algebras to questions of module theory. The idea of using the universal module goes as far back as [2] which was proved some propeties of $\Omega_1(R)$. The universal modules of higher differential operators of an algebra were introduced by [4]. During the recent years, subject of universal modules of high order differential operators has studied by [1]. Some notion has emerged in Heyneman and Sweedler in [3]. They mentioned differential operators on a commutative algebra which spread out the notion of derivations. Throughout this paper, unless the contrary is stated explicitly, we will let R be a commutative algebra over an algebraically closed field k with charasteristic zero and by a k-algebra, we mean commutativr ring which contains a field k of characteristic zero. All modules will be unitary. We use the following notation: The universal module of n-th order derivations of a ring R will be denoted by $\Omega_n(R)$.

Key Words: Universal Modules, Exterior derivations, Symmetric derivations.

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On the Bernstein Polynomials and Their Properties in (p,q)-Calculus

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ABSTRACT

Many researchers in different fields are keenly interested in the study of Bernstein polynomials [2] from the day which it was invented to today. These studies involve both pure and applied branches as mathematics, statistics, numerical analysis, computer aided geometric design, physics, engineering and so on. After, about a century later, the definition of Bernstein polynomials and the generating function of these polynomials was obtained by Acikgoz and Araci [1]. Identifying the generating functions is of major importance in mathematics and its fields such as number theory, combinatorics and so on. By using the generating function, the Bernstein polynomials's many properties were obtained. Recently, the Bernstein polynomials have been moved into *(p,q)*-calculus ([3] and [4]) and are studied by many researchers. Mursaleen et. al. [5] defined Bernstein polynomials depend on *(p,q)*- integers and derived their approximation properties and rate of convergance.

In this study, we will describe a new type Bernstein type polynomials. Also, we obtain the generating function, derivative and symmetric properties, recurrence relations, linear combinations and some interesting properties for these polynomials. Furthermore, we will give some results as visually. Finally, we will obtain some relations between these polynomials and special numbers and polynomials.

Key Words: (p,q)-Calculus, Bernstein polynomials, Generating functions, Stirling numbers, Bernoulli polynomials.

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On the bounds of eigenvalues of matrixs for bi-periodic Fibonacci and bi-periodic Lucas recurrence relations

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ABSTRACT

In the literature, there are many generalizations of Fibonacci and Lucas sequences. One of them given by Edson and Yayenie [1] was called the bi-periodic Fibonacci sequence and defined as a, b for nonzero real numbers and $q_0 = 0,\ q_1 = 1,$

$$q_n = \begin{cases} aq_{n-1} + q_{n-2}, if\ n\ is\ even \\ bq_{n-1} + q_{n-2}, if\ n\ is\ odd \end{cases}, \text{for } n \geq 2$$

, the other one given by Bilgici [2] was called the bi-periodic Lucas sequence and defined as a, b for nonzero real numbers and $l_0 = 2,\ l_1 = a$

$$l_n = \begin{cases} bl_{n-1} + l_{n-2}, if\ n\ is\ even \\ al_{n-1} + l_{n-2}, if\ n\ is\ odd \end{cases}, \text{for } n \geq 2.$$

In [5], the authors defined the bi-periodic Fibonacci matrix sequence and the nth general term of the matrix sequence via bi-periodic Fibonacci numbers as follows

$$\mathcal{F}_n(a,b) = \begin{cases} a\mathcal{F}_{n-1}(a,b) + \mathcal{F}_{n-2}(a,b), if\ n\ is\ even \\ b\mathcal{F}_{n-1}(a,b) + \mathcal{F}_{n-2}(a,b), if\ n\ is\ odd \end{cases}$$

with initial conditions

$\mathcal{F}_0(a,b) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathcal{F}_1(a,b) = \begin{bmatrix} b & b/a \\ 1 & 0 \end{bmatrix}$ and a, b are nonzero real numbers.

For any integer $n \geq 0$, nth general term of the matrix sequence is

$$\mathcal{F}_n(a,b) = \begin{bmatrix} (b/a)^{\varepsilon(n)}\, q_{n+1} & (b/a)q_n \\ q_n & (b/a)^{\varepsilon(n)}\, q_{n-1} \end{bmatrix}$$

, where $\varepsilon(n)$ is partial function which

$$\varepsilon(n) = \begin{cases} 1, & n \text{ is odd} \\ 0, & n \text{ is even} \end{cases} \qquad (1)$$

In [6], the author defined the bi-periodic Lucas matrix sequence

$$\mathcal{L}_n(a,b) = \begin{cases} a\mathcal{L}_{n-1}(a,b) + \mathcal{L}_{n-2}(a,b), \text{if } n \text{ is odd} \\ b\mathcal{L}_{n-1}(a,b) + \mathcal{L}_{n-2}(a,b), \text{if } n \text{ is even} \end{cases}$$

with initial conditions

$\mathcal{L}_0(a,b) = \begin{bmatrix} a & 2 \\ 2(a/b) & -a \end{bmatrix}$, $\mathcal{L}_1(a,b) = \begin{bmatrix} a^2 + 2(a/b) & a \\ a^2/b & 2(a/b) \end{bmatrix}$ and a, b are nonzero real numbers.

The nth general term of this matrix sequence as for any integer $n \geq 0$

$$\mathcal{L}_n(a,b) = \begin{bmatrix} (a/b)^{\varepsilon(n)}\, l_{n+1} & l_n \\ (a/b)\, l_n & (a/b)^{\varepsilon(n)}\, l_{n-1} \end{bmatrix}$$

, where $\varepsilon(n)$ is as in the equation(1).

In this study, a new bound was obtained for the maximum and minimum eigenvalues of the matrix representation of bi-periodic Fibonacci and bi-periodic Lucas sequences.

Key Words: bi-periodic Fibonacci sequence , bi-periodic Lucas sequence , bounds of eigenvalues .

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On the Konhauser Bıorthogonal Polynomials

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ABSTRACT

In this study, we establish new properties of the $Y_n^{\alpha}(x;k)$ Konhauser biorthogonal polynomials. We give new properties and various families for Konhauser biorthogonal polynomials. These polynomials are deal with some new properties of the Konhauser biorthogonal polynomials. We give a theorem about the summation formula for Konhauser biorthogonal polynomials. First of all, the generating function of Konhauser biorthogonal polynomials have been derived using some features of Pochhammer symbol. Then, in accordance with this purpose, some results have been obtained by taking derivative of the generating functions with respect to different variables. After then some corollary and remarks have been presented by applying this generating functions to the some theorems. In order to show these theorems, some properties of summation formula have been used. These theorems would help to define new and different kind of properties of Konhauser biorthogonal polynomials. Later on some values of n such as 1, 2, 3, were used and the corresponding expressions were obtained. A graph was drawn for some specific values of Konhauser biorthogonal polynomials.The results obtained here include various families of multilinear and multilateral generating functions, miscellaneous properties and also some special cases for these polynomials. In [2] and [3] contain the similar study presented here.

Key Words: Konhauser Biorthogonal Polynomials, generating function, multilinear and multilateral generating function, recurrence relation.

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On the Domination Number and the Total Domination Number of Fibonacci Cubes and Lucas Cubes

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ABSTRACT

In this work we deal with the domination number and the total domination number of Fibonacci cubes and Lucas cubes. We obtain improved lower bounds and upper bounds for these numbers by the recursive structure of these graphs.

An n-dimensional hypercube Q_n is a simple graph having 2^n vertices. The vertex set of Q_n can be labeled with all binary sequences having length n as

$$V(Q_n) = \{v_1 v_2 \dots v_n \mid v_i \in \{0,1\}, for\ i = 1,2,\dots,n\}.$$

In this representation there is an edge between two different vertices if the Hamming distance of these vertices is 1. By removing the vertices having two consecutive 1s we obtain a subgraph of Q_n, which is called an n-dimensional Fibonacci cube and is denoted by Γ_n. The number of vertices of Γ_n becomesF_{n+2}, where F_n are the Fibonacci numbers defined as $F_0 = 0, F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. By removing the vertices having 1 in its first and last positions in the vertex set of Γ_n ($n \geq 1$) we obtain the subgraphs called Lucas cubes, which is denoted by Λ_n. The number of vertices in Λ_n is L_n where L_n are the Lucas numbers defined as $L_0 = 2, L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 2$.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. $D \subseteq V(G)$ is called a *dominating set* if every vertex of G either belongs to D or is adjacent to some vertex in D. The *domination number* of G is defined as the minimum cardinality of a dominating set of the graph G. In a similar manner, $D \subseteq V(G)$ is called *total dominating set* if every vertex of G is adjacent to some vertex in D and the *total domination number* of G is defined as the minimum cardinality of a total dominating set of the graph G.

Key Words: Fibonacci cube, Lucas cube, Domination number.

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On the Geometric Circulant Matrices with the Some Special Numbers

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ABSTRACT

The circulant and $r-$circulant matrix have important applications in numerical analysis, probability, coding theory, and so on. An $n \times n$ matrix C_r is called an $r-$circulant matrix if it is defined as follows:

$$C_r = \begin{pmatrix} c_0 & c_1 & c_2 & \dots & c_{n-2} & c_{n-1} \\ rc_{n-1} & c_0 & c_1 & \dots & c_{n-3} & c_{n-2} \\ rc_{n-2} & rc_{n-1} & c_0 & \dots & c_{n-4} & c_{n-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ rc_1 & rc_2 & rc_3 & \dots & rc_{n-1} & c_0 \end{pmatrix}.$$

An $n \times n$ matrix C_{r^*} is called an geometric circulant matrix if it is defined as follows:

$$C_{r^*} = \begin{pmatrix} c_0 & c_1 & c_2 & \dots & c_{n-2} & c_{n-1} \\ rc_{n-1} & c_0 & c_1 & \dots & c_{n-3} & c_{n-2} \\ r^2c_{n-2} & rc_{n-1} & c_0 & \dots & c_{n-4} & c_{n-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ r^{n-1}c_1 & r^{n-2}c_2 & r^{n-3}c_3 & \dots & rc_{n-1} & c_0 \end{pmatrix}.$$

In particular, for $r=1$, C_r and C_{r^*} turn into

$$C = \begin{pmatrix} c_0 & c_1 & c_2 & \dots & c_{n-2} & c_{n-1} \\ c_{n-1} & c_0 & c_1 & \dots & c_{n-3} & c_{n-2} \\ c_{n-2} & c_{n-1} & c_0 & \dots & c_{n-4} & c_{n-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_1 & c_2 & c_3 & \dots & c_{n-1} & c_0 \end{pmatrix}$$

and C is called a circulant matrix.

The sequence of the Fibonacci numbers is one of the most well-known sequences, it has many applications to different fields such as mathematics, statistics and physics. The Fibonacci and Lucas sequences are defined by the following recurrence relations: for $n \geq 0$, $F_{n+2} = F_{n+1} + F_n$ and $L_{n+2} = L_{n+1} + L_n$,

where $F_0 = 0$, $F_1 = 1$, $L_0 = 2$ and $L_1 = 1$, respectively. The generalized Fibonacci and Lucas sequences, $\{U_n\}$ and $\{V_n\}$, are defined by the following recurrence relations: for $n \geq 0$ and any nonzero integer p,

$$U_{n+2} = pU_{n+1} + U_n \text{ and } V_{n+2} = pV_{n+1} + V_n,$$

where $U_0 = 0$, $U_1 = 1$, $V_0 = 2$ and $V_1 = p$.

On the other hand, recently in [7], Tuğlu et al. defined harmonic and hyperharmonic Fibonacci numbers and gave some combinatorics identities of these numbers. Also they obtained spectral and Euclidean norms of circulant matrices involving harmonic and hyperharmonic Fibonacci numbers.

Circulant and $r-$circulant matrices with the special numbers have been studied by many researchers in last decade [1-7].

In this paper, we define a new circulant matrix which is called geometric circulant matrix and give some properties of these matrices with the generalized Fibonacci numbers.

Key Words: Generalized Fibonacci numbers, spectral norm, circulant matrix.

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On the Numerical Semigroups with Generated by Two Elements with Multiplicity 3

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ABSTRACT

Throughout this study, we assume that $\mathbb{N}$ and $\mathbb{Z}$ be the sets of nonnegative integers and integers, respectively. The subset S of $\mathbb{N}$ is a numerical semigroup if $0 \in S$, $x+y \in S, \text{ for all } x, y \in S$, and $Card(\mathbb{N} \backslash S) < \infty$ (this condition is equivalent to $\gcd(S) = 1$, $\gcd(S) =$ greatest common divisor the element of S).

Let S be a numerical semigroup, then $F(S) = \max(\mathbb{Z} \backslash S)$ and $m(S) = \min\{s \in S : s > 0\}$ are called Frobenius number and multiplicity of S, respectively. Also, $n(S) = Card(\{0,1,2,...,F(S)\} \cap S)$ is called the number determine of S. If S is a numerical semigroup such that $S = < a_1, a_2, ..., a_r >$, then we observe that $S = < a_1, a_2, ..., a_r > = \{s_0 = 0, s_1, s_2, ..., s_{n-1}, s_n = F(S)+1, \rightarrow ...\}$ where $s_i < s_{i+1}$, $n = n(S)$, and the arrow means that every integer greater than $F(S)+1$ belongs to S, for $i = 1, 2, ..., n = n(S)$.

If $a \in \mathbb{N}$ and $a \notin S$, then a is called gap of S. We denote the set of gaps of S, by $H(S)$, i.e, $H(S) = \mathbb{N} \backslash S$. The $G(S) = Card(H(S))$ is called the genus of S. Also, It is known that $G(S) = F(S) + 1 - n(S)$. Let S be a numerical semigroup and $m \in S, m > 0$. Then $Ap(S,m) = \{x \in S : x - m \notin S\}$ is called Apery set of S according to m.

A numerical semigroup S is Arf if $a + b - c \in S$, for all $a, b, c \in S$ such that $a \geq b \geq c$. The intersection of any family of Arf numerical semigroups is again an Arf

numerical semigroup. Thus, since ¥ is an Arf numerical semigroup, one can consider the smallest Arf numerical semigroup containing a given numerical semigroup. The smallest Arf numerical semigroup containing a numerical semigroup S is called the Arf closure of S, and it is denoted by $Arf(S)$.

In this presentation, we will give some results about gaps, the determine number, Apery set and Arf closure of S numerical semigroup such that $S = \langle 3, x \rangle$.

Key Words: Frobenius number, telescopic numerical semigroup, genus.

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On the $r-$Circulant Matrices with the Binomial Coefficients

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ABSTRACT

The circulant and $r-$ circulant matrices have a connection to probability, coding theory, numerical analysis and many other areas. The $r-$ circulant matrices of order n, $C_r = \left(c_{ij}\right)$ are defined by

$$c_{i,j} = \begin{cases} c_{j-i}, & i \le j \\ rc_{n+j-i}, & i > j \end{cases} \tag{1}$$

where $r \in \square - \{0\}$ and $i, j = 1, 2, \ldots, n$. Also, the matrix C_r is determined by its first row elements and r, we denote it shortly $C_r = Circ\left\{ {}_r\left(c_0, c_1, \ldots, c_{n-1}\right)\right\}$. The $r-$circulant matrices are a generalization of circulant, negacyclic and semicirculant matrices. Namely, taking $r=1$ in (1), we have the classical circulant matrix. Using $r=-1,0$ in (1), we obtain the negacyclic (skew circulant) and semicirculant matrices, respectively. Circulant and $r-$circulant matrices with the special numbers have been studied by many researchers in last decade (see [1,5,6,8]). In [3,4], the author consider the circulant matrices with the binomial coefficients.

In this paper, we consider the $r-$circulant matrices with the binomial coefficients. Namely, we consider the C_r matrix as follows

$$C_r = Circ\left\{ {}_r\left(\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n-1}\right)\right\} \tag{2}$$

Afterwards, we investigate the eigenvalues, determinant and norms (Euclidean, maximum column and row sum norm, spectral norm) of the $r-$ circulant matrices

with the binomial coefficients. Taking $r=1$ in (2), we have the binomial circulant matrices in [3,4]. Also, if we take $r=-1, r=0$ in (2), we have the negacyclic and semicirculant matrices with the binomial coefficients, respectively. Thus, we give the special cases of our results for the circulant, negacyclic and semicirculant matrices with the binomial coefficients.

Key Words: Binomial coefficient, $r-$Circulant matrix, Matrix norm

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Plastic Number and Inverse, Factorization and Determinantal Representation of Cordonnier and Perrin Matrices

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ABSTRACT

The plastic number is a real number which is the unique real solution of the equation $x^3 = x+1$. This equation is the characteristic equation associated with the Perrin and Cordonnier sequence. The name plastic number was given to this real number in 1928 by Dom Hans van der Laan. Like golden number, plastic number is used in art and architecture. In [3, 4] Richard Padovan gave some properties of plastic number in architecture and mathematics. Christopher Bartlett found a relation with significant number of paintings canvas sizes and plastic number [2]. In [1] Şahin defined generalized Cordonnier and generalized Perrin matrices using associated polynomials Cordonnier and Perrin numbers. Şahin gave some relationships between generalized Perrin and Cordonnier matrices. Şahin also obtained the inverse of generalized Cordonnier and generalized Perrin matrices with the aid of determinants of some Hessenberg matrices which obtained from a part of these matrices, gave factorization of generalized Cordonnier matrices and gave some determinantal representation of associated polynomials Cordonnier numbers. In this study, we present some properties and applications of plastic numbers and give relationship with Cordonnier numbers and plastic number. Then we give some properties of generalized Cordonnier and generalized Perrin matrices and generalize results in [1].

Key Words: Plastic number, Cordonnier numbers, Cordonnier matrices.

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Rings of Soft Sets

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ABSTRACT

Molodtsov[1] introduced the theory of soft sets, which can be seen as an effective mathematical tool to deal with uncertainties, since it is free from the diffuculties that the usual theoretical approaches have troubled. It associates a set with a set of parameters and thus free from the diffuculties effecting existing methods. Soft set theory has rich potential applications most of which have already been demonstrated by Molodtsov. In many fields such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory and measurement theory. Soft set theory has attracted much attention since its introduction by Molodtsov. Sezgin and Atagün [6] discussed the basic properties of operations on soft sets such as intersection, extended intersection, restricted union, restricted diffference defined in [2] and they illustrated their interconnections between each other. Soft set theory has continued to experience tremendous growth in the mean of algebraic structures since Aktaş and Çağman [5] defined and studied soft groups, soft subgroups, normal soft subgroups, soft homomorphisms, adopting the definition of soft sets. In [4], the same authors introduced two new perations on soft sets, called inverse production and characteristic production depending on the relation forms of soft sets and obtained two isomorphic abelian groups called "the inverse group of soft sets" and "the characteristic group of soft sets". In this study, we redefine the operations inverse and characteristic products of soft sets without using relation forms of soft sets. This leads to simplicity and brevity. Also, we construct two ring structure consisting the representations of uncertain objects as elements.

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Key Words: Soft sets, group structure, ring structure, inverse product, characteristic product

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Rough Sets And On Decısıon-Makıng Methods

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ABSTRACT

Many fields deal daily with the uncertain data that may not be successfully modeled bye the classical mathematics. The volume and complexity of the collected data in our modern society is growing rapidly.There often exist various types of uncertainties in those date related to complex problems in biology, economics, engineering, envorimentel science , medial science, social science, and many other fields.İn order to describe and extract the useful information hidden in uncertain data, researchers in mathematics, computer science an related areas have proposed a number of theories such as probability theory, fuzzy set theory, intuitionistic fuzzy set theory , rough set theory, vague set theory and interval mathematics. Molodstov, introduce the concept of soft sets that can be seen as a new mathematical theory for dealing with uncertainty. The soft theory has been applied to many different fields with great success. Chen at al. proposed a new definition of soft set parametrization reduction, and compared it with the related concept of attributes reduction in rough set theory, and Kong et al. presented the normal parametrization reduction of soft sets. Maji et al. worked on theoretical study of soft sets in detail., and presented an application of soft set in the decision making problem using the reduction of rough sets. Rough set theory was prosed by Pawlak in [8] 1982 for dealing with the vagueness, granularity and uncertainty in information systems, where it classifies information through indiscernibility relations, which have been used widely in artificial intelligence, data mining, machine learning, and other fields. Liu et al. combined the logistic regression and the decision–theoricrough set into a new classification approach, which can effectively reduce the misclassification rate. Yu et al. applied decision-theoric rough set model for automatically determining the number of clusters with much smaller time cost. In this study, we establish an connection between two mathematical

approaches to vagueness; rough sets and soft sets. And applications of rough sets on decision-making methody are studied.

Key Words: Soft sets, rough sets, rough soft sets, decision-making theory

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r-Semisimple Modules

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ABSTRACT

Let R be a ring with identity. We define and characterize r-semisimple modules and rings which are generalizations of semisimple and second modules and rings. A module M_R is called a semisimple module if every submodule is a direct summand of M_R. A module M_R is a second module if for every ideal I of R either MI = 0 or MI = M. A module M_R is called an r-semisimple module if for any right ideal I of R, MI is a direct summand of M_R (or equivalently for every ideal I, MI is a direct summand of M_R). It is clear that every semisimple and second modules are r-semisimple but the converse may not be true. If R_R is r-semisimple, then it is semiprimitive, that is J(R) = 0. Every direct summand and direct sum of r-semisimple modules are also r-semisimple. We investigate when an r -semisimple ring is semisiple and simple. Moreover, any module M_R need not be r-semisimple although every proper submodule is r-semisimple. We show that a ring R with the number of the proper right ideals < 5 and J(R) = 0 is r-semisimple. We investigate the structure of ring and modules whenever R is an r-semisimple ring. We obtain that R is an r-semisimple ring if and only if it is a direct sum of simple rings and we prove that every injective module M_R is r-semisimple and if R_R is also self-injective and noetherian, then R_R is semisimple . If R is an r-semisimple and right Kasch ring , then R is semisimple. For any unexplained terminology, we refer the reader to [[1], [7], [2]] .

Key Words: Semisimple Modules, Second Modules, Injective Modulles.

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Sabit b. Qurra and (821-901) Mathematics

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ABSTRACT

Harran is one of the most important places of science and thought centers where Islamic civilization spreads in history. After entering the dominion of the Islamic state, the Harranans contributed to the formation of Islamic science by translating translations of Greek and Syriac boks as well as copyright works. Here is one of these scientists is Sabit b. Qurra who is from Harran (Death, 288/901). His real name is Abu'I-Hasan Sabit b. Qurra, he is born in Harran in 821, he has spent his childhood and youth here. It is at the head of the Harranian mathematicians who contributed to the formation of Islamic mathematics. In addition to the Syriac, the Arabic and the Greek who knows very well. Furthermore, he compiled around 150 Arabicand 15 Syriac copyrighted works. Mathematics plays an important role in Qurra's scientific activities. He worked on almost every branch of mathematics. Especially the works of Archimedes were translated by Sabit b. Qurra in the field of mathematics. Since many of the works of Archimedes are now lost as Greek originals, We are informed from them by the translation of Qurra's Arabic translations. He produced original copyright works in fields such as mathematics, arithmetic (number theory), algebra, geometry, conic sections and trigonometry. He also wrote comments on thesymbol elements of Euclid and Almagest of Ptolemy. Qurra'sbook of Mafrûdat gained much fame in the Middle Ages. Nasreddin al-Tûsî, in the transition books, also included this book alongside Elemankar and Almagest. The work covered 36 recommendations on geometry and geometric algebra, including 20 problems related to construction and a geometric problem related to the solution of (a + x) x = b quadratic equations. Makâle fi istihrâc al-a'dâd al-mütahâbba bi suhûlet al-maslak to zâlika (an article on the discovery of friendly numbers) contained 10 theorems related to the number theory.

Key Words: Harran, Sabit, Qurra

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Setlling Of Fuzzy Logic Controlling According To Electrical Angle

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ABSTRACT

Until now, many methods for controlling systems have been explored and tested. These control methods include linear and non-linear methods. Linear control methods can systematically control systems while non-linear control methods respond quickly to sudden changes on the system. This allows the desired results are obtained in undesirable situations. In this article, a study is conducted for fuzzy logic control, which is nonlinear control. First, when the voltage is generated on the load, the operating characteristic of the drive system controlling the load is determined by the pulse width modulations. While pulse width modulation determines the way to control the switches on the drive system, it directs the step behaviour of positive and negative periods of the alternating voltage with electrical angle. The rule table for fuzzy logic control is created and there are rules on this table to decide how to manner the undesirable situations which will occur in the output values. In order to execute the rules in this table, the current flowing on the load is taken into consideration and compared with the reference current value. Two values are created, such as the error and the change of error, depending on the change between these two compared values. The formation of these values is described by the formation of differential and second order differential equations.

Key Words: differential equations, pulse width modulation, non-linear control.

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Some Algebraic Properties of Arithmetic Fuchsian Groups

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ABSTRACT

In this paper we investigate some properties and examples of Fuchsian groups. The most interesting and important ones are the so-called arithmetic Fuchsian groups, which are discrete subgroups of $PSL(2,\mathbb{R})$ obtained by some arithmetic constructions. If we choose all matrices of $SL(2,\mathbb{R})$ with integer coefficients, then the corresponding elements of $PSL(2,\mathbb{R})$ form the modular group $PSL(2,\mathbb{Z})$. The same construction, restriction of scalars to integers, allows us to obtain arithmetic subgroups of larger matrix groups, e.g. $SL(n,\mathbb{Z})$ in $SL(n,\mathbb{R})$.

In [1,2,3] we study some important classes of its subgroups of finite index. And also we gave some properties of suborbital graphs for congruence subgroups of modular group. Indeed, the idea of suborbital graphs corresponding to non-trivial suborbits of a group $\mathcal{G}$ acting on a non empty set Ω was first introduced by C.C. Sims [4] when he studied graphs and finite permutation groups. We can say significance of graphs as representation theory is both an application of the group concept and an important concept for a deeper understanding of groups. Given a group action, representation gives further means to study the object being acted upon, yielding more information about the group. Thus group representations are an organizing principle in the theory of finite groups [5,6]. The studies will also be of help to various fields. To combinatorics, calculation of cycle index formulas will enhance processes by which we organize sets so that we can interpret and apply the data they contain. To graph theorists, graphs are used to model many types of relations and processes in physical, biological, social and information systems. We know that many practical problems can be represented by graphs.

Let $\alpha \longrightarrow \Psi(\alpha)$ be a finite dimensional representation of the group $PSL(2,\mathbb{R})$. The elements of $PSL(2,\mathbb{R})$ which correspond to matrices $\Psi(\alpha)$ with integer coefficients form a discrete subgroup of $PSL(2,\mathbb{R})$. All subgroup of $PSL(2,\mathbb{R})$

therefore obtained and also their subgroups of finite index are called arithmetic Fuchsian groups [7]. This definition is slightly different from a commonly used one of an arithmetic subgroup of a semi-simple Lie group. It is rather hard to check, except for trivial cases, whether or not a given Fuchsian group is arithmetic according to the above definition. However, it follows from results of [8] on classification of classical groups that the list of all arithmetic subgroups of $SL(2,\mathbb{R})$ is exhausted up to commensurability by Fuchsian groups derived from quaternion algebras over totally real number fields.

Key Words: Fuchsian groups, quaternion algebra, suborbital graphs.

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Some Generalized Suborbital Graphs

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ABSTRACT

It is known that the graph of a group provides a method by which a group can be visualized; in many cases it suggests an economical algebraic proof for a result and it gives same information but in a much more efficient way. In this view, the idea of suborbital graph has been used mainly by finite group theorists [7].

The normalizer Nor(n) of the congruence subgroup $\Gamma_0(n)$ of the modular group Γ in the projective special linear group PSL(2,R) acts on the upper half plane and on the extended set of rationals as a group of Möbiüs transformations[4]. This normalizer has acquired significance because it is related to the Monster simple group. It is also played an important role in work on Weierstrass points on the Riemann surfaces associated to $\Gamma_0(n)$, and on Modular forms [1].

Its action is neither transitive nor primitive. Finding the maximal subset on which Nor(n) acts transitively, it can be defined a Nor(n)-invariant equivalence relation on it with equivalence classes. This provide the notion of suborbital graphs of Nor(n) (introduced in 1967 by Sims [8] for finite permutations groups). Some special cases of these graphs were obtained in [2, 3, 5, 6]. The general statement is still an open problem.

In this study, we give some calculations to reach this generalization.

Key Words: Finite permutation groups, suborbital graphs.

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Some New Identities Concerning the Horadam Sequence and its Companion Sequence

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ABSTRACT

Many number sequences can be defined, characterized, evaluated, and classified by linear recurrence relations with certain orders. In this paper, we consider the sequences defined by linear recurrence relations with second order. The best known of these sequences is called the Horadam sequence, which was introduced in 1965 by Horadam. The Horadam sequence $\{W_n\} = \{W_n(a,b;P,Q\}$ is defined by $W_0 = a, W_1 = b \ and \ W_n = PW_{n-1} + QW_{n-2} \ for \ n \geq 2,$ where $a, b, P, and \ Q$ are real numbers with $PQ \neq 0 \ and \ (a,b) \neq (0,0)$. Particular cases of $\{W_n\}$ are the Lucas sequence of the first kind $\{U_n(P,Q)\} = \{W_n(0,1;P,Q\}$ and the Lucas sequence of the second kind $\{V_n(P,Q)\} = \{W_n(2,P;P,Q\}$. If we define the sequence $\{X_n\} = \{X_n(a,b;P,Q\}$ by $X_0 = a, X_1 = b \ and \ X_n = PX_{n-1} + QX_{n-2} \ for \ n \geq 2,$ then it is convenient to consider it to be a companion sequence of $\{W_n\}$, in the same way that $\{V_n(P,Q)\}$ is the companion of $\{U_n(P,Q)\}$. Many identities concerning the terms of the Lucas sequence of the first and second kind can be proved by using Binet formulae, induction and matrices. In this study, we give a general relation between the Horadam sequence and nxn matrices satisfying the relation $X^2 = PX + QI.$ Then we obtain some new identities between the Horadam sequence and its companion sequence using $2x2$ matrices satisfying the relation $X^2 = PX + QI.$ Moreover, we solve some Diophantine equations by the help of these identities. Lastly, we make an application of these sequences to trigonometric functions and get some new angle addition formulas such as

$$sinr\theta \sin(m+n+r)\theta = \sin(m+r)\theta \sin(n+r)\theta - sinm\theta sinn\theta,$$

$$cosr\theta \cos(m+n+r)\theta = \cos(m+r)\theta \cos(n+r)\theta - sinm\theta sinn\theta,$$

and

$$cosr\theta \sin(m+n)\theta = \cos(n+r)\,\theta sinm\theta + \cos(m-r)\theta sinn\theta.$$

Key Words: Horadam Sequence, Second-Order Recurring Sequences.

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Some Notes On Locally Internal Uninorm On Bounded Lattices

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ABSTRACT

Uninorms, introduced by Yager and Rybalov [7] and studied by Fodor et al. [4], are special aggregation operators that have proven useful in many fields like fuzzy logic, expert systems, neural networks, aggregation, and fuzzy system modeling [3,6].

In [5], it is studied that locally internal, monotonic operations on unit interval [0,1] with fixed neutral element and description of these operators. Furthormore, characterization of locally internal monotonic, associative operations with a neutral element is given.

In [1], it is showed that considering an arbitrary bounded lattice *L*, the existence of idempotent uninorms on *L* for any element e in L \ {0,1} playing the role of a neutral element. Moreover, the smallest idempotent uninorm and the greatest idempotent uninorm with the neutral element e in L \ {0,1} is obtained.

In [2], it is proposed that in any bounded lattice idempotent uninorms need not be internal extending definition of the term ”internal”. By giving a sufficient condition for the bounded lattice to any idempotent uninorm be internal, some properties of idempotent uninorm is investigated.

In this study, we introduce definiton of locally internal uninorm on arbitrary bounded latice L. We examine some properties of idempotent and locally internal uninorm on arbitrary bounded latice L and investigate relationship between these operators. And, some illustrative examples are added to clearly showing connection between idempotent and locally internal uninorm.

Key Words: Bounded Lattice, Uninorm, Idempotent Uninorm, Locally Internal.

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Some Notes on Ordering Based On Uninorms

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ABSTRACT

Uninorms are one of the important aggregation operators. In addition to these features of aggregation functions that contain boundary conditions and monotonicity properties, uninorms also contain neutral element and associativity properties. Additionally, uninorms are generalization of t-norms and t-conorms.

Uninorms were defined on unit interval [0,1] by Yager and Rybalov [6]. Then, they were defined and studied on more general algebraic structures such as bounded lattices [2,4]. In recent years, another area of interest for researchers is the orders obtained from logical operators. These studies started with defining the partially ordered relation from t-norms and it has been subject to many studies in the following [5].

It is quite natural for such a work to be done for uninorms if it is thought to be more general structures involving t-norms and t-conorms. In this study, a relation is derived from uninorms defined on bounded lattice, and it is shown that this relation is a partially ordered relation on bounded lattice. Moreover, the relationship between order obtained from uninorms and the natural order on given bounded lattice is investigated. It has been observed and exemplified that even if it is a lattice (chain) according to natural order, it may not be lattice (chain) according to the order obtained from uninorms.

Key Words: Uninorm, t-norm, t-conorm, bounded lattice.

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Some properties of bi-periodic Fibonacci and Lucas sequences

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ABSTRACT

In this study, we consider the generalized bi-periodic Fibonacci sequences $\{w_n\}$ which are emerged as a generalization of the best known sequences in the literature, such as bi-periodic Fibonacci and bi-periodic Lucas sequence, classical Fibonacci and classical Lucas sequence, Pell and Pell-Lucas sequence, $k-$Fibonacci and $k-$Lucas sequences, etc. Edson and Yayenie [2], introduced the generalized bi-periodic Fibonacci sequences $\{w_n\}$ by the recurrence $w_n = aw_{n-1} + w_{n-2}$ if n is even, $w_n = bw_{n-1} + w_{n-2}$ if n is odd with arbitrary initial conditions w_0, w_1 and nonzero numbers a, b. Note that by allowing arbitrary initial conditions, each new choice of a and b produces a distinct sequence. For example, if we take the initial conditions $w_0 = 0$ and $w_1 = 1$ we get the bi-periodic Fibonacci sequences, and it is obvious that under these conditions by taking $a = b = 1$, we get the classical Fibonacci sequences, by taking $a = b = 2$, we get the Pell sequences, and by taking $a = b = k$, for some positive integer k, we get the $k-$Fibonacci sequences. Analogously, if we take the initial conditions $w_0 = 2$ and $w_1 = b$, by switching a and b, we get the bi-periodic Lucas sequences in [1]. By motivating Horadam's results in [3], we give some basic properties of the sequence $\{w_n\}$ which generalize the known results for the bi-periodic Fibonacci and Lucas sequences [6]. Also, we define a new matrix identity for the bi-periodic Fibonacci and bi-periodic Lucas numbers, and by using the matrix method, we give simple proofs of several properties of these sequences [4,5]. Moreover, we obtain a new binomial sum formula for the bi-periodic Fibonacci and bi-periodic Lucas numbers which generalize the former results in the literature.

Key Words: bi-periodic Fibonacci sequences, bi-periodic Lucas sequence. generalized Fibonacci sequence.

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Some Properties of the Fuzzy Sheaf of the Fundamental Groups

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ABSTRACT

The concept of a fuzzy set was discovered by Zadeh [6] and one of its earliest branches, the theory of fuzzy topology, was developed by Chang [1] and others. Recently, Zheng [7] and Wuyts [5] introduced the concept of a fuzzy path. Using this concept, Salleh and Md Tap [4] constructed the fundamental group of a fuzzy topological space. Gumus and Yildiz [2] formed an algebraic fuzzy sheaf by means of the fuzzy topological group. Guner and Balci [3] gave some characterizations concerned with the fuzzy sheaf of the fundamental groups. The sheaves constructed over topological spaces, which are horizontally topological and vertically algebraic structures, are very interesting spaces.

Let X be a fuzzy path connected topological space and H_{a_λ} be the fundamental group of X based for any $a_\lambda \in X$, that is $H_{a_\lambda} = \pi_1(X, a_\lambda)$ [12]. Let $X = (X, x_p)$ be a pointed fuzzy topological space for an arbitrary fuzzy fixed point $x_p \in X$. Let us denote the disjoint union of all fundamental groups obtained for each $a_\lambda \in X$ by H, i.e., $H = \bigvee_{a_\lambda \in X} H_{a_\lambda}$. H is a set over X and the mapping $\psi : H \to X$ defined by

$$\psi(\sigma_{a_\lambda}) = \psi([\alpha(A)]_{a_\lambda}) = a_\lambda$$

for any $\sigma_{a_\lambda} = [\alpha(A)]_{a_\lambda} \in H_{a_\lambda} \subset H$ is onto.

Now, let $W \subset X$ be a open fuzzy set. Define a mapping $s : W \to H$ such that

$$s(a_\lambda) = [\gamma^{-1}(H) * \alpha(A) * \gamma(G)]_{a_\lambda}$$

for each $a_\lambda \in W$, where $[\alpha(A)]_{x_p} \in H_{x_p}$ is any element and $[\gamma(G)]$ is an arbitrary fixed fuzzy homotopy class defines an isomorphism between H_{a_λ} and H_{x_p}. Then the change of s depends on only the change of $\sigma_{x_p} = [\alpha(A)]_{x_p}$. Furthermore, $\psi \circ s = 1_W$. Let us denote the totality of the mappings s defined on W by $\Gamma(W,H)$.

If B is a fuzzy base for X, then $B^* = \{s(W) : W \in B, \ s \in \Gamma(W,H)\}$ is a fuzzy base for H. The mappings ψ and s are fuzzy continuous in this topology. Moreover ψ is a locally fuzzy topological mapping. Then (H,ψ) is a fuzzy sheaf over X. (H,ψ) or only H is called " the fuzzy sheaf of the fundamental groups" over X .

The fuzzy sheaf H satisfies the following properties:

1. Let $W \subset X$ be an open fuzzy set. Then, a fuzzy section over W can be extended to a global fuzzy section over X.
2. Any two stalks of H are isomorphic with each other.
3. Let $W_1, W_2 \subset X$ be any two open fuzzy sets, $s_1 \in \Gamma(W_1,H)$ and $s_2 \in \Gamma(W_2,H)$. If $s_1(x_0) = s_2(x_0)$ for any fuzzy point $x_0 \in W_1 \cap W_2$, then $s_1 = s_2$ over the whole $W_1 \cap W_2$.
4. Let $W \subset X$ be an open fuzzy set and $s_1, s_2 \in \Gamma(W,H)$. If $s_1(x_0) = s_2(x_0)$ for any fuzzy point $x_0 \in W$, then $s_1 = s_2$ over the whole W.

In this talk,the fuzzy sheaf of fundamental groups over fuzzy path connected topological spaces is shown a fuzzy covering space. Furthermore, General Lifting Theorem and its some results are adapted to the fuzzy sheaves .

Key Words: Fuzzy Path Connected Topological Space, Fuzzy Sheaf, Fuzzy Covering Space, Fuzzy Lifting Theorem.

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Some Results in Triply –Generated Telescopic Numerical Semigroups

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ABSTRACT

In this study, we assume that $\mathbb{N} = \{0,1,2,...,n,...\}$, $S \subseteq \mathbb{N}$ and $\mathbb{Z}$ integer set. S is a numerical semigroup if $a+b \in S$, for $a,b \in S$, $0 \in S$ and $\mathbb{N} \backslash S$ is finite. Let S be a numerical semigroup and $X \subset \mathbb{N}$. X is called minimal system of generators of S if $S = <X>$ and there is not any subset $Y \subset X$ such that $S = <Y>$. $S = <s_1, s_2, s_3>$ is called a triply-generated telescopic numerical semigroup if $s_3 \in <\frac{s_1}{d}, \frac{s_2}{d}>$ where $d = \gcd(s_1 s_2)$ (Here, $\gcd(S)$ = greatest common divisor the element of S). Let S be a numerical semigroup, then $m(S) = \min\{x \in S : x > 0\}$ is called as multiplicity of S. The number $F(S) = \max(\mathbb{Z} \backslash S)$ is called Frobenius number of S. A numerical semigroup S can be written the form $S = <a_1, a_2, ..., a_n> = \{s_0 = 0, s_1, s_2, ..., s_{n-1}, s_n = F(S)+1, \rightarrow ...\}$, where $s_i < s_{i+1}$, $n = n(S)$ and the arrow means that every integer greater than $F(S)+1$ belongs to S for $i = 1,2,...,n = n(S)$.

If $x \in \mathbb{N}$ and $x \notin S$, then x is called gap of S. We denote the set of gaps of S, $H(S) = \mathbb{N} \backslash S$. The number $G(S) = \#(H(S))$ is called the genus of S. Let S be a numerical semigroup and $m \in S$,$m > 0$. Then $Ap(S,m) = \{x \in S : x - m \notin S\}$ is called Apery set of S according to m. $Ap(S,m)$ is formed by the smallest elements of S belonging to the different congruence classes mod m. Thus $\#(Ap(S,m)) = m$

and the Frobenius number of S equal to $\max(Ap(S,m)) - m$ (where #(A) stands for Cardinality (A)). The number $n(S) = \#(\{0,1,2,...,F(S)\} \cap S)$ is called determine of numerical semigroup S. Also, It is known that $G(S) = F(S) + 1 - n(S)$.

In this study, we will give some results about gaps, fundamental gaps and the determine number of S triply-generated telescopic numerical semigroups with arbitrary multiplicity.

Key Words: Frobenius number, telescopic numerical semigroup, genus.

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226 (2000), 479-487.

Some Results on Arf Numerical Semigroups with Multiplicity 8

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ABSTRACT

A numerical semigroup is a subset of the set of nonnegative integers (denoted here by $\mathbb{N}$) closed under addition, containing the zero element and with finite complement in $\mathbb{N}$. Note also that up to isomorphism the set of numerical semigroups classify the set of all submonoids of $(\mathbb{N},+)$**.** Let *S* be submonoid of $\mathbb{N}$, the condition of having finite complement in $\mathbb{N}$ is equivalent to saying that the greatest common divisor (gcd for short) of its elements is one.

Those positive integers which do not belong to S are called gaps of S. The number of gaps of S is called the genus of S and it is denoted by $G(S)$. The largest gaps of S is $F(S)$ if S is different from $\mathbb{N}$.

$m(S)=\min\{s\in S: s>0\}$ are called multiplicity of S, respectively. Also, $n(S)=Card\left(\{0,1,2,...,F(S)\}\cap S\right)$ is called the number determine of S.

If $a\in\mathbb{N}$ and $a\notin S$, then a is called gap of S. We denote the set of gaps of S, by $H(S)$, i.e, $H(S)=\mathbb{N}\backslash S$. The $G(S)=Card(H(S))$ is called the genus of S. Also, It known that $G(S)=F(S)+1-n(S)$.

A numerical semigroup *S* is called Arf if $x+y-z\in S$ for all $x,y,z\in S$, where $x\geq y\geq z$. This definition was first given by C. Arf in 1949.

In the study, we intend to examine the Arf numerical semigroup with multiplicity eight and fixed conductor. We will also able to compute the notable elements and special sets of these numerical semigroups.

Key Words: Arf numerical semigroups, Pseudo-Frobenius number, Genus,

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Some weighted geometric averaging operators and weighted arithmetic operators on SVNSs

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ABSTRACT

As a variation of fuzzy sets and intuitionistic fuzzy sets, neutrosophic sets have been developed to represent uncertain, imprecise, incomplete and inconsistent information that exists in the real world. The neutrosophic sets may express more abundant and flexible information as compared with the fuzzy sets and intuitionistic fuzzy sets. In this paper, this article introduces an approach to handle multi-criteria decision making (MCDM) problems under the single valued neutrosophic sets. Therefore, we develop some new geometric and arithmetic aggregation operators, such as the single valued neutrosophic weighted arithmetic (SVNWA) operator, the single valued neutrosophic ordered weighted arithmetic (SVNOWA) operator, the single-valued neutrosophic sets hybrid ordered weighted arithmetic (SVNSHOWA) operator, the single-valued neutrosophic weighted geometric (SVNWG) operator and the single-valued neutrosophic ordered weighted geometric (SVNOWG) operator and the single-valued neutrosophic hybrid ordered weighted geometric (SVNHOG) operator, which extend the intuitionistic fuzzy weighted geometric and intuitionistic fuzzy ordered weighted geometric operators to accommodate the environment in which the given arguments are single valued neutrosophic sets which are characterized by a membership function, an indeterminacy-membership function and a non-membership function. Some numerical examples are given to illustrate the developed operators. Finally, a numerical example is used to demonstrate how to apply the proposed approach.

Key Words: Neurosophic set, Single valued neutrosophic set, Single valued neutrosophic weighted geometric (SVNWG) operator.

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Statistical Convergence order β Defined By a Modulus Function for Sequences of Fuzzy Numbers

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ABSTRACT

In order to generalize the concept of convergence of real sequences, the notion of statistical convergence was introduced by Fast [5] and Schoenberg [8], independently. Matloka [6] defined the notion of fuzzy sequence and introduced bounded and convergent sequences of fuzzy real numbers and studied their some properties. After then, Nuray and Savaş [7] defined the notion of statistical convergence for sequences of fuzzy numbers.

Çolak [4] generalized the statistical convergence by ordering the interval [0,1] and defined the statistical convergence of order α and strong p-Cesàro summability of order α, where $0<\alpha\leq 1$ and p is a positive real number. Altinok et al. [2] introduced the concepts of statistical convergence of order β and strong p-Cesàro summability of order β for sequences of fuzzy numbers. Aizpuru et al. [1] defined the f-density of the subset A of $\mathbb{N}$ by using an unbounded modulus function. After then, Bhardwaj [3] introduced f-statistical convergence of order α and strong Cesaro summability of order α with respect to a modulus function f for real sequences. The purpose of this paper is to generalize the study of Bhardwaj [3] and Çolak [4] applying to sequences of fuzzy numbers so as to fill up the existing gaps in the summability theory of fuzzy numbers.

In this study, we generalize and examine the concepts of statistically convergence of order β and strong Cesàro summability of order β with respect to an unbounded modulus function f for sequences of fuzzy numbers and give some inclusion theorems for different values of β.

Key Words: Fuzzy number, sequence of fuzzy numbers, statistical convergence, Cesàro summability, modulus function.

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Sugeno and Mamdani based Fuzzy Inference Approaches for Aircraft Pitch Control

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ABSTRACT

In this paper, we will give a control design for a fixed wing aerial vehicle using fuzzy Proportional-Integral-Derivate (PID) design methods. For the sake of simplicity and detailing the design of controllers, we simplify the full aircraft kinematics and dynamics and use the longitudinal model of the aerial vehicle. Short period approximation is also performed at longitudinal dynamics. In general, PID controllers are effective for linear and time invariant systems but they are not fully convenient and appropriate for nonlinear systems with uncertain conditions. Real-time fuzzy self-tuning PID controllers are able to robustly stabilize for uncertain system dynamics. There are many Fuzzy Inference Systems in the literature, such as Sugeno, Mamdani, Tsukamoto, and Şen [1]. In this paper, we examine Sugeno and Mamdani Fuzzy Inference Systems for controlling fixed wing aerial vehicle. The Mamdani and Sugeno membership functions and PID coefficients are developed in order to identify the efficiency of the methods [2,3]. Besides, the stability of the Sugeno and Mamdani based tuning of PID coefficients are compared with a specific reference line without any disturbances. Then, Sugeno based PID tuning is analysed under the availability of lumped disturbances. White noise is given to the system and the results are compared.

Key Words: Aircraft Pitch Model, Fuzzy PID Control, Sugeno and Mamdani Fuzzy Inference Systems

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The Number of Irreducible Polynomials Over Finite Fields with first two fixed coefficients

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ABSTRACT

In this study we determine the number of some special irreducible polynomials over Finite Fields. The aim of this paper is to count the number of irreducible polynomials over F_3 with prescribed, first and second coefficients. This is a generalization of Hansen Mullen Conjecture. According to Hansen Mullen primitivity conjecture, there exists a primitive polynomial with any one of its coefficient prescribed to any value (except for some genuine cases). But it is still unknown how to express these polynomials explicitly and what is their number over fixed Finite Field. Kuzmin [4] and Cattell [1] studied the number of irreducible polynomials of degree n over F_q and F_2 respectively, with first two coefficients prescribed. Some authors gave an estimated value for the number of irreducible polynomials [3], with two prescribed coefficients over F_3. We find the exact number of irreducible polynomials of degree n over F_3, with first two coefficients prescribed, i.e. the coefficients of x_{n-1} and x_{n-2}.

Theorem 1: [2] Let q be a prime power and F_q be the finite field of order *q* and μ be the mobius function. The number of irreducible polynomials over F_q is $f_q(n) = \frac{1}{n}\sum_{d|n} \mu(d) q^{n/d}$.

Theorem 2: [1] Let $P(n)$ be the set of all irreducible polynomials of degree n over F_q. Let $t_1, t_2 \in F_q$, then the number of elements in F_{q^n} with trace t_1 and subtrace t_2 is given by $F(n, t_1, t_2) = \sum_{d|n} \frac{n}{d} \left| \left\{ p \in P\left(\frac{n}{d}\right) : T_j(p^d) = t_j, j = 1,2 \right\} \right|$

Theorem3: [3] For $t_j \in F_3, 1 \leq j \leq 2$, the number of elements in F_{3^n} with trace and subtrace prescribed to t_1 and t_2 respectively, is given by

$$F(n,t_1,t_2) = \sum_{\substack{d|n \\ d\equiv 0}} \frac{n}{d}\left|\left\{p \in P\left(\frac{n}{d}\right): t_j = 0, j = 1,2\right\}\right| + \sum_{\substack{d|n \\ d\equiv 1}} \frac{n}{d}\left|\left\{p \in P\left(\frac{n}{d}\right): T_j(p) = t_j, j = 1,2\right\}\right|$$
$$+\sum_{\substack{d|n \\ d\equiv 2}} \frac{n}{d}\left|\left\{p \in P\left(\frac{n}{d}\right): 2T_1(p) = t_1,\ \ T_1^2(p) + 2T_2(p) = t_2\right\}\right|$$

Theorem 4: [1] Let G be a function of integers. For $F(n,t_1,t_2)$ it is approximated that

$$F(n/d\,, t_1, t_2) = 3^{n/d-2} + G(n/d\,, t_1, t_2)$$

Theorem 5:

1. If $n/d = 2m$ is even, then the values of $G(n/d\,, t_1, t_2)$ is given by

	(t_1,t_2)							
m (mod6)	0,1	0,2	1,0	1,1	1,2	2,0	2,1	2,2
0	3^{m-1}	3^{m-1}	0	0	0	0	0	0
1	3^{m-1}	-3^{m-1}	-3^{m-1}	-1	3^{m-1}	-3^{m-1}	-1	3^{m-1}
2	3^{m-1}	-3^{m-1}	-1	3^{m-1}	-3^{m-1}	-1	3^{m-1}	-3^{m-1}
3	-3^{m-1}	-3^{m-1}	0	0	0	0	0	0
4	-3^{m-1}	3^{m-1}	3^{m-1}	-1	-3^{m-1}	3^{m-1}	-1	-3^{m-1}
5	-3^{m-1}	3^{m-1}	-1	-3^{m-1}	3^{m-1}	-1	-3^{m-1}	3^{m-1}

2. If $n/d = 2m + 1$ is odd, then the values of $G(n/d, t_1, t_2)$ is given by

	(t_1, t_2)							
m (mod 6)	0,1	0,2	1,0	1,1	1,2	2,0	2,1	2,2
0	-3^{m-1}	-3^{m-1}	$2.3^{m-1}(\sim)$	-3^{m-1}	-3^{m-1}	$2.3^{m-1}(\sim)$	-3^{m-1}	-3^{m-1}
1	-3^m	3^m	0	0	0	0	0	0
2	3^{m-1}	3^{m-1}	3^{m-1}	$-2.3^{m-1}(\sim)$	3^{m-1}	3^{m-1}	$-2.3^{m-1}(\sim)$	3^{m-1}
3	3^{m-1}	3^{m-1}	3^{m-1}	3^m	3^{m-1}	$-2.3^{m-1}(\sim)$	3^{m-1}	3^{m-1}
4	3^m	-3^m	0	0	0	0	0	0
5	-3^{m-1}	-3^{m-1}	-3^{m-1}	$2.3^{m-1}(\sim)$	-3^{m-1}	-3^{m-1}	$2.3^{m-1}(\sim)$	-3^{m-1}

Keywords: Irreducible polynomials, Hansen Mullen Conjecture, Trace, Mobius Inversion Formula.

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The Pell, Modified Pell Identities via Orthogonal Projection

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ABSTRACT

For $n \geq 2$, the Pell numbers and modified Pell numbers are defined by the recurrence relations

$$P_n = 2P_{n-1} + P_{n-2}$$

with the initial condition $P_0 = 0,\ P_1 = 1$ and

$$q_n = 2q_{n-1} + q_{n-2} \tag{1}$$

with the initial condition $q_0 = 1,\ q_1 = 1$. The modified Pell numbers are connected with the Pell and Pell-Lucas numbers. These numbers have been studied in several papers (see [1,4]).

On the other hand in [2], the authors consider the two dimensional space of second order recurrence sequences $\Re(a,b)$ and give the properties of this space. In [3, 6], the authors obtain the orthogonal bases onto the space $\Re(k,1)$. Afterwards, they have the orthogonal projection matrix onto the space $\Re(k,1)$ which is Hankel matrix with the $k-$Fibonacci numbers.

In this paper, we consider the orthogonal bases onto the space $\Re(2,1)$. This orthogonal bases are connected with the modified Pell numbers, Pell numbers and roots of the characteristic equation of recurrence relation (1). For all even n, Using this orthogonal bases, we obtain the orthogonal projection matrix onto the $\Re(2,1)$ as follows

$$H_P = \frac{2}{P_n}\begin{pmatrix} P_{-n+1} & P_{-n+2} & \cdots & P_0 \\ P_{-n+2} & P_{-n+3} & \cdots & P_1 \\ \vdots & \vdots & \ddots & \vdots \\ P_0 & P_1 & \cdots & P_{n-1} \end{pmatrix}.$$

The H_P matrix is Hankel matrix with the Pell numbers. Similarly, taking the different orthogonal bases for all odd n, we have the orthogonal projection matrix associated with the Hankel matrix H_P. By using the orthogonal projection matrices, we obtain the identities for the Pell, Pell-Lucas and modified Pell numbers.

Key Words: Orthogonal projection, Pell numbers , Modified Pell numbers

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The Subalgebra Membership Problem for Free Poisson Algebras

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ABSTRACT

There are many important applications of Gröbner bases. One of them [3] gives the decidability of the subalgebra membership problem for polynomial algebras, i.e. let $k[X]=k[x_1, x_2,\dots,x_n]$ be the polynomial algebra in the variables $x_1, x_2,\dots,x_n$ over a constative field k. There exists an effective algorithm which for any finite sequence of elements $f,f_1,f_2,\dots,f_m \in k[X]$ determines whether f belongs to the subalgebra $< f_1,f_2,\dots,f_m>$ generated by elements $f_1,f_2,\dots,f_m$ or not. Another application of Gröbner bases (see, for example [2]) gives decidability of the ideal membership problem for k[X], i.e., there exists an effective algorithm which for any finite sequence of elements $f,f_1,f_2,\dots,f_m \in k[X]$ determines whether f belongs to the ideal $(f_1,f_2,\dots,f_m)$ generated by elements $f_1,f_2,\dots,f_m$ or not.

Generally, the ideal membership problem for free algebras is called the word problem for corresponding variety of algebras. The word problem is undecidable for associative algebras and Lie algebras [7].

The subalgebra membership problem is decidable for free algebra Nielsen-Schreier of algebras [5]. In particular, a Shirsov-Witt Theorem states that the subalgebra of free Lie algebras are free [1,4]. This result easily imply the decidability of the subalgebra membership problem for free Lie algebras. The subalgebra membership problem is algorithmically undecidable for free associative algebras and free Jordan algebras [8].

Using a method of interpreting the ideal membership problem from [8], we prove that the subalgebra membership problem is algorithmically undecidable for free Poisson algebras. Recall that the ideal membership problem for Lie algebra is undecidable [6], which implies the undecidability of the ideal membership problem for free Poisson algebras.

This is a joint work with U. Umirbaev.

Key Words: Free Poisson algebras, The ideal membership problem, The subalgebra membership problem.

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The Theory of Topdemir's Numbers

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ABSTRACT

In addition to the numbers of Fermant, Smith, Kaprekar, harshad, ... etc, which have been obtained with different disciplines, Topdemir Numbers were found in the result of the multiplication process of some numbers in the direction of a certain discipline.

Topdemir Numbers: Numbers of at least two digits are replaced by the first and last numbers (numbers) of the new number formed as a result of multiplication by itself, and if the sum of the numbers constituting this number is the sum of any number, these numbers are called "Topdemir Numbers".

ab.ba = cdc => c + d+ c = x2

ab. ba = cdecd => c + d + e + c + d = x2

abc.cba = cdefg…cd => c + d + e + f + g +…… + c + d = x2

Few examples:

11 . 11 = 121 => 1 + 2 + 1 = 4 = 2^2

12 . 21 = 252 => 2 + 5 + 2 = 9 = 3^2

22 . 22 = 484 => 4 + 8 + 4 = 16 = 4^2

59 . 95 = 565 => 5 + 6 + 5 = 16 = 4^2

101 . 101 = 10201 => 1 + 2 + 1 = 4 = 2^2

102 . 201 = 20502 => 2 + 5 + 2 = 9 = 3^2

Resolved in the form of; 11, 12. 22, 59, 101, 102, 104, 111, 112, 113, 121, 122, 131, 202, 203, 212, 405, ... are the number of Topdemir.

This study aims to make mathematics loveable, easy to understand and functional, while supporting causal relation to learning by contrasting techniques.

Key Words: TOPDEMIR'S Numbers, The Theory of Numbers, The Revolution of Numbers

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The Trace Functions on Mantaci Reutenauer Algebra

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ABSTRACT

Let (W_n, S_n) be a Coxeter system of type B_n and $\mathbb{C}Irr(W_n)$ be the algebra generated by all the irreducible characters of W_n. A family of orthogonal primitive idempotents of $\mathbb{C}Irr(W_n)$ is defined as $e_\lambda = \sum_{\lambda \in BP(n)} u_{\lambda\mu}\varphi_\mu$ and each of these orthogonal primitive idempotents is also a characteristic class function of the group W_n, where $BP(n)$ stands for the set of all double partitions of n. For every $\lambda \in BP(n)$, the orthogonal primitive idempotent e_λ is not an irreducible character of $\mathbb{C}Irr(W_n)$. If e_λ is extended by linearity to group algebra $\mathbb{C}W_n$, the action of e_λ on d_A is $e_\lambda(d_A) = \sum_{x \in D_A} e_\lambda(x) = |C_\lambda \cap D_A|$, where C_λ denotes the conjugate class of W_n correspondig to double partition λ of n. Furthermore, for every $\lambda \in BP(n)$ each characteristic class function e_λ is a trace function of Mantaci-Reutenauer algebra $MR(W_n)$, which is generated by the set $\{d_A : A \in SC(n)\}$, where $SC(n)$ denotes the collection of all signed composition of n. In addition, Mantaci-Reutenauer algebra $MR(W_n)$ contains Solomon's descent algebras of Coxeter groups of type A_n and B_n. As a vector space, the dimension of $MR(W_n)$ is $2\cdot 3^{n-1}$. Although the multiplication of basis elements of Mantaci-Reutenauer d_A and d_B is not commutative, the images of $d_A d_B$ and $d_B d_A$ under e_λ are equal to each other. For each $A \in SC(n)$, the unique linear map $\psi : MR(W_n) \to \mathbb{C}Irr(W_n)$, which is defined by the rule $\psi(d_A) = \mathrm{Ind}_{W_A}^{W_n} 1_A$, is a surjective morphism of algebras. $\mathrm{Ker}\psi$ is also the radical of the Mantaci-Reutenauer algebra $MR(W_n)$. Since $\mathrm{Ker}\psi$ includes non zero elements, then the Mantaci-Reutenauer algebra $MR(W_n)$ is not semisimple. Since the map ψ is surjective and e_λ for each $\lambda \in BP(n)$ is an orthogonal primitive idempotent of $\mathbb{C}Irr(W_n)$, there is an orthogonal primitive idempotent E_λ of Mantaci-Reutenauer algebra $MR(W_n)$ such that $\psi(E_\lambda) = e_\lambda$. The commutator subspace of Mantaci-Reutenauer algebra is $[MR(W_n), Mr(W_n)] = \mathrm{Ker}\psi$. Since the dimension of the trace function space of Mantaci-Reutenauer algebra is $|BP(n)|$, it exactly coincide with the algebra $\mathbb{C}Irr(W_n)$.

Key Words: Coxeter Group, Mantaci-Reutenauer Algebra, Orthogonal Primitive Idempotents, Trace Functions

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Type-2 Fuzzy Number And Its Applications To Probability Distributions

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ABSTRACT

Type-1 fuzzy sets (T1FSs) and their memberships are influential tools for reasoning under uncertainty and vagueness. However, the degrees of membership of T1FSs are crisp numbers. There is a growing interest in type-2 fuzzy sets to express uncertainty in T1FSs. Type-2 fuzzy set (T2FS) was initially given by Zadeh (1975) as an extension version of an ordinary fuzzy set, i.e., a T1FS. There are some studies on T2FSs [Hamrawi (2011), Karnik and Mendel (2001a), Karnik and Mendel (2001b)]. Also, there are many application areas of T2FSs [Tao et al. (2012), Turksen (2002), Wagenkneckt and Hartmann (1988)]. Some basic operations on T2FSs were studied by different researchers [Karnik and Mendel (2001a), Karnik and Mendel (2001b)]. Mathematical operations on type-2 fuzzy numbers (T2FNs) are necessary to make inference on T2FNs and related application areas. There are limited number of studies related to the operations on T2FNs (Kardan et. al., 2014). Besides, the mathematical operators given in the literature can be quite difficult to apply to T2FNs. For this reason, we propose several mathematical operators on T2FNs which can be applied quite easy. Also, we give some applications to probability distributions when some parameters of the distributions are defined as the type-2 fuzzy numbers.

Key Words: Type-2 fuzzy number, membership function, fuzzy probability distribution, type-2 fuzzy parameters.

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(α,β,ϒ) Interval Cut Set Of Interval Valued Neutrosophic Sets

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ABSTRACT

Neutrosophic sets (NS) proposed by Smarandache [1], is a powerful tool to deal with incomplete, indeterminant and inconsistent information in the real world. It is the generalization of the theory of fuzzy sets and intuitionistic fuzzy sets ([2], [3]). Neutrosophic sets are characterized by truth membership function (T), in derminancy function (I) and falsity membership function (F). This theory is very important in many application areas since in determinacy is quantified explicitly and the truth membership function, in determinacy membership function and falsity membership functions are independent. Wang.et al. [4], introduced the concept of single valued neutrosophic set. This can independently express truth membership degree, in determinacy membership degree and falsity membership degree which can deal with incomplete, indeterminate and inconsistent information very well. Single valued neutrosophic set has been developing rapidly due to its wide range of application areas. The Smaradanche's neutrosophic set theory has become a popular topic of investigation in the fuzzy and intuitionistic community. However, there is less investigation on the cut sets. In this paper, different types of interval cut-set of interval-valued neutrosophic sets (IVNSs), the complement of these cut-sets are defined. Some properties of those cut-set of IVNSs are investigated.

Key Words: Neutrosophic set, Interval Valued Neutrosophic Sets, Interval Cut Set.

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Δ_λ^m-Statistical Convergence with respect to a Modulus Function for Sequences of Fuzzy Numbers

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ABSTRACT

The difference sequence spaces were defined by Kızmaz [5] and have been studied many authors. On the other hand, Mursaleen [7] investigated the λ-statistical convergence for complex sequences. After then, Altinok et al [1] extended the concept of λ-statistical convergence using generalized difference sequences of fuzzy real numbers. Altinok et al [2] examined the relations between λ- and μ-statistical convergence, in case of $\lambda_n \le \mu_n$ in sequences of fuzzy real numbers. Subsequently, Cakan and Altin [3] defined $S_F(\Delta,f)$, the set of all Δ-statistically convergent by a modulus function sequences of fuzzy numbers.

In the present paper, we extend the notion of statistical convergence defined by a modulus function of sequences of fuzzy numbers using generalized difference operator Δ^m such that $\Delta^m X_k = \Delta^{m-1} X_k - \Delta^{m-1} X_{k+1}$ for (m=1,2,3,...) and nondecreasing sequence $\lambda = (\lambda_n)$ of positive real numbers such that $\lambda_{n+1} \le \lambda_n + 1$, $\lambda_1 = 1$, $\lambda_n \to \infty$ and give some inclusion relations by helping examples so as to fill up the existing gaps in the theory of generalized statistical convergence of fuzzy numbers.

Let f be a modulus function, $\lambda = (\lambda_n)$ be a nondecreasing sequence of positive real numbers such that $\lambda_{n+1} \le \lambda_n + 1$, $\lambda_1 = 1$, $\lambda_n \to \infty$ $(n \to \infty)$ and Δ^m be generalized difference operator. A sequence $X = (X_k)$ of fuzzy numbers is said to be Δ_λ^m-statistically convergent by f modulus function if for every ε>0 there is a fuzzy number $X_0 \in L(\mathbb{R})$ such that $\lim_{n\to\infty}(1/(\lambda_n))|\{k \le n: f\,[d(\Delta^m X_k, X_0)] \ge \varepsilon\}| = 0$, (m=1,2,3,...).
In this case we write $X_k \to X_0(S_F^\lambda(\Delta^m, f))$ or $S_F^\lambda(\Delta^m, f)\text{-}\lim X_k = X_0$. We shall use $S_F^\lambda(\Delta^m, f)$ to denote the set of all Δ_λ^m -statistically convergent by a modulus function sequences of fuzzy numbers.

Key Words: Sequence of fuzzy numbers, statistical convergence, modulus function, difference sequence.

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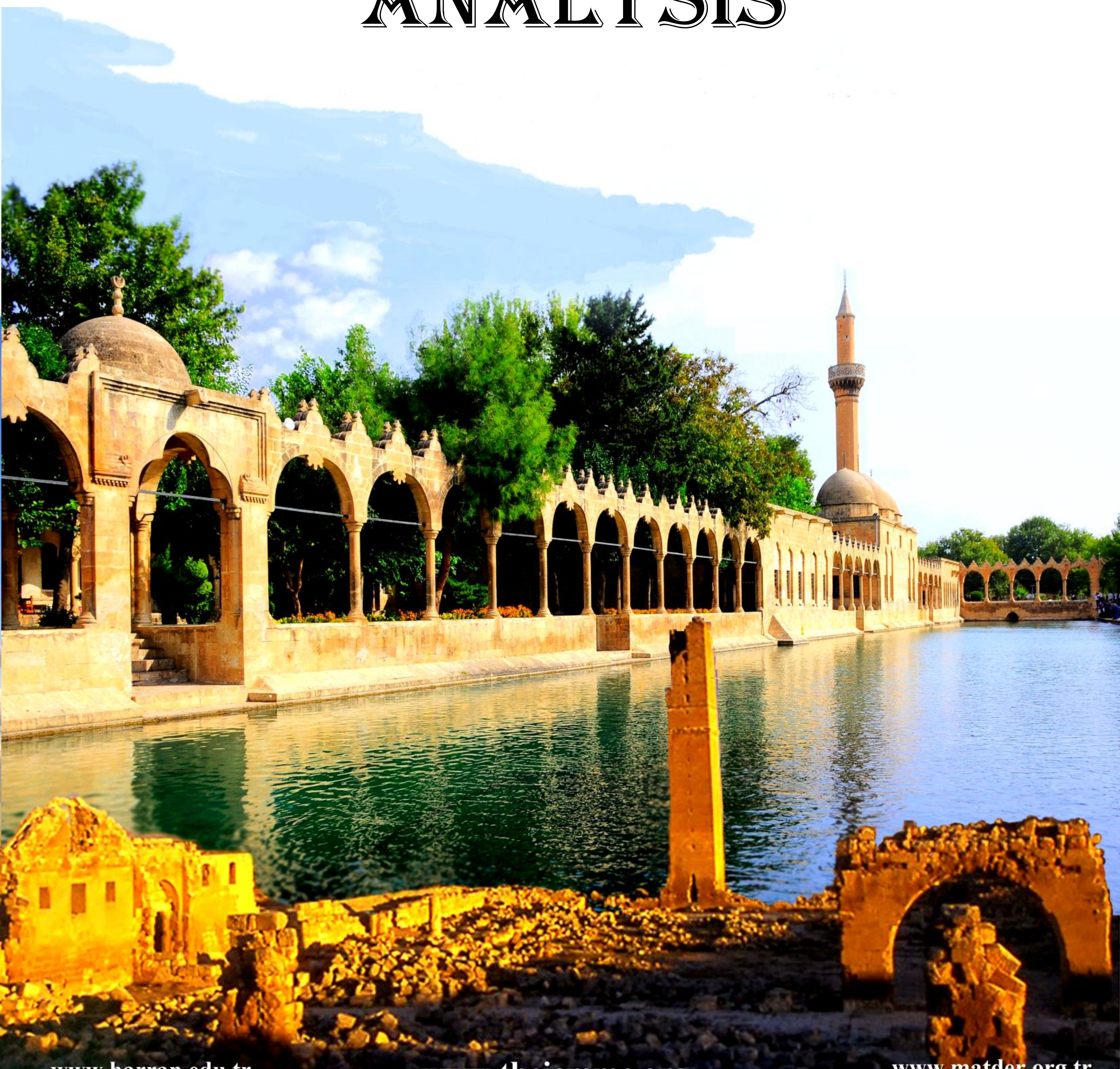
ANALYSIS



A Decomposition Formula For Bivariate Hypergeometric-Trigonometric Series

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ABSTRACT

In this work, a general identity is presented for bivariate hypergeometric-trigonometric series, which can be considered as a decomposition formula for the aforementioned series. Some special examples are also given in this sense.

We have recently introduced two bivariate series in [5] as

$$C_\alpha(f^*; r,\theta,m,n) = \sum_{k=0}^{\infty} a^*_{nk+m}\, r^k \cos(\alpha+k)\theta \,, \tag{3}$$

and

$$S_\alpha(f^*; r,\theta,m,n) = \sum_{k=0}^{\infty} a^*_{nk+m}\, r^k \sin(\alpha+k)\theta \,, \tag{4}$$

where r,θ are real variables, $\alpha \in \square$, $n \in \square$ and $m \in \{0,1,...,n-1\}$, and showed that they are convergent if the reduced series $\sum_{k=0}^{\infty} a^*_{nk+m}\, r^k$ is convergent.

Now, assume in (3) and (4) that $a^*_k = \frac{(a_1)_k (a_2)_k ... (a_p)_k}{(b_1)_k (b_2)_k ... (b_q)_k}$ are hypergeometric terms where $(r)_k = \prod_{j=0}^{k-1} (r+j)$ denotes the well-known Pochhammer symbol [1]. Then

$${}_pC_q\left(\begin{matrix} a_1,\, a_2,\, ... & a_p \\ b_1,\, b_2,\, ... & b_q \end{matrix}\middle| (r,\theta);\alpha\right) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k ... (a_p)_k}{(b_1)_k (b_2)_k ... (b_q)_k} \frac{r^k}{k!} \cos(\alpha+k)\theta, \tag{5}$$

and

$$ {}_pS_q\left(\begin{matrix} a_1,\, a_2,\, \dots & a_p \\ b_1,\, b_2,\, \dots & b_q \end{matrix}\middle| (r,\theta);\alpha\right) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_p)_k}{(b_1)_k (b_2)_k \dots (b_q)_k} \frac{r^k}{k!} \sin(\alpha + k)\theta, $$

(6)

are called bivariate hypergeometric-trigonometric series [6]. It is clear from (5) and (6) that

$$ \begin{aligned} {}_pC_q\left(\begin{matrix} a_1,\, a_2,\, \dots & a_p \\ b_1,\, b_2,\, \dots & b_q \end{matrix}\middle| (r,\theta);\alpha\right) &= \cos\alpha\theta\, {}_pC_q\left(\begin{matrix} a_1,\, a_2,\, \dots & a_p \\ b_1,\, b_2,\, \dots & b_q \end{matrix}\middle| (r,\theta);0\right) \\ &\quad - \sin\alpha\theta\, {}_pS_q\left(\begin{matrix} a_1,\, a_2,\, \dots & a_p \\ b_1,\, b_2,\, \dots & b_q \end{matrix}\middle| (r,\theta);0\right), \end{aligned} $$

(7)

and

$$ \begin{aligned} {}_pS_q\left(\begin{matrix} a_1,\, a_2,\, \dots & a_p \\ b_1,\, b_2,\, \dots & b_q \end{matrix}\middle| (r,\theta);\alpha\right) &= \sin\alpha\theta\, {}_pC_q\left(\begin{matrix} a_1,\, a_2,\, \dots & a_p \\ b_1,\, b_2,\, \dots & b_q \end{matrix}\middle| (r,\theta);0\right) \\ &\quad + \cos\alpha\theta\, {}_pS_q\left(\begin{matrix} a_1,\, a_2,\, \dots & a_p \\ b_1,\, b_2,\, \dots & b_q \end{matrix}\middle| (r,\theta);0\right). \end{aligned} $$

(8)

Many ordinary hypergeometric series (when $\theta = 0$) and Fourier trigonometric series (when r is fixed and pre-assigned) can be represented in terms of the series (5) or (6). See also [2,7].

A general identity for bivariate hypergeometric-trigonometric series

First, for any arbitrary series that we clearly have

$$ \sum_{k=0}^{\infty} u_k = \sum_{j=0}^{\infty} u_{2j} + \sum_{j=0}^{\infty} u_{2j+1} = \sum_{j=0}^{\infty} u_{3j} + \sum_{j=0}^{\infty} u_{3j+1} + \sum_{j=0}^{\infty} u_{3j+2} = \dots = \sum_{j=0}^{\infty} u_{mj} + \sum_{j=0}^{\infty} u_{mj+1} + \dots + \sum_{j=0}^{\infty} u_{mj+m-1}, $$

(9)

where m is a natural number. By recalling the Pochhammer symbol and noting the series (5) and (6), if

$$u_k = \frac{(a_1)_k (a_2)_k ... (a_p)_k}{(b_1)_k (b_2)_k ... (b_q)_k (1)_k} r^k \begin{Bmatrix} \cos(\alpha + k)\theta \\ \sin(\alpha + k)\theta \end{Bmatrix},$$

is substituted in the last equality of (9), then we have

$$\begin{aligned} {}_p\begin{Bmatrix} C \\ S \end{Bmatrix}_q \left(\begin{matrix} a_1, a_2, ... & a_p \\ b_1, b_2, ... & b_q \end{matrix} \middle| (r,\theta);\alpha \right) &= \sum_{j=0}^{\infty} \frac{(a_1)_{mj}(a_2)_{mj}...(a_p)_{mj}}{(b_1)_{mj}(b_2)_{mj}...(b_q)_{mj}(1)_{mj}} r^{mj} \begin{Bmatrix} \cos(\alpha + mj)\theta \\ \sin(\alpha + mj)\theta \end{Bmatrix} \\ &+ \sum_{j=0}^{\infty} \frac{(a_1)_{mj+1}(a_2)_{mj+1}...(a_p)_{mj+1}}{(b_1)_{mj+1}(b_2)_{mj+1}...(b_q)_{mj+1}(1)_{mj+1}} r^{mj+1} \begin{Bmatrix} \cos(\alpha + mj+1)\theta \\ \sin(\alpha + mj+1)\theta \end{Bmatrix} \\ &+ ... + \sum_{j=0}^{\infty} \frac{(a_1)_{mj+m-1}...(a_p)_{mj+m-1}}{(b_1)_{mj+m-1}...(b_q)_{mj+m-1}(1)_{mj+m-1}} r^{mj+m-1} \begin{Bmatrix} \cos(\alpha + mj+m-1)\theta \\ \sin(\alpha + mj+m-1)\theta \end{Bmatrix}. \end{aligned}$$

(10)

On the other hand, since the two following identities hold

$$(a)_{mk} = m^{mk} \prod_{j=0}^{m-1} (\frac{a+j}{m})_k,$$

and

$$(a)_{mj+i} = (a+i)_{mj} (a)_i,$$

relation (10) can be re-written as

$$\begin{aligned} {}_p\begin{Bmatrix} C \\ S \end{Bmatrix}_q \left(\begin{matrix} a_1, a_2, ... & a_p \\ b_1, b_2, ... & b_q \end{matrix} \middle| (r,\theta);\alpha \right) &= \sum_{j=0}^{\infty} \frac{\prod_{r=0}^{m-1}(\frac{a_1+r}{m})_j ... \prod_{r=0}^{m-1}(\frac{a_p+r}{m})_j}{\prod_{r=0}^{m-1}(\frac{b_1+r}{m})_j ... \prod_{r=0}^{m-1}(\frac{b_q+r}{m})_j \prod_{r=0}^{m-1}(\frac{1+r}{m})_j} \left(m^{(p-q-1)m} r^m\right)^j \begin{Bmatrix} \cos(\frac{\alpha}{m}+j)(m\theta) \\ \sin(\frac{\alpha}{m}+j)(m\theta) \end{Bmatrix} \\ &+ \frac{(a_1)_1(a_2)_1...(a_p)_1}{(b_1)_1(b_2)_1...(b_q)_1} \frac{r}{(1)_1} \sum_{j=0}^{\infty} \frac{\prod_{r=0}^{m-1}(\frac{a_1+1+r}{m})_j ... \prod_{r=0}^{m-1}(\frac{a_p+1+r}{m})_j}{\prod_{r=0}^{m-1}(\frac{b_1+1+r}{m})_j ... \prod_{r=0}^{m-1}(\frac{b_q+1+r}{m})_j \prod_{r=0}^{m-1}(\frac{2+r}{m})_j} \left(m^{(p-q-1)m} r^m\right)^j \begin{Bmatrix} \cos(\frac{\alpha+1}{m}+j)(m\theta) \\ \sin(\frac{\alpha+1}{m}+j)(m\theta) \end{Bmatrix} + ... \\ &+ \frac{(a_1)_{m-1}(a_2)_{m-1}...(a_p)_{m-1}}{(b_1)_{m-1}(b_2)_{m-1}...(b_q)_{m-1}} \frac{r^{m-1}}{(1)_{m-1}} \sum_{j=0}^{\infty} \frac{\prod_{r=0}^{m-1}(\frac{a_1+m-1+r}{m})_j ... \prod_{r=0}^{m-1}(\frac{a_p+m-1+r}{m})_j \left(m^{(p-q-1)m} r^m\right)^j}{\prod_{r=0}^{m-1}(\frac{b_1+m-1+r}{m})_j ... \prod_{r=0}^{m-1}(\frac{b_q+m-1+r}{m})_j \prod_{r=0}^{m-1}(\frac{m-1+r}{m})_j} \begin{Bmatrix} \cos(\frac{\alpha+m-1}{m}+j)(m\theta) \\ \sin(\frac{\alpha+m-1}{m}+j)(m\theta) \end{Bmatrix}, \end{aligned}$$

which eventually leads to the main theorem.

Theorem. For any natural number m, the two series (5) and (6) satisfy the relation

$$
{}_p\begin{Bmatrix} C \\ S \end{Bmatrix}_q \left(\begin{matrix} a_1, \, a_2, \, \ldots & a_p \\ b_1, \, b_2, \, \ldots & b_q \end{matrix} \middle| (r,\theta);\alpha \right)
$$

$$
= \sum_{k=0}^{m-1} \frac{(a_1)_k \ldots (a_p)_k}{(b_1)_k \ldots (b_q)_k} \frac{r^k}{k!} \, {}_{(mp+1)}\begin{Bmatrix} C \\ S \end{Bmatrix}_{(mq+m)} \left(\begin{matrix} \vec{A}_{1,k}, \, \vec{A}_{2,k}, \, \ldots & \vec{A}_{p,k}, \, 1 \\ \vec{B}_{1,k}, \, \vec{B}_{2,k}, \, \ldots & \vec{B}_{q,k}, \vec{I}_{1,k} \end{matrix} \middle| (m^{(p-q-1)m} r^m, m\theta); \frac{\alpha+k}{m} \right),
$$

(11)

where

$$
\vec{A}_{j,k} = \left(\frac{a_j+k}{m}, \frac{a_j+1+k}{m}, \ldots, \frac{a_j+m-1+k}{m}\right) \quad (j=1,2,\ldots,p),
$$

$$
\vec{B}_{j,k} = \left(\frac{b_j+k}{m}, \frac{b_j+1+k}{m}, \ldots, \frac{b_j+m-1+k}{m}\right) \quad (j=1,2,\ldots,q),
$$

and

$$
\vec{I}_{1,k} = \left(\frac{1+k}{m}, \frac{2+k}{m}, \ldots, \frac{m+k}{m}\right).
$$

This theorem can be interpreted as a decomposition formula for many hypergeometric-trigonometric series of type (5) and (6).

Key Words: Bivariate hypergeometric-trigonometric series, Fourier trigonometric series, decomposition formulae.

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A new Generalized Fractional Derivatives and Fractional Integral

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ABSTRACT

In recent years, fractional calculus has led to significant improvements in many areas. Based on the wide applications in engineering and sciences such as physics, mechanics, chemistry, and biology, research on fractional ordinary or partial differential equations and other relative topics is active and extensive around the world.

In the past few years, the increase of the subject is witnessed by hundreds of research papers, several monographs, and many international conferences.

For the history and main results on fractional derivatives and fractional integrals, we refer the reader to [1, 6, 7].

The main aim of this work is to introduced a new general definition of fractional derivative and fractional integral, which depends on an unknown kernel. By using these definitions, we obtain the basic properties of fractional integral and fractional derivative such as Product Rule, Quotient Rule, Chain Rule, Roll's Theorem, Mean Value Theorem, Integration by parts and Inverse property.

In this work, we introduce a new fractional derivative which is generalized the results obtained in [2-5].

The limit definition of the derivative,

$$D^{\alpha}(f)(t) = \lim_{\varepsilon \to 0} \frac{f\left(t - k(t) + k(t)e^{\varepsilon \frac{k(t)^{-\alpha}}{k'(t)}}\right) - f(t)}{\varepsilon},$$

is the most natural generalization that uses the limit approach.

We have defined a new generalized fractional integral which is inspired by the definition of derivative.

The α −fractional integral of f is defined by,

$$I^{\alpha}(f)(t) = \int_{a}^{t} \frac{k'(x)f(x)}{k(x)^{1-\alpha}}\,dx.$$

Key Words: Fractional Integral, Fractional Derivative, Riemann-Liouville Fractional integral, Classical Calculus, Fractional Differential Equations.

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A New Gradient Projection Algorithm for Convex Minimization Problem

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ABSTRACT

Consider convex minimization problem:

$$\min_{x \in C} f(x) \tag{1}$$

where $f : C \to \mathbb{R}$ is a convex mapping, C is a closed and convex subset of a Hilbert space H. Let P_C be a projection from H onto C and f be Frechet differantiable. It is known that a minimum of $f(x)$ in C is a fixed point of $P_C(I - \lambda \nabla f)$, that is $x^* = P_C(x^* - \lambda \nabla f(x^*))$. This relation between x^* and $P_C(I - \lambda \nabla f)$ has developed idea of approximating a solution of (1) through iteration which is often used to approach fixed point of a map in the fixed point theory. Here, this approach named gradient projection algorithm (shortly GPA) is a powerful tool to solve this minimization problem (1). GPA starts with any point of C and generates iteration with $\{x_n\}_{n=0}^{\infty}$ as the follows:

$$x_{n+1} = P_C(x_n - \lambda \nabla f(x_n)) \tag{2}$$

where $\lambda > 0$ is a stepsize, P_C is a projection from H onto C and ∇f is gradient of f. The iteration (2) can be generalized as

$$x_{n+1} = P_C(x_n - \lambda_n \nabla f(x_n)). \tag{3}$$

Levitin and Polyak [1] gave in the following theorem related with weak convergece of (2) to a solution of (1):

Theorem: Assume that problem (1) is is solvable and the gradient ∇f satisfies the Lipschitz condition. Let $\{\lambda_n\}_{n=0}^{\infty}$ satisfy $0 < \lim_{n\to\infty} \inf \lambda_n < \lim_{n\to\infty} \sup \lambda_n < \frac{2}{L}$. Then (3) converges weakly to a minimizer of (1).

In 2011, Xu gived an alternative operator-oriented approach to the (3) named an averaged mapping approach. Xu's this approach has attracted many researcher (see [3-6]). He showed that $P_C(I - \lambda \nabla f)$ can be written as

$$P_C(I - \lambda \nabla f) = \frac{2 - \lambda L}{4} I + \frac{2 + \lambda L}{4} T = (1 - \beta) I + \beta T$$

where T is nonexpansive, ∇f is L-Lipschitz map, $0 < a \le \lambda \le b < \frac{2}{L}$.

The focus of this presentation is the weak convergence analysis of a new the projected gradient algorithm using Xu's approach for solving constrained convex minimization problems in the Hilbert spaces. We give an example to illustrate our result. Also, we apply it to solving the split feasibility problem.

Key Words: Gradient projection algorithm, convex optimization problem, fixed point.

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A Note on Approximation of Nonlinear Singular Integral Operators

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ABSTRACT

In [7] and [8], Taberski investigated the singular integrals of the convolution form $I(.;\lambda, f) = f * K(.;\lambda)$ for $f \in C_{2\pi}$ or $f \in L^p_{2\pi}$, $p \geq 1$ where $\lambda \in \Lambda \subset \mathbb{R}$, $\lambda \to \lambda_0$ (λ_0 is the accumulation point of set E) and $\{K(.;\lambda)\}_{\lambda \in \Lambda}$ is the kernel of the integral I satisfies appropriate conditions. He analysed the pointwise convergence of integrable functions and the order of convergence of these integrals with respect to norms in the spaces $C_{2\pi}$ and $L^p_{2\pi}$. In [7] Taberski generalized classical theorems given by Romanovskii and Faddeev. These studies were extended by some authors for example S. Siudut [5] and Rydzewska [4]. In 1981 [3], Musielak investigated the approximation of convolution type nonlinear integral operators therefore he constructed the linear concept to cover the case of nonlinear integral operators. He extended the singularity assumption via replacing the linearity property of the integral operators by an assumption of Lipschitz condition for the kernel of the nonlinear integral operators

After this work some papers have written that are devoted to the research of the role played by the nonlinear integral operators in approximation theory, for example the studies of Swiderski and Wachnicki [6] and Bardaro and Vinti [1], [2].

It is our goal in this paper to give some theorems on pointwise approximation of nonlinear double singular integrals in the space of Lebesgue integrable functions.

Key Words: Nonlinear singular integrals, pointwise convergence, Lebesgue points

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A Novel Layout For Almost Convergent Sequence Spaces

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ABSTRACT

The tendency to build a new sequence spaces by means of the matrix domain of a particular limitation method has recently been used by several authors. Two of them are Fibonacci Matrix and generalized weighted means. Firstly, let us recall them.

The famous Fibonacci sequence is obtained by the recursive formula, for $n \geq 2$,

$$f_n = f_{n-1} + f_{n-2},$$

where $f_0 = f_1 = 1$. Kara [2] defined Fibonacci difference matrix $F = (f_{nk})$ by means of the famous Fibonacci sequence as

$$f_{nk} = \begin{cases} -\dfrac{f_{n+1}}{f_n}, & k = n-1, \\ \dfrac{f_n}{f_{n+1}}, & k = n, \\ 0, & 0 \leq k < n-1 \ or \ k > n, \end{cases}$$

for each $n, k \in \mathbb{N}$ and introduced the Fibonacci difference sequence spaces $l_p(F)$ and $l_\infty(F)$, for $1 \leq p < \infty$. Başarır et al. [1] introduced the sequence spaces $l_p(F)$, $c_0(F)$ and $c(F)$ and established some inclusion relations.

The other matrix as known generalized weighted mean is as follows:

Let U be the set of all sequences $u = (u_k)$ such that $u_k \neq 0$ for all $k \in \mathbb{N}$. For $u \in U$; let $\frac{1}{u} = \left(\frac{1}{u_k}\right)$. Let $u, v \in U$ and define the matrix $G(u,v) = (g_{nk})$ by

$$g_{nk} = \begin{cases} u_n v_k, & k < n \\ u_n v_n, & k = n \\ 0, & k > n \end{cases}$$

for all $k \in \mathbb{N}$, where u_n depends only on n and v_k only on k. The matrix $G(u,v)$ defined above is called as generalized weighted mean or factorable matrix.

Some of studies done by using the aforesaid matrix are: The sets $f(G)$ and $f_0(G)$ which are derived by the generalized weighted mean have recently been studied in [6], the spaces $f(u,v,\Delta)$, $f_0(u,v,\Delta)$ and $fs(u,v,\Delta)$ were presented in [8]. Also see [3], [4], [5], [7].

The aim of the present study is to introduce the sequence spaces $\hat{c}(G,F)$, $\hat{c_0}(G,F)$ and $\widehat{cs}(G,F)$, where G is generalized weighted means and F is a Fibonacci matrix. We describe α −and β −duals of the spaces $\hat{c}(G,F)$ and $\widehat{cs}(G,F)$. Further, we characterize the infinite matrices $(\hat{c}(G,F):\mu)$ and $(\mu:\hat{c}(G,F))$, where μ is an arbitrary sequence space.

Key Words: Fibonacci numbers, difference matrix, sequence spaces.

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A Study on Nevanlinna Theory and Its Applications

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ABSTRACT

Nevanlinna theory (also called value distribution theory) is a very useful theory which was devised by the Finnish Mathematician, Rolf Nevanlinna in 1920. Rolf Nevanlinna, in his works between 1920 and 1925, added "Nevanlinna Theory", which is called by his name, to the theory of functions. The development of this theory which has been very rapid and has reached dimensions that can not be followed. Recently, many researchers have quite a few studies in different branches of mathematics such as projective geometry, applied mathematics and complex analysis in this sense. This theory deals with meromorphic functions, that are analytic in a region B, except for poles in this region and it plays a very important role in value distribution theory [1]. In addition, this theory studies the notations with related to poles and zeros of meromorphic functions and the distribution of roots of the equation f(z)=a, where f is entire or meromorphic function and a is any complex number.. After giving a brief review and some fundamental facts such as Poisson-Jensen formula, positive logarithmic, characteristic of meromorphic functions and first and second main theorems about Nevanlinna theory and the related topics, we will give some concepts such as proximate order, finite logarithmic order and finite double logarithmic order of functions and the relations between them [2]. Our main purpose in this study is to investigate some relations between finite logarithmic order and related topics such as an integral criterion for a non negative increasing function to be of finite logarithmic and double logarithmic order in a finite order in the sense of Nevanlinna theory and its applications in the content of the papers in [2] and [5].

Key Words: Meromorphic function, Value distribution, Proximate order, Finite logarithmic order and finite double logarithmic order, Poisson Jensen formula.

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An Introduction to Generated Measure Theory

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ABSTRACT

In 1972 Grossman and Katz established a new calculus, called non-Newtonian calculus, thought to be an alternative to the classical calculus [1]. They created this calculus by the help of functions called generators. As a special case by taking generator as exponential function, they formed multiplicative calculus which is also called geometric calculus. Quite a few authors studied on the multiplicative calculus. Bashirov, Kurpınar, Özyapıcı restated some classical properties of derivatives and integrals in the realm of multiplicative calculus [2]. In the non-Newtonian calculus only increasing generators are chosen. In this study we chose not only increasing but also decreasing ones. Moreover, we claimed that generators need not to be monotone. By this point of view we tried to investigate the building blocks of non-Newtonian calculus which are systems called α-arithmetic in a deeper perspective and tried to enrich them with examples. After that we present some principal results of measure theory in the non-Newtonian sense however, we used term generated instead of non-Newtonian due to choice of generators. We defined generated measure and generated outer measure also, some properties of these measures are mentioned. Generated Lebesgue measure is defined and the concept of integrability is scrutinized. Besides, Levi's Theorem, Fatou's Lemma and the Lebesgue Dominated Convergence Theorem, which have important roles in integral theory, are interpreted according to these new integral. Furthermore, generated Riemann integral is defined and finally the relation between generated Riemann integral and generated Lebesgue integral is examined. While creating a new type of measure and integral theory, we benefited from the book of Reel Analysis written by Aliprantis and Burkinshaw [3] and Handbook of Measure Theory compiled by Endre Pap [4].

Key Words: Generated measure, generated Lebesgue integral, α-arithmetic.

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Approximation To Generalized Derivatives By Integral Operator Families

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ABSTRACT

Theory of functions and differentials of positive integral operators. We can come across in many problems of the theory of equations. Fourier some collection methods of the series are expressed by such integrals. Furthermore, the limit value for the solution of the Dirichlet problem and the heat equations the solution of the problem is given in the form of positive core integrals.

Hence, we use generalized operators examination of the problem of approach to derivatives is both theoretical and practicalls of great importance. The more general problem than the problem of finding the approach speed, asymptotic value. Integral operator's family Asymptotic value of the x-point approach to f function mathematicians Convolute type problems such as Weierstrasse, Gauss, Perron, Landau, Picard, Lebesgue, Faddeev, Romanovsky, Natanson, Korovkin, Butzer Have been studied by mathematicians. In the mentioned studies approach to the derivation of a f function from a n th order to a point The problem of speed is investigated Our approach to Taylor derivatives generalized by integral operator families we will examine

Theorem

Let $f(x)$ differentiable function meaning of Taylor. And let integral operator families,

$$L_\lambda(f;x_0)=\int_{-\infty}^{\infty} f(x_0+t)K_\lambda(t)dt$$

$$\Delta_\lambda = \int_0^\infty \frac{t^4}{2} K_\lambda(t)dt \to 0 \quad , \quad (\lambda \to \infty)$$

At the same time kernel function and with above condition provide following conditions:

i) $K_\lambda(t) > 0 \quad , \quad \lambda \geq 0$

ii) $K_\lambda(-t) = K_\lambda(t)$

iii) $\int_{-\infty}^{\infty} K_\lambda(t)dt = 1$

Be on the point of being $\int_{-\infty}^{\infty} K_\lambda(t)dt = 2\int_0^\infty K_\lambda(t)dt$

$$\lim_{\lambda \to \infty} \frac{L_\lambda(f, x_0) - f(x_0)}{\Delta_\lambda} = f^{(2)}(x_0)$$

The state of equation is right.

Key Words: Taylor derivative, Kernel function, Differentiable function, Operator theory.

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Approximation and Its Several kinds on 2-Structures

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ABSTRACT

In the year 1965, Gahler Siegfried in his sixties, introduced the concept of 2-normed spaces and he had many important properties and examples for these spaces. From Gahler's time to the present, this topic has been extensively studied and developed by many different mathematicians by different view and they have dealt mainly with some 2-structures involving 2-normed spaces, generalized 2-normed sets, 2-Banach spaces and other related areas. Approximation is an old and also an important notion in mathematical analysis and it has many applications in many areas, especially in engineering. For example, recent investigations including approximations in uniformly convex linear 2-normed spaces by Sivadasan et al. [5], approximation theory in 2-Banach spaces by Gürdal et al. [4], best approximation in sets in linear 2-normed spaces by Elumalai et al. [1], best simultaneous approximation in linear 2-normed spaces by Elumalai and et al. [3], best approximation in real linear 2-normed spaces by Vijayaragavan et al., have been widely-investigated. Acikgoz also considered the best simultaneous approximation in linear 2-normed spaces. Acikgoz et al., derived types of approximation in the sense of 2-structures. After giving a brief survey on the 2-structures, our first purpose of in this talk is to give some relations between the notions of best and 2-best approximation, best simultaneous approximation, ε-approximation, ε-best approximation and set type of approximation in the content of linear 2-normed spaces.

Key Words: Best approximation in real linear 2-normed spaces, 2-Banach spaces, approximation theory in 2-Banach spaces.

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Approximation By Composition Of Q-Szasz-Mirakyan And Q-Durrmeyer-Chlodowsky Operators

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ABSTRACT

Approximation theory has been used in the theory of approximation of continuous functions by means of sequences of positive linear operators and still remains as a very active area of research. Since Korovkins famous theorem in 1950, the study of the linear methods of approximation given by sequences of positive and linear operators has became a firmly entrenched part of approximation theory. Due to the importance of polynomials, a variety of their generalizations and related topics have been studied (see, e.g. [1]-[6]). An intensive research has been conducted on polynomials and operators based on q-integers, see [1], [2]. During the last two decades, the applications of q −calculus have emerged as a new area in the field of approximation theory. The first q −analogue of the well-known Bernstein polynomials was introduced by Lupaş by applying the idea of q−integers. Since approximation studied by q −Bernstein polynomials is better than classical one under convenient choice of q, many authors introduced q-generalization of various operators and investigated several approximation properties. In recent years, the q−Bernstein polynomials, introduced by Phillips, have attracted a great deal of interest because of their potential applications in approximation theory and numerical analysis, and many properties of these polynomials have been discovered. While for $q=1$ these polynomials coincide with the classical ones, for $q\neq1$ we obtain new polynomials possesing interesting properties. Recently, Gupta and Heping introduced and studied the q-analogues of usual and discretly defined Durrmeyer operators.The

literature stuied the local approximation and the Voronovskaja type theorem. for the q-Szasz-Mirakyan operators, the rate of the convergence, the weighted of Korovkin-type theorem and the weighted approximation were given.

In the present paper, we introduce q-Szasz-Mirakyan operators by taking the weight function of q-Chlodowsky-Durrmeyer operators on $C[0,\infty)$ and investigate their approximation properties. We give weighted approximation theorem. We also establish the rate of convergence for q-Szasz-Mirakyan operators.

Key Words: q-calculus, q-Szas-Durrmeyer operators, q-Durrmeyer-Cholowsky operators, Korovkin approximation.

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Approximation by Double Cesàro Submethods of Double Fourier Series for Lipschitz Functions

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ABSTRACT

In this paper, we study the rate of uniform approximation by double submethods of the rectangular partial sums of the double Fourier series of a function $f(\mathrm{x},\mathrm{y})$ belonging the class Lip_α, $0 \le \alpha \le 1$, on the two-dimensional torus $-\pi < x, y \le \pi$. As a special case we obtain the rate of uniform approximation by double submethods.

Let the index sequences $\lambda(n)$ and $\mu(m)$ be strictly increasing single valued sequences of positive integers and $x = (x_{nm})$ be a double sequence. Then the Cesàro submethod $C_{\lambda,\mu} := (C_{\lambda,\mu},1,1)$ is defined to be

$$\left(C_{\lambda,\mu}x\right)_{nm} = \frac{1}{\lambda(n)\mu(m)} \sum_{j=1,k=1}^{\lambda(n),\mu(m)} x_{jk}$$

where $\sum_{j=1}^{\lambda(n)} \sum_{k=1}^{\mu(m)} x_{jk} = \sum_{k=1}^{\mu(m)} \sum_{j=1}^{\lambda(n)} x_{jk}$. Since $\left\{\left(C_{\lambda,\mu}x\right)_{nm}\right\}$ is a subsequence of $\left\{\left(C_1 x\right)_{nm}\right\}$, then the method $C_{\lambda,\mu}$ is RH-regular for any λ and μ [3]. Motivating by Moricz and Rhoades [2], we make the following definition:

Let $\left\{p_{jk} : \mathrm{j},\mathrm{k} = 0,1,...\right\}$ be a double sequence of nonnegative numbers, $p_{00} > 0$. The $C_\lambda^\mu N_{nm}$-submethod is defined as

$$C_\lambda^\mu N_{nm} = \frac{1}{P_{\lambda(n)\mu(m)}} \sum_{j=0}^{\lambda(\mathrm{n})} \sum_{k=0}^{\mu(\mathrm{m})} p_{\lambda(\mathrm{n})-j,\mu(\mathrm{m})-k} x_{jk} \qquad \left(n,m = 0,1,...\right),$$

where $\left\{x_{jk} : j,k = 0,1,...\right\}$ is a double sequence of real or complex numbers and

$$P_{\beta(n,m)}=\sum_{j=1}^{\lambda(n)}\sum_{k=1}^{\mu(m)}p_{jk}\qquad (n,m=0,1,\ldots).$$

Let $f(x,y)$ be a complex valued function defined on the two-dimensional real torus $\Gamma:-\pi<x\le\pi,-\pi<y\le\pi.$ If $f\in L^1(\Gamma)$, then its double series is

$$f(x,y)\approx\sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}c_{jk}e^{i(jx+ky)}\tag{1}$$

where

$$c_{jk}=\frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}f(u,v)e^{-i(ju+kv)}dudv\qquad (j,k=\ldots,-1,0,1,\ldots).$$

We associate with (1) the double sequence of (symmetric) rectangular partial sums

$$s_{nm}(x,y)=\sum_{j=-m}^{m}\sum_{k=-n}^{n}c_{jk}e^{i(jx+ky)}\qquad (n,m=0,1,\ldots).$$

Now, the $C_\lambda^\mu N_{nm}$ - means for (1) are defined as those for the sequence $\{s_{nm}(x,y)\}$:

$$C_\lambda^\mu N_{nm}(x,y)=\frac{1}{P_{\lambda(n)\mu(m)}}\sum_{j=0}^{\lambda(n)}\sum_{k=0}^{\mu(m)}p_{\lambda(n)-j,\mu(m)-k}s_{jk}(x,y)\qquad (n,m=0,1,\ldots).$$

The representation

$$C_\lambda^\mu N_{nm}(x,y)=\frac{1}{\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}f(x+u,y+v)C_\lambda^\mu K_{\lambda(n)\mu(m)}(u,v)dudv$$

plays a central role, where the $C_\lambda^\mu N_{nm}$ -kernel $C_\lambda^\mu K_{\beta(n,m)}(u,v)$ is defined by

$$C_\lambda^\mu K_{\lambda(n)\mu(m)}(u,v)=\frac{1}{P_{\lambda(n)\mu(m)}}\sum_{j=0}^{\lambda(n)}\sum_{k=0}^{\mu(m)}p_{\lambda(n)-j,\mu(m)-k}D_j(u)D_k(v)\qquad (n,m=0,1,\ldots),$$

where $D_j(u)$ and $D_k(v)$ are the Dirichlet kernels in terms of u and v, respectively.

Key Words: Double Fourier Series, Double Cesàro submethods, Nörlund Means

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Approximation by modified Bernstein-Chlodowsky operators on weighted spaces

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ABSTRACT

There are many original constructions of sequences of linear positive operators, satisfying the conditions of Korovkin's theorem and therefore converging to continuous functions on a finite interval. Many examples can be given in this regard.

In 1912, for a function $f \in C[0,1]$, Russian mathematician Sergei N. Bernstein [4] defined the following sequences of operators,

$$B_n(f;x) = \sum_{k=0}^{n} \binom{n}{k} (x)^k (1-x)^{n-k} f\left(\frac{k}{n}\right)$$

Bernstein operators are generalized, modified a studied by many author.

In 1937 Chlodowsky generalized Bernstein polynomials for functions on an unbounded interval as following;

$$B_n(f;x) = \sum_{k=0}^{n} \binom{n}{k} \left(\frac{x}{b_n}\right)^k \left(1-\frac{x}{b_n}\right)^{n-k} f\left(\frac{k}{n} b_n\right)$$

where $\{b_n\}$ is a positive increasing sequence with the conditions

$$\lim_{n\to\infty} b_n = \infty, , \qquad \lim_{n\to\infty} \frac{b_n}{n} = 0 .$$

Bernstein-Chlodowsky operators are studied by many authors.

In 2011, Sahai [3] defined primal and dual variant of Bernstein polynomials, respectively as;

$$B_n^P(f;x) = \sum_{k=0}^{n} \binom{n}{k} \left(\frac{1+x}{2}\right)^k \left(\frac{1-x}{2}\right)^{n-k} f\left(\frac{k}{n}\right)$$

and

$$B_n^D(f;x) = \sum_{k=0}^{n} \binom{n}{k} \left(\frac{1-x}{2}\right)^k \left(\frac{1+x}{2}\right)^{n-k} f\left(\frac{k}{n}\right)$$

In 2012 Cilo [1] for a function $f \in C[-1,1]$ studied the following operators for her master thesis;

$$C_n(f;x) = \frac{1}{2^n} \sum_{k=0}^{n} \binom{n}{k} (1+x)^k (1-x)^{n-k} f\left(\left(\frac{2k}{n} - 1\right)\right)$$

In this paper, the weighted approximation and the order of approximation of continuous functions by Bernstein-Chlodowsky polynomials and their generalizations are studied.

$$H_n(f;x) = \frac{1}{2^n} \sum_{k=0}^{n} \binom{n}{k} \left(1 + \frac{x}{b_n}\right)^k \left(1 - \frac{x}{b_n}\right)^{n-k} f\left(\left(\frac{2k}{n} - 1\right) b_n\right)$$

where $\{b_n\}$ is a positive increasing sequence with the conditions

$$\lim_{n\to\infty} b_n = \infty, \qquad \lim_{n\to\infty} \frac{{b_n}^2}{n} = 0$$

$H_n(f;x)$ to $f(x)$ as $n \to \infty$ in weighted space of functions f continuous on positive semi-axis and satisfying the condition $\lim_{n\to\infty} f(x)/(1+x^2) = M_f < \infty$ is established.

Key Words: Bernstein-Chlodowsky polynomials in weighted space, Linear positive operators.

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Approximation of the Family of Solutions of the Urysohn Integral Equation

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ABSTRACT

Integral equations arise in different areas of theory and applications (see, e.g. [1-3] and references therein). Pointing out the importance of the integral equations, W. Heisenberg in his well known ”Physics and Philosophy” writes: ”The final equation of motion for matter will probably be some quantized nonlinear wave equation... This wave equation will probably be equivalent to rather complicated sets of integral equations...” (see, [4]). In this presentation a family of solutions of the Urysohn type integral equation including a parameter is studied. It is assumed that parameter is a integrally constrained function from the space of p-integrable functions. Some topological properties of the family of solutions of given integral equations are investigated. It is shown that the family of solutions is a compact subset of the space of continuous functions and it depends on constraint parameter continuously. Step by step way the family of solutions is replaced by a set that consists of a finite number of solutions. It is proved that the family of solutions can be approximated by a set consisting of a finite number of solutions. An error estimation between the family of solutions and its approximation is obtained. Note that an approximation of the family of solutions can be used for construction of solutions with prescribed properties.

Key Words: Integral equation, family of solutions, approximation

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A-Strongly Statistical Convergence in Metric Spaces

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ABSTRACT

In this work, we introduce A-strongly statistical convergence in an (X,d) metric space and investigate relations between A-strong convergence and this new concept of convergence, where A is an infinite matrix of nonnegative real numbers.

A sequence (x_n) in a metric space (X,d) is called A-strongly statistical convergent to L if $\lim_{n\to\infty}\frac{1}{S_n}\left|\left\{k\leq S_n:\sum_{i=1}^{\infty}a_{k,i}d(x_i,L)\geq\varepsilon\right\}\right|=0$ for all $\varepsilon>0$, where $A=\left(a_{n,k}\right)_{n,k=1}^{\infty}$ is a nonnegative infinite matrix and $S_n=\sum_{k=1}^{n}\sum_{i=1}^{\infty}a_{k,i}$.

We show providing that A is a nonnegative matrix, (x_n) is a sequence, $L\in X$ and $A_k=\sum_{i=1}^{\infty}a_{ki}d(x_i,L)$, then (x_n) is A-strongly statistical convergent to L if and only if there exist two nonnegative sequences $(B_k),(C_k)$ such that $A_k=B_k+C_k$, $\lim_n B_n=0$ and $\delta_{S_n}(\{k\in\mathbb{N}:C_k\neq0\})=0$.

Besides, for the uniqueness of A-strongly statistical limit of a sequence, we obtain some conditions for the matrix A. A nonnegative sequence (r_n) is dense positive provided that the limit superior of subsequence (r_{m_n}) is positive all (m_n) with the density of 1. A-strongly statistical limit is unique if and only if (r_n) is S_n-dense positive, where $r_n=\sum_{k=1}^{\infty}a_{nk}$.

We introduce A-strongly statistical Cauchy sequence in an (X,d) metric space and it is proved that if a sequence (x_n) is A-strongly statistical convergent sequence then it is A-strongly statistical Cauchy sequence.

Furthermore, necessary condition for linearity of A-strongly statistical limit is given in an $(X,\|.\|)$ normed space.

Key Words: Infinite matrix, density, summability, statistical convergence.

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Asymptotically f-Lacunary Statistical Equivalence of Set Sequences in Wijsman Sense

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ABSTRACT

In this work we introduce a generalization of asymptotically lacunary statistical equivalence of set sequences in Wijsman sense by using modulus functions and we give the definition of strongly asymptotically f-lacunary equivalence respectively as follows:

Let (X,ρ) be a metric space, θ be a lacunary sequence and $f:[0,\infty)\to[0,\infty)$ be an unbounded modulus function. For any non-empty closed subsets $\{A_k\}$, $\{B_k\}\subseteq X$ such that $d(x,A_k)>0$ and $d(x,B_k)>0$ for each $x\in X$, we say that the sequences $\{A_k\}$ and $\{B_k\}$ are asymptotically f-lacunary statistical equivalent of multiple L in Wijsman sense if for every $\varepsilon>0$ and for each $x\in X$,

$$\lim_r \frac{1}{f(h_r)} f\left(\left|\left\{k\in I_r: \left|\frac{d(x,A_k)}{d(x,B_k)}-L\right|\geq\varepsilon\right\}\right|\right)=0$$

and

Let (X,ρ) be a metric space, θ be a lacunary sequence and f be an unbounded modulus function. For any non-empty closed subsets $\{A_k\}$, $\{B_k\}\subseteq X$ such that $d(x,A_k)>0$ and $d(x,B_k)>0$ for each $x\in X$, we say that the sequences $\{A_k\}$ and $\{B_k\}$ are strongly asymptotically f-lacunary equivalent of multiple L in Wijsman sense if for every $\varepsilon>0$ and for each $x\in X$,

$$\lim_{r} \frac{1}{h_r} \sum_{k \in I_r} f\left(\left|\frac{d(x,A_k)}{d(x,B_k)} - L\right|\right) = 0.$$

We obtain some inclusion results related to asymptotically f-lacunary statistical equivalence of set sequences in Wijsman sense with the notion of strongly asymptotically f-lacunary equivalence.

We show that if the sequences $\{A_k\}$ and $\{B_k\}$ are asymptotically f-lacunary statistical equivalent of multiple L in Wijsman sense then the sequences $\{A_k\}$ and $\{B_k\}$ are asymptotically lacunary statistical equivalent of multiple L in Wijsman sense.

For an unbonded modulus $f:[0,\infty) \rightarrow [0,\infty)$ satisfying the inequality $f(xy) \geq cf(x)f(y)$ with some $c \in (0,\infty)$ for all $x,y \in [0,\infty)$, if $liminf_r q_r$>1 and the sequences $\{A_k\}$ and $\{B_k\}$ are asymptotically statistical equivalent of multiple L in Wijsman sense then the sequences $\{A_k\}$ and $\{B_k\}$ are asymptotically lacunary statistical equivalent of multiple L in Wijsman sense and the reverse of its when the condition $limsup_r q_r < \infty$ holds.

Key Words: Asymptotically statistical equivalent sequence, lacunary sequence, set sequences in Wijsman sense.

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Asymptotically Lacunary Statistical Equivalent Sequences spaces defined by ideal convergence and an Orlicz function

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ABSTRACT

We introduce the concepts of generalized asymptotically lacunary statistical equivalent of order α to multiple L, where I is an ideal of the subset of positive integers and M is an Orlicz function. In addition to these definitions, natural inclusion theorems where presented.

Let s; l_∞ ; c denote the spaces of all real sequences, bounded, and convergent sequences,respectively. Any subspace of s is called a sequence space. Following Freedman et al.[4], we call the sequence $\Theta = (k_r)$ lacunary if it is an increasing sequence of integers such that $k_0=0$; $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$ .The intervals determined by Θ will be denoted by $I_r = (k_{r-1} ; k_r]$ and $q_r = k_r / k_{r-1}$ .These notations will be used troughout the paper. The sequence space of lacunary strongly convergent sequences N_θ was defined by Freedman et al.[4]

Orlicz [7] used the idea of Orlicz function to construct the space L^M. An Orlicz function is a function $M : [0, \infty) \to [0, \infty)$ which is continuous, nondecreasing and convex with M(0)=0; M(x)> 0 and $M(x) \to \infty$ as $x \to \infty$.
An Orlicz function M is said to satisfy the Δ_2 -condition for all values of u, if there exists constant K > 0, such that $M(2u) \leq KM(u) (u \geq 0)$. It is also easy to see that always K > 2. The Δ_2-conditionis equivalent to the satisffies the inequality $M(Lu) \leq KLM(u)$ for all values of u and L > 1.

Marouf presented definitions for asymptotically equivalent sequences and asymptotic regular matrices in [6].

Kostyrko et al. [5] introduced the notion of I-convergence with the help of an admissible ideal I,which denotes the ideal of subsets of N, which is a generalization of statistical convergence. Quite recently, Das et al. [2,3] unifed these two approaches to introduce new concepts such as I- statistical convergence and I-lacunary statistical convergence and investigated some of their consequences.

More investigations in this direction and more applications of asymptotically equivalent sequence can be found in [1, 2, 3, 8].

In this paper we introduce the concepts of generalized asymptotically lacunary statistical equivalent of order α to multiple L, I is an ideal of the subset of positive integers and M is an Orlicz function and also some inclusion theorems are proved.

Key Words: Asymptotically equivalence, Ideal convergence, Lacunary sequence, Orlicz function, Statistical convergence of order α.

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Bessel Type Kolmogorov Inequalities On Lebesgue Spaces

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ABSTRACT

In this paper, it is considered a class of singular integral operators generated by generalized shift operator. Singular integral operators, such that Riesz-Bessel transforms, are bounded on $L_{p,\gamma}(\mathbb{R}_+^n)$ Lebesgue spaces for $1 < p < \infty$, but these operators are not bounded on Lebesgue spaces for $0 < p < 1$. For this reason, it is necessary to work in Hardy Spaces to get the boundedness of such operators for $0 < p < 1$. To show that convolution type Calderón-Zygmund singular integrals with sufficient regularity are bounded on $H_{p,\gamma}(\mathbb{R}_+^n)$, there are two main techniques which are the finite atomic decomposition and weighted norm inequalities. Thus, the most important method to show the boundedness of the Riesz-Bessel transforms in $H_{p,\gamma}(\mathbb{R}_+^n)$ Hardy spaces is atomic decomposition. The most important inequality used when atomic decomposition is applied is Bessel type Kolmogorov inequality. Therefore, in this paper, Bessel type Kolmogorov inequalities, which is necessary to demonstrate the boundedness of Riesz-Bessel transforms in the Hardy spaces, will be proved in weighted Lebesgue spaces. We say that $R_\gamma^{(k)}\mathrm{f} = \mathrm{K} * \mathrm{f}$ is a convolution-type singular integral operator with regularity of order k if the distribution K coincides with a function on $\mathbb{R}_+^n \setminus \{0\}$ and has the properties that are hold$F_B(K) \in L_{\infty,\gamma}(\mathbb{R}_+^n)$ and, $\partial^\beta K(x) \le \frac{C}{|x|^{n+k+\gamma+|\beta|}}$ for all multi-indices $0 \le |\beta| \le k+1$ and $x \ne 0$. Therefore, singular integrals that satisfy above conditions are bounded on $L_{p,\gamma}(\mathbb{R}_+^n)$, $1 < p < \infty$. More importantly, the pointwise smoothness conditions guarantee that they satisfy weighted norm inequalities. In this work, we will obtain weighted Kolmogorov inequality.

Keywords: Generalized shift operator, Hardy spaces, Laplace-Bessel operator, Lebesgue spaces, Kolmogorov inequality, Weight.

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Certain New Hermite-Hadamard Type Inequalities For Convex Functions Via Fractional Integrals

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ABSTRACT

Convexity is an important concept in many branches of mathematics. In particular, many important integral inequalities are based on a convexity assumption of a certain function. For example, the following famous inequality is one of them. Let $f: I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex function defined on the interval I of real numbers and $a, b \in I$ with $a < b$. The following inequality holds

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}.$$

It was firstly discovered by Ch. Hermite in 1881 in the journal Mathesis. But this inequality was nowhere mentioned in the mathematical literature and was not widely known as Hermite's results. E.F. Beckenbach wrote that this result was proven by J. Hadamard in 1893. In 1974, D.S. Mitrinovic found Hermite's note in Mathesis. This inequality known as Hadamard's inequality is now commonly referred as the Hermite-Hadamard inequality. Hermite-Hadamard inequality is playing a very important role in all the fields of mathematics. Thus such inequalities were studied extensively by many researchers and a number of the papers have been written on this inequality providing new proofs, noteworthy extensions, generalizations and numerous applications. In recent years, one more dimension has been added to this studies, by introducing various integral inequalities involving fractional integral operators like Riemann-Liouville, Hadamard, Erdelyi-Kober, Katugampola fractional operators and fractional operator with exponential kernel. The aim of this paper is to establish

generalized Hermite-Hadamard type integral inequalities for convex function via fractional integral operators.

Key Words: Convex functions, Hermite-Hadamard inequality, fractional integrals

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Chebyshev Polynomial Coefficient Estimates for Analytic Bi-Univalent Functions

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ABSTRACT

A function is said to be bi-univalent on the open unit disk if both the function and its inverse are univalent in the unit disc. In this paper, using the Chebyshev Polynomials, we obtain coefficient expansions for analytic bi-univalent functions in some subclasses and determine coefficients for such functions. We also demonstrate the unpredictable behavior of the initial coefficients of subclasses of bi-univalent functions. For some special cases, also we show that our class is generalization class of them. Also, we give Fekete-Szegö inequalities for these function classes.

If $F = f^{-1}$ is the inverse of a function $f \in S$, then F has a Maclaurin series expansion in some disk about the origin. A function $f \in S$ is said to be bi-univalent in D if both f and its inverse $F = f^{-1}$ are univalent in D. Let Σ denote the class of analytic and bi-univalent functions in D given by the Taylor-Maclaurin series expansion given by $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$.

Firstly, Lewin studied the class of bi-univalent functions, obtaining the bound $|a_2| \leq 1.51$. Subsequently, Brannan and Clunie conjectured Lewin's result to $|a_2| \leq \sqrt{2}$ for $f \in \Sigma$. Accordingly, Netanyahu showed that $|a_2| \leq 4/3$. Brannan and Taha introduced certain subclasses of biunivalent function class Σ similar to the familiar subclasses. In fact, the aforecited work of Srivastava et al. essentially revived the investigation of various subclasses of biunivalent function class Σ in recent years. Recently, many authors investigated bounds for various subclasses of bi-univalent functions. Not much is known about the bounds on general coefficient $|a_n|$ for $n \geq 4$.This is because the bi-univalency requirement makes the behavior of the coefficients of the function f and its inverse $F = f^{-1}$ unpredictable. In the literature, only few works determine general coefficient bounds $|a_n|$ for the analytic bi-univalent functions.

Chebyshev polynomials, which is used by us in this presentation, play a considerable act in numerical analysis. We know that the Chebyshev polynomials are four kinds. The most of books and research articles related to specific orthogonal polynomials of Chebyshev family, contain essentially results of Chebyshev polynomials of first and second kinds $T_n(x)$ and $U_n(x)$ and their numerous uses in different applications.

In this presentation, motivating by the earlier work of Dziok et al. [5], Ding et al. [2] and study of Frasin and Aouf [6], Srivastava et al.[7], we define certain subclasses $\sum$ and determine initial Taylor- Maclaurin coefficients.

Key Words: Bi-univalent functions, Analytic functions, Chebyshev polynomials, Coefficient estimates.

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Coefficient Bounds for Some Subclasses of m-fold Symmetric Bi-univalent Functions

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ABSTRACT

A function is said to be univalent if it never takes the same value twice, that is $f(z_1)=f(z_2) \quad if \quad z_1 \neq z_1$.

Let A denote the class of functions f of the form

$$f(z)=z+\sum_{k=2}^{\infty} a_k z^k \tag{1}$$

which are analytic in the open unit disc $U=\left\{z:\ z\in\mathbb{C};\ |z|<1\right\}$ and normalized by the conditions $f(0)=f'(0)-1=0$. We also denote by S the subclass of all functions in the class of normalized analytic function (see for [3]). It is well known that every function $f\in S$ has an inverse f^{-1}, which is defined by

$$f^{-1}\left(f(z)\right)=z; \quad z\in U$$

and

$$f\left(f^{-1}(w)\right)=w; \quad |w|<r_0(f) \quad and \quad r_0(f)\geq\frac{1}{4},$$

where the inverse function f^{-1} is given by

$$g(w)=f^{-1}(w)=w-a_2w^2+\left(2a_2^2-a_3\right)w^3-\left(5a_2^3-5a_2a_3+a_4\right)w^4+\ldots=w+\sum_{n=2}^{\infty}b_nw^n \ . \tag{2}$$

Let $f\in A$. The function f is said to be bi-univalent in U if both f and $f^{-1}$are univalent in U. Let Σ denote the class of bi-univalent functions in U given by (1).

We can accept that the beginning of estimating bounds for the coefficients of classes of bi-univalent functions is the date 1967 [5]. Later the papers of Brannan and Taha [1] and Srivastava et al. [6] were picked up the interest on the coefficient bounds of bi-univalent functions. Not much is known about the bounds on general coefficient $|a_n|$ for $n \geq 4$.This is because the bi-univalency requirement makes the behavior of the coefficients of the function f and its inverse $F = f^{-1}$ are unpredictable. In the literature, only few works determine general coefficient bounds $|a_n|$ for the analytic bi-univalent functions (see [2] and [4]).

Let $m \in \mathbb{N}$. A domain E is said to be *m-fold symmetric* if a rotation of E about the origin through an angle $2\pi/m$ carries E on itself. It follows that, a function $f(z)$ analytic in D is said to be m-fold symmetric if

$$f(e^{2\pi i/m} z) = e^{2\pi i/m} f(z).$$

In particular every f is one-fold symmetric and every odd f is two-fold symmetric. S_m denote the class of m-fold symmetric univalent functions in U.

$f \in S_m$ is characterized by having a power series of the form

$$f(z) = z + \sum_{k=1}^{\infty} a_{mk+1} z^{mk+1} \qquad (z \in U,\ m \in \mathbb{N}). \tag{2}$$

In [7] Srivastava et al. defined m-fold symmetric bi-univalent functions analogues to the concept of m-fold symmetric univalent functions. They introduce some important results, such as each function $f \in \Sigma$ generates an m-fold symmetric bi-univalent function for each $m \in \mathbb{N}$, they obtained the series expansion for f^{-1} as following:

$$\begin{aligned} F(w) = w - a_{m+1} w^{m+1} + [(m+1)a_{m+1}^2 - a_{2m+1}] w^{2m+1} \\ - \left[\frac{1}{2}(m+1)(3m+2)a_{m+1}^3 - (3m+2)a_{m+1}a_{2m+1} + a_{3m+1}) \right] w^{3m+1} + \ldots \end{aligned}$$

where $f^{-1}=F$. We denote by $\sum_m$ the class of m-fold symmetric bi-univalent functions in U.

In this study, we give two new subclasses of the bi-univalent functions; both f and f^{-1} are m-fold symmetric analytic functions. Among other results belonging to these subclasses, upper coefficients bounds $|a_{m+1}|$ and $|a_{2m+1}|$ are obtained in this study.

Key Words: Analytic functions, univalent functions, bi-univalent functions.

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Commutators of Maximal Operator on Generalized Orlicz-Morrey Spaces

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ABSTRACT

A natural step in the theory of functions spaces was to study Orlicz-Morrey spaces where the "Morrey-type measuring" of regularity of functions is realized with respect to the Orlicz norm over balls instead of the Lebesgue one. In [1], Deringoz *et al.* introduced generalized Orlicz-Morrey spaces as an extension of generalized Morrey spaces. Note that, generalized Orlicz-Morrey spaces unify Orlicz and generalized Morrey spaces. It is well-known that commutators of classical operators of harmonic analysis play an important role in various topics of analysis and PDE. Unfortunately commutators in Morrey-type spaces were studied in a less generality in comparison with other spaces. Let M be the Hardy-Littlewood maximal function and b be a locally integrable function. Denote by M_b and $[b,M]$ the maximal commutator and the commutator of M, respectively. Although operators M_b and $[b,M]$ essentially differ from each other (for example, M_b is a positive and a sublinear operator, but $[b,M]$ is neither a positive nor a sublinear), but if b satisfies some additional conditions, then operator M_b controls $[b,M]$. The aim of this talk is to present criteria for the boundedness of commutators of maximal operator on generalized Orlicz-Morrey spaces.

Key Words: generalized Orlicz-Morrey spaces, maximal operator, commutator.

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Convolution and Best Approximations in Weighted Variable Exponent Lebesgue Spaces

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ABSTRACT

We investigate relationship between best approximation number and convolution operators in weighted Lebesgue spaces with variable exponent. A function $\omega:[0,2\pi]\to[0,\infty]$ will be called a weight if ω is Lebesgue measurable and almost everywhere positive on $[0,2\pi]$. By $L_{\omega}^{p(\cdot)}$ we denote the class of 2π periodic Lebesgue measurable functions f, with the finite norm $\|f\|_{p(\cdot),\omega} := \inf\left\{\lambda>0:\int_0^{2\pi}\left|f(x)\omega(x)/\lambda\right|^{p(x)}dx\le 1\right\}$. Let $\hat{\beta}_{2\pi}$ be the class of Lebesgue measurable functions $p(\cdot):[0,2\pi]\to[1,\infty)$ such that

$$1<p_-:=\underset{x\in[0,2\pi]}{essinf}\,p(x)\le p_+:=\underset{x\in[0,2\pi]}{esssup}\,p(x)<\infty$$

and $|p(x)-p(y)|\le -c_0/\log(|x-y|)$ for all $x,y\in[0,2\pi]$ with $|x-y|\le 1/2$. By $A_{p(\cdot)}$ we denote the class of weights ω, satisfying the condition

$$\left\|\omega^{p(x)}\right\|_{A_{p(\cdot)}} := \sup_{I\in\hat{I}}\frac{1}{|I|^{p_I}}\left\|\omega^{p(x)}\right\|_{L^1(I)}\left\|\frac{1}{\omega^{p(x)}}\right\|_{L^{(p'(\cdot)/p(\cdot))}(I)}<\infty .$$

Here $p_I := \left(1/|I|\int_I 1/p(x)dx\right)^{-1}$, $\hat{I}$ is the class off all intervals I in $[0,2\pi]$ and $p'(x)$ is the conjugate exponent of $p(x)$, defined as $p'(x):=p(x)/(p(x)-1)$.

Weighted Lebesgue space with variable exponent is noninvariant with respect to the usual shift operator $f\to f(\cdot+h)$, which used for construction of the classical convolution operators. Therefore, using Steklov mean value function

$$(\sigma_h f)(x,u) := \frac{1}{2h}\int_{-h}^{h} f(x+tu)\,dt, \quad 0<h<\pi,\ x\in[0,\pi],\ -\infty<u<\infty,$$

for every $f \in L_{\omega}^{p(\cdot)}$, we define the convolution type operator

$$\int_{-\infty}^{\infty}(\sigma_h f)(x,u)\,d\sigma(u)$$

with a bounded variation function $\sigma(u)$ on the real line R. Let

$$D_{\omega}(f,\sigma,h,p(\cdot)) := \left\|\int_{-\infty}^{\infty}(\sigma_h f)(\cdot,u)\,d\sigma(u)\right\|_{p(\bullet),\omega}.$$

The best approximation number $E_n(f)_{p(\cdot),\omega}$ is defined $E_n(f)_{p(\cdot),\omega} := \inf_{T_n}\|f-T_n\|_{p(\cdot),\omega}$ where inf is taken on the class of trigonometric polynomials $T_n(x)=\sum_{k=0}^{n} c_k e^{ikx}$ of degree at most n.

Theorem Let $p(\cdot)\in\hat{\beta}_{2\pi}$ and $\omega^{-p_0}\in A_{p'(\cdot)/p_0}$ for some $p_0\in(1,p_-)$. Then there exists a positive constant $c(p)$ such that for every $f\in L_{\omega}^{p(\cdot)}$ and $m\in N$

$$D_{\omega}(f,\sigma,h,p(\cdot)) \le c_{p(\cdot)}\left[\sum_{k=0}^{m} E_{2^k-1}(f)_{p(\cdot),\omega}\,\delta_{2^k,h} + E_{2^{m+1}}(f)_{p(\cdot),\omega}\right]$$

where

$$\delta_{2^k,h} := \sum_{l=2^k}^{2^{k+1}-1}\left|\hat{\sigma}(lh)-\hat{\sigma}((l+1)h)\right| + \left|\hat{\sigma}(2^k h)\right|$$

$$\hat{\sigma}(x) := \int_{-\infty}^{\infty}\frac{\sin(ux)}{ux}\,d\sigma(u), \quad 0<h<\pi.$$

This problem for variable exponent Lebesgue spaces was investigated $[1]$. On the other hand, for the classical convolution operators in the classical Lebesgue spaces was investigated $[2]$. Later, in the non-weighted and weighted Orlicz spaces it was studied in $[3]$ and $[4]$, respectively.

Key Words: Convolution, Variable exponent space, Best approximation.

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Core Theorems in the Generalized Statistical Sense

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ABSTRACT

Kolk have introduced the concept of generalized statistical convergence by considering a sequence of infinite matrices in [1] as follows:

An index set K is said to have B-density by $\delta_B(K)$ equals to d, if the characteristic sequence of K is B-summable to d, i.e.

$$\lim_n \sum_{k\in K} b_{nk}^{(i)} = d, \quad uniformly\ in\ i,$$

where by an index set we mean a set $K = \{k_i\} CIN$, $k_i < k_{i+1}$ for all i.

A sequence $x = (x_k)$ is called generalized statistically convergent to the number l, if for every ε>0

$$\delta_B(\{k: |x_k - l| \geq \varepsilon\}) = 0$$

and we write $st_B - limx = l$.

In particular if $B = (C_1)$, the Cesàro matrix, then generalized statistical convergence is reduced to the usual statistical convergence.

In this study we extend the concept of A-statistical core to generalized statistical core. We also examine relation between the Knopp core and the generalized statistical core. The Knopp core theorem has been studied by many authors with various directions. The theorems proved here are generalized statistical versions of theorems about the Knopp core concerning its containment or invariance under a matrix transformation. Also we consider some matrix transformations between the space of all generalized statistical convergent sequences and a sequence space. We give some characterizations for such matrix transformations.

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Key Words: Knopp Core, generalized statistical convergence.

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Euler-Riesz Difference Sequence Spaces

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ABSTRACT

Let us define the Euler mean $E_1 = (e_{nk})$ of order one and Riesz mean $R_q = (r_{nk})$ of order one by

$$e_{nk} = \begin{cases} \frac{\binom{n}{k}}{2^n}, 0 \leq k \leq n, \\ 0, \quad k > n, \end{cases} \qquad r_{nk} = \begin{cases} \frac{q_k}{Q_n}, 0 \leq k \leq n, \\ 0, \quad k > n, \end{cases}$$

for all $k, n \in \mathbb{N}$ and where (q_k) be a sequence of positive numbers and $Q_n = \sum_{k=0}^{n} q_k$. We define the matrix $\tilde{B} = (\tilde{b}_{nk})$ by the composition of the matrices E_1, R_q and Δ as

$$\tilde{b}_{nk} = \begin{cases} \frac{\binom{n}{k} q_k}{2^n Q_n}, 0 \leq k \leq n, \\ 0, \quad k > n, \end{cases}$$

for all $k, n \in \mathbb{N}$.

Now, we will give some new sequence spaces defined as;

$$[c_0]_{e.r} = \left\{ x = (x_k) \in w : \lim_{n \to \infty} \sum_{k=0}^{n} \frac{\binom{n}{k} q_k}{2^n Q_n} x_k = 0 \right\},$$

$$[c]_{e.r} = \left\{ x = (x_k) \in w : \lim_{n \to \infty} \sum_{k=0}^{n} \frac{\binom{n}{k} q_k}{2^n Q_n} x_k \ exists \right\},$$

$$[\ell_\infty]_{e.r} = \left\{ x = (x_k) \in w : \sup_{n \in \mathbb{N}} \left| \sum_{k=0}^{n} \frac{\binom{n}{k} q_k}{2^n Q_n} x_k \right| < \infty \right\}.$$

Basar and Braha [F. Başar and N. L. Braha, Euler- Cesàro Difference Spaces of Bounded, Convergent and Null Sequences, Tamkang J. of Math. 47(4), (2016), 405-420.], introduced the sequence spaces $\check{\ell}_\infty$, $\check{c}$ and $\check{c}_0$ of Euler- Cesàro bounded, convergent and null difference sequences and studied their some properties.

The main purpose of this study is to introduce the sequence spaces $[\ell_\infty]_{e.r}$, $[c]_{e.r}$ and $[c_0]_{e.r}$ of Euler- Riesz bounded, convergent and null difference sequences by using the composition of the Euler mean E_1 and Riesz mean R_q with backward difference operator Δ. Furthermore, the inclusions $c_0 \subset [c_0]_{e.r}$, $c \subset [c]_{e.r}$ and $\ell_\infty \subset [\ell_\infty]_{e.r}$ strictly hold, the basis of the sequence spaces $[c_0]_{e.r}$ and $[c]_{e.r}$ is constucted and alpha-, beta- and gamma-duals of these spaces are determined. Finally, the classes of matrix transformations from the Euler- Riesz difference sequence spaces to the spaces ℓ_∞, c and c_0 are characterized.

Key Words: Euler sequence spaces, Riesz sequence spaces, matrix domain, matrix transformations.

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f-Lacunary Statistical Convergence And Strong *f*-Lacunary Summability Of Order α

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ABSTRACT

Aizpuru et al. [1] defined the f-density of the subset $E\subset\mathbb{N}$ (natural numbers) for any unbounded modulus f by

$$d^f(E)=\lim_{n\to\infty}[f(|\{k\le n:k\in E\}|)/f(n)]$$

in case this limit exists and defined the f-statistically convergence for any unbounded modulus f by

$$d^f(\{k\in\mathbb{N}: |x_k-\ell|\ge\varepsilon\})=0$$

and we write it as $S^f\text{-}\lim x_k=\ell$. Every f-statistically convergent sequence is statistically convergent, but a statistically convergent sequence need not be f-statistically convergent for every unbounded modulus f.

The main object of this presentation is to introduce the concepts of f- lacunary statistical convergence of order α of sequences of real numbers where f is an unbounded modulus and strong f-lacunary summability of order α of sequences of real numbers where *f* is an unbounded modulus and we will give some inclusion relations between these spaces.

Definition 1. Let $\theta=(k_r)$ be a lacunary sequence and $0<\alpha\le1$ be given. We say that the sequence $x=(x_k)$ is f-lacunary statistically convergent of order α where f is an unbounded modulus if there is a real number ℓ such that

$$\lim_{r\to\infty}[1/f(h_r)^\alpha]\, f(|\{k\in I_r: |x_k-\ell|\ge\varepsilon\}|)=0$$

where $I_r=(k_{r-1},k_r]$ and $f(h_r)^\alpha$ denotes the αth power $[f(h_r)]^\alpha$ of $f(h_r)$.

Definition 2. Let f be an unbounded modulus, $p=(p_k)$ be a sequence of strictly positive real numbers and $0<\alpha\le1$ be real numbers. We say that the sequence $x=(x_k)$ is strongly f-lacunary summable to ℓ (a real number) such that for $k\in I_r$

$\lim_{r\to\infty}[1/f(h_r)^{\alpha}]\ \sum [\ f(|x_k-\ell|)]^p=0.$

Key Words: Lacunary sequence, Modulus function, Statistical convergence.

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Generalized Fractional Integral Operators On Vanishing Generalized Local Morrey Spaces

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ABSTRACT

The vanishing Morrey space $VM_{p,\lambda}(R^n)$ of the classical Morrey Spaces $M_{p,\lambda}(R^n)$ was introduced by Vitanza in [8] and applied there to obtain a regularity result for elliptic partial differential equations. Later Vitanza proved an existence theorem for a Dirichlet problem, under weaker assumptions then those introduced by Miranda in 1963, and a $W^{3,2}$ regularity result assuming that the partial derivatives of the coefficients of the highest and lower order terms belong to vanishing Morrey spaces depending on the dimension. Also M.A. Ragusa [5] obtained a sufficient condition for commutators of fractional integral operators to belong to vanishing Morrey spaces $VM_{p,\lambda}(R^n)$. The vanishing generalized Morrey space $VM_{p,\varphi}(R^n)$ and vanishing generalized local Morrey space $VLM_{p,\varphi}^{\{x_0\}}(R^n)$ was introduced by N. Samko ([6], [7]). The boundedness of the multi-dimensional Hardy type operators, maximal, potential and singular operators in these spaces were proved in ([6], [7]).

Let $f \in L_1^{loc}(R^n)$. The generalized fractional integral operator I_ρ is defined by

$$I_\rho f(x) = \int_{R^n} \frac{\rho(|x-y|)}{|x-y|^n} f(y)dy,$$

where $\rho: (0,\infty) \to (0,\infty)$ is a suitable function. If $\rho(t) \equiv t^\alpha$, then $I_\alpha \equiv I_{t^\alpha}$ is the Riesz potential operator.

The generalized fractional integral operator I_ρ was initially investigated in [1], [2], [4]. Nakai [4] proved the boundedness of I_ρ from the generalized Morrey spaces $M_{1,\varphi}$ to the spaces $M_{1,\psi}$ for suitable functions φ and ψ. Nowadays many authors have been culminating important observations about I_ρ especially in connection with Morrey spaces (see [3]).

In this prsentation, we prove the Spanne-Guliyev type boundedness of the operator I_ρ from the vanishing generalized local Morrey spaces $VLM_{p,\varphi_1}^{\{x_0\}}$ to $VLM_{p,\varphi_2}^{\{x_0\}}$, $1 < p < q < \infty$, and from the space $VLM_{p,\varphi_1}^{\{x_0\}}$ to the weak space $VWLM_{p,\varphi_2}^{\{x_0\}}$, $1 < q < \infty$.

We also prove the Adams-Guliyev type boundedness of the operator I_ρ from the vanishing generalized Morrey spaces $VM_{p,\varphi^{\frac{1}{p}}}$ to $VM_{q,\varphi^{\frac{1}{q}}}$, $1 < p < q < \infty$, and from the space $VM_{1,\varphi}$ to the weak space $VWM_{q,\varphi^{\frac{1}{q}}}$, $1 < q < \infty$.

Key Words: Generalized fractional integral operator; vanishing generalized Morrey space; vanishing generalized local Morrey space.

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Inequalities Involving the (α,k) −Gamma and (α,k) −Beta Functions

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ABSTRACT

The classical Euler gamma function or Euler integral of the second kind is given by

$$\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt.$$

The rising factorial $x^{(n)}$, sometimes also denoted or x^{n}, is defined by

$$x^{(n)}=x(x+1)\ldots(x+n-1).$$

This function is also known as the rising factorial power and frequently called the Pochhammer symbol in the theory of special functions. The rising factorial is implemented in the Wolfram Language as Pochhammer $[x,n]$.

In recently, Diaz and Pariguan give a new definition for the function of variable x as follows [7]

$$(x)_{n,k}=x(x+k)(x+2k)\ldots(x+(n-1)k),$$

and they called the Pochhammer k −symbol. Setting $k=1$ one obtains the usual Pochhammer symbol $(x)_n$.

More recently in [5], Sarikaya et al. gave a new definition for the function of variable p as follows

$$(p)_{n,k}^{\alpha}=(p+\alpha-1)(p+\alpha-1+\alpha k)(p+\alpha-1+2\alpha k)\ldots(p+\alpha-1+(n-1)\alpha k),$$

and they called the Pochhammer symbol $(p)_{n,k}^{\alpha}$. Setting $k\to 1$ and $\alpha\to 1$ one obtains the usual Pochhammer symbol $(p)_n$.

The Conformable gamma function $\Gamma_k^{\alpha}(x)$ is defined by

$$\Gamma_k^{\alpha}(x)=\int_0^\infty t^{x-1}e^{-\frac{t^{\alpha k}}{\alpha k}}d_{\alpha}t.$$

The (α,k) −Beta function $B_k^{\alpha}(p;q)$ is given the by formula $\mathrm{B}_k^{\alpha}(p,q)=\frac{1}{\alpha k}\int_0^1 t^{\frac{p}{\alpha k}-1}(1-t)^{\frac{q}{\alpha k}-1}d_\alpha t.$

For the history and main results on Gamma function and Conformable Fractional Calculus, we refer the reader to $[1-4,6]$.

In this work, we present some fundamental relations for (α,k) −gamma and (α,k) −beta functions introduced by the Sarikaya et al. in $[5]$.

Also, some inequalities involving the n −th derivative of the (α,k) −Gamma function are established.

Here, the n −th derivative of the $\Gamma_{\alpha,k}(x)$ is given by

$$\Gamma_{\alpha,k}^{(n)}(x)=\int_0^\infty (lnt)^n t^{x-1}e^{-\frac{t^{\alpha k}}{\alpha k}}d_\alpha t.$$

When $(\alpha,k)\to(1,1)$, it turns out to be the usual gamma function and beta function.

Key Words: Gamma Function, Beta Function, Special Functions, Inequalities.

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Isometry of $l_p(\Delta_q^m)$ Sequence Space Generated by Δ_q^m Difference Operator

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ABSTRACT

Let l_∞, c and c_0 be the Banach spaces of bounded, convergent and null sequences $x=(x_k)_1^\infty$ respectively.

Kızmaz [1] presented the difference sequence spaces, $X(\Delta)=\left\{x=(x_k):\Delta x\in X\right\}$ for $X=l_\infty, c$ and c_0 where $\Delta x=(\Delta x_k)=(x_k-x_{k+1})$. These are Banach spaces with the norm $\|x\|_\Delta=|x_1|+\|\Delta x\|_\infty$. He also studied their topological properties. Recently Et and Çolak [2] extended the spaces $X(\Delta)$ and $X(\Delta^2)$ to the spaces $X(\Delta^m)$. Let X be any sequence spaces and defined $X(\Delta^m)=\left\{x=(x_k):\Delta^m x\in X\right\}$, where $m\in\mathbb{N}$ and $\Delta^m x=\left((\Delta\circ\Delta^{m-1})x_k\right)$ for all $k\in\mathbb{N}$. The following sequence spaces have been given by Karakaş et al.[3]. $X(\Delta_q{}^m)=\left\{x=(x_k):\Delta_q{}^m x\in X\right\}$ for $X=l_\infty, c$ and c_0, where $q,m\in\mathbb{N}$,

$\Delta_q^m x_k=(\Delta_q^{m-1}x_k-\Delta_q^{m-1}x_{k+1})$ and so $\Delta_q^m x_k=\sum_{v=0}^{m}(-1)^v\binom{m}{v}q^{m-v}x_{k+v}$.

Recently, Peralta [4] studied $l_p(\Delta^m)$ and examined the topological properties of this space.

All throughout this paper we take $p\in[1,\infty)$. By ω, we shall define the space of all sequences $x=(x_k)$, where $x_k\in\mathbb{R}$ for all $k\in\mathbb{N}$. Given $x\in\omega$, describe

$$\|x\|_p=\left(\sum_{k=1}^{m}|x_k|^p\right)^{1/p}$$

and let

$$l_p = \left\{ x = (x_k) : \|x\|_p < \infty \right\}.$$

We obtain the linear difference operator $\Delta : \omega \to \omega$ which maps a sequence $x \in \omega$ into a sequence $\Delta x = (\Delta x_k) \in \omega$ having components $\Delta x_k = x_k - x_{k+1}$.

For $m \geq 2$, the linear operator $\Delta^m : \omega \to \omega$ is given recursively as the composition $\Delta^m = \Delta \circ \Delta^{m-1}$. One can easily see that for $m \geq 1$ and $x \in \omega$ the following Binomial representation

$$\Delta_q^m x_k = \sum_{v=0}^{m} (-1)^v \binom{m}{v} q^{m-v} x_{k+v}$$

for all $k \in \mathbb{N}$.

Given $m \in \mathbb{N}$ we find the sequence space as;

$$l_p(\Delta_q^m) = \left\{ x = (x_k) : \Delta_q^m x \in l_p \right\}$$

and for $x \in l_p(\Delta_q^m)$ we get

$$\|x\|_{p,\Delta_q^m} = \left(\sum_{i=1}^{m} |x_i|^p + \left\|\Delta_q^m\right\|_p^{\,p} \right)^{1/p}$$

It can be easily proven that the pair $\left(l_p(\Delta_q^m), \|\cdot\|_{p,\Delta_q^m} \right)$ is a normed space. The sequence space $l_p(\Delta_q^m)$ equipped with the norm

$$\|x\|_{p,\Delta_q^m} = \left(\sum_{i=1}^{m} |x_i|^p + \left\|\Delta_q^m\right\|_p^{\,p} \right)^{1/p}$$

is a Banach space. We show that, if $l_p(\Delta_q^m)$ is equipped with an appropriate norm then this sequence space is linearly isometric to the usual sequence space l_p.

Key Words: Sequence spaces, Difference sequence spaces, Isometric sequence spaces.

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Jackson and Stechkin type inequalities of trigonometric approximation in $A_{w,\vartheta}^{p,q(.)}$

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ABSTRACT

The space $A_{w,\vartheta}^{p,q(.)}$ is the intersection of the well known weighted Lebesgue space L_w^p and the variable exponent weighted Lebesgue space $L_\vartheta^{q(.)}$, i.e., $A_{w,\vartheta}^{p,q(.)} = L_w^p \cap L_\vartheta^{q(.)}$. In the weighted Lebesgue spaces and the variable exponent Lebesgue spaces, the direct and inverse theorems of trigonometric approximation were proved in the papers [1,2,3,5,7]. In literature, the variable exponent Lebesgue spaces were first time investigated by Orlicz [4]. Also, in weighted Lorentz spaces L_w^{pq} which is the generalization of L_w^p the similar problems were investigated in the papers [6,8]. In this paper, we prove the direct and inverse theorems of trigonometric approximation in $A_{w,\vartheta}^{p,q(.)}$. Our main results are following:

Theorem 1. Let $f \in A_{w,\vartheta}^{p,q(.)}, w \in A_p, \vartheta \in A_{q(.)}, 1 < p < \infty, q(.) \in \wp\,(T)$ and $n, r \in \mathbb{N}$. Then

$$E_n(f)_{A_{w,\vartheta}^{p,q(.)}} \lesssim \Omega_r\left(f,\left(\frac{1}{n}\right)\right)_{A_{w,\vartheta}^{p,q(.)}}$$

holds where some constant depending only on $r, p, q(.)$ and C_{A_p}.

Theorem 2. Let $f \in A_{w,\vartheta}^{p,q(.)}, w \in A_p, \vartheta \in A_{q(.)}, 1 < p < \infty, q(.) \in \wp\,(T)$, $w \in A_p$, $\vartheta \in A_{q(.)}$ and $n, r \in \mathbb{N}$. Then

$$\Omega_r\left(f,\frac{1}{n}\right)_{A_{w,\vartheta}^{p,q(.)}} \lesssim \frac{1}{n^{2r}}\sum_{k=0}^{n}(k+1)^{2r-1}E_n(f)_{A_{w,\vartheta}^{p,q(.)}}$$

holds where some constant depending only on $r, p, q(.)$ and C_{1A_p} of w and $C_{2A_{q(.)}}$ of ϑ.

From Theorem 1 and Theorem 2, we obtain the following Marchaud type inequality.

Corollary 3. Let $f \in A_{w,\vartheta}^{p,q(.)}$,$w \in A_p, \vartheta \in A_{q(.)}, 1 < p < \infty, q(.) \in \wp\ (T)$, $w \in A_p$, $\vartheta \in A_{q(.)}$. Then we have

$$\Omega_r\left(f,\frac{1}{n}\right)_{A_{w,\vartheta}^{p,q(.)}} \lesssim \delta^{2r}\int_\delta^1 u^{-2r-1}\,\Omega_k(f,u)_{A_{w,\vartheta}^{p,q(.)}}du,\ \ 0<\delta<1$$

for $r, k \in \mathbb{N}$ with $r < k$.

Corollary 4. Let $f \in A_{w,\vartheta}^{p,q(.)}$,$w \in A_p, \vartheta \in A_{q(.)}, 1 < p < \infty, q(.) \in \wp\ (T)$. If

$$E_n(f)_{A_{w,\vartheta}^{p,q(.)}} \lesssim n^{-\alpha}, \qquad n \in \mathbb{N}$$

for some $\alpha > 0$, then, for a given $r \in \mathbb{N}$, we have the estimations

$$\Omega_r\left(f,\frac{1}{n}\right)_{A_{w,\vartheta}^{p,q(.)}} = \begin{cases} \delta^{\alpha} & , \ r > \alpha/2; \\ \delta^{2r}log(1/\delta) & , \ r = \alpha/2; \\ \delta^{2r} & , \ r < \alpha/2. \end{cases}$$

If we define the generalized Lipschitz class $Lip(\alpha, A_{w,\vartheta}^{p,q(.)})$ for $\alpha > 0$ as

$$Lip(\alpha, A_{w,\vartheta}^{p,q(.)}) := \{f \in A_{w,\vartheta}^{p,q(.)} : \Omega_k(f,\delta)_{A_{w,\vartheta}^{p,q(.)}} \lesssim \delta^{\alpha}, \delta > 0\},$$

then by virtue of Theorem 1 and Corollary 3 we obtain the following result which gives a constructive characterization of the Lipschitz classes $Lip\left(\alpha, A_{w,\vartheta}^{p,q(.)}\right)$.

Corollary 5. Let $f \in A_{w,\vartheta}^{p,q(.)}$,$w \in A_p, \vartheta \in A_{q(.)}, 1 < p < \infty, q(.) \in \wp\ (T)$. The following assertions are equivalent.

$$(i) f \in Lip(\alpha, A_{w,\vartheta}^{p,q(.)});\ \ (ii) E_n(f)_{A_{w,\vartheta}^{p,q(.)}} \lesssim n^{-\alpha},\ n \in \mathbb{N}.$$

Key Words: Modulus of smoothness, Trigonometric polynomials, Muckenhoupt weight, Sharp direct and inverse theorems.

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Logarithmic Statistical Convergence Of Order α

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ABSTRACT

The idea of statistical convergence goes back to the first edition of monograph of Zygmund [8]. This notion was first defined for real sequences by Steinhaus [7] and Fast [4]. Schoenberg [6] has defined from a sequence to sequence summability method called *D*-convergence which, as he has shown, implies statistical convergence. Different mathematicians studied properties of statistical convergence and applied this concept in various fields such as measure theory, trigonometric series, approximation theory, locally convex spaces, Banach spaces, fuzzy set theory, fuzzy logic, interval analysis, set valued analysis, etc .

The order of statistical convergence of a sequence was previously given by Gadjiev and Orhan [3] and after then statistical convergence of order α and strong $p-$ Cesàro summability of order α studied by Çolak [2]. Moricz [5] studied the theorems relating to statistical harmonic summability and ordinary convergence of slowly decreasing or oscillating sequences. Alghamdi *et al*.[1] introduced the notion logarithmic statistical convergence. Furthermore, some connections between logarithmic statistical convergence and $[H,1]_q$ - summability are given.

In this study, we explain the notions of logarithmic α- density and logarithmic statistical convergence of order α for complex or real sequence for $0<\alpha\leq 1$.

Moreover, some connections between logarithmic statistical convergence of order α and strong $[H,1]^{\alpha}{}_q$ - summability of order α are explained.

Key Words: Density, statistical convergence, logarithmic density, statistical summability (H,1)

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Multilinear Calderón Zygmund Operators on the Vanishing Generalized Morrey Space with variable exponent

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ABSTRACT

It is well known that the Morrey space plays very important role in the harmonic analysis and also in the study of partial differential equations. In classical harmonic analysis, the vanishing Morrey space was first introduced by Vitanza [7] to obtain the regularity results for elliptic partial differential equations. Ragusa [4] studied the boundedness of some classical operators in vanishing Morrey spaces. The vanishing generalized Morrey spaces were first introduced by Samko [5] in 2013. This study considered the boundedness of a class of sublinear operators, including Hardy-Littlewood maximal operator and Calderón-Zygmund singular operators with standard kernel. The vanishing generalized weighted Morrey spaces were first introduced by the authors in [1,2] and the boundedness of some classical operators and their commutators in these spaces was studied in.

In very recent years, many papers have been published on variable exponent Lebesgue spaces and on variable exponent Morrey spaces. On the other hand, in last decade many studies have been published on the boundedness of multilinear singular integral operators. Tao et al. [6] proved the boundedness of Calderón-Zygmund type multilinear singular integrals on product of variable exponent Lebesgue spaces and on product of variable exponent Morrey spaces. In [8] the authors proved the boundedness of the multilinear maximal functions, multilinear singular integrals and multilinear Riesz potential on the product of generalized Morrey spaces. Long and Han [3] gave some characterizations for the boundedness of potential operators, maximal operators and singular integral operators on the vanishing generalized Morrey spaces with variable exponent.

In this talk, we shall present the boundedness conditions for the multilinear singular integral operators of Calderón-Zygmund type on the product of vanishing generalized Morrey spaces with variable exponent.

Key Words: multilinear singular integral, variable exponent, vanishing Morrey space

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New Distinguished Subspaces of FK Spaces

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ABSTRACT

The concept of FK spaces was investigated by Zeller in [1] and the properties of sectional convergence (AK), weak sectional convergence (SAK), and functional sectional convergence (FAK) in FK spaces were investigated by Zeller in [2]. BK spaces in which all sequences have bounded sections (AB) were studied by Sargent in [3]. Also, the notions of convergent and bounded sections in FK spaces were studied by Garling in [4, 5]. In [6], Sember viewed unconditional sections in sequence spaces and characterized some properties. In addition to this, more general notions of Cesaro section boundedness and convergence and T-section boundedness and convergence were investigated and shown to be significant in [7]. In this talk, we present new distinguished subspaces of FK spaces and some properties of them. In second section, we give brief information about sectional property and α-, β-, γ- duals and also we give some definitions and lemmas which are need in the our theorems. In third section, we construct new important subspaces of a locally convex FK space X containing Φ. Then, we give new definitions of the distinguished subspace. Also, we show the relation among these subspaces. That is, we prove an inclusion theorem among these new subspaces of FK spaces. We introduce some inclusions which are similar to given in [8]. Furthermore, we study new properties in accordance with previous investigations on related sectional properties such as [6, 7, 8]. Finally, we give some theorems regarding new distinguished subspaces of FK spaces.

Key Words: FK spaces, Sequence spaces, α-, β-, γ- duals.

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New Inner Product Quasilinear Spaces on Interval Numbers

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ABSTRACT

Aseev [2] launched a new branch of functional analysis by introducing the concept of quasilinear spaces which is generalization of classical linear spaces. He proceeds, in a similar way to linear functional analysis on quasilinear spaces by introducing the notions of norm, with quasilinear operators and functional. We know that any inner product space is a normed space and any normed space is a particular class of normed quasilinear space. Hence, this relation and Aseev's work motivated us to quasilinear counterparts of inner product space in classical analysis in [7]. Generally in [7], we give some consistent quasilinear counterparts of fundamental definitions and theorems in linear functional analysis.

In this work, we examine a new type of a quasilinear space, namely, "IR^n" interval space. We obtain some new theorems and results related to this new quasilinear space. After giving some new notions of quasilinear dependence-independence and basis on quasilinear functional analysis [5], we obtain some results on "IR^n" interval space related to these concepts. Furthermore, we present the concepts of "Is, Ic_0, Il_∞ and Il_2" interval sequence spaces as new examples of quasilinear spaces. Moreover, we obtain some theorems and results related to these new spaces which provide us with improving the elements of the quasilinear functional analysis.

Key Words: Quasilinear Space, Quasilinear Inner Product Space, Interval Space.

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On Asymptotically Deferred Statistical Equivalent Sequences of Order α

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ABSTRACT

The statistical convergence of real or complex valued sequences was first introduced by Fast [3], but the idea of statistical convergence goes back to Zygmund [8] in the first edition of his monograph published in Warsaw in 1935. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, Ergodic theory, Number theory, Measure theory, Trigonometric series, Turnpike theory and Banach spaces. In recent years, statistical convergence has become popular research area for many mathematicians [2, 4, 7] etc.

The concept of deferred density was given by Küçükaslan and Yılmaztürk [5] such as:

Let K be a subset of $\mathbb{N}$ and denote the set $\{k:p(n)<k\leq q(n),\ k\in K\}$ by $K_{p,q}(n)$. Deferred density of K is defined by

$$\delta_{p,q}(K)=\lim_{n\to\infty}[1/(q(n)-p(n))]\ |K_{p,q}(n)|, \text{ provided the limit exists,} \qquad (1)$$

where $\{p(n)\}$ and $\{q(n)\}$ are sequences of non-negative integers satisfying

$$p(n)<q(n) \quad \text{and} \quad \lim_{n\to\infty} q(n)=\infty.$$

The vertical bars in (1) indicate the cardinality of the set $K_{p,q}(n)$. If $q(n)=n$ and $p(n)=0$, then deferred density coincides natural density of K.

Definition Let $\{p(n)\}$ and $\{q(n)\}$ be two sequences as above and $\alpha\in(0,1]$ be any real number. Two nonnegative sequence $x=(x_k)$ and $y=(y_k)$ are said to be asymptotically statistical equivalent of order α if

$$\lim_{n\to\infty}[1/(q(n)-p(n))]^{\alpha}\ |\{p(n)<k\leq q(n): |(x_k/y_k)-L|\geq\varepsilon\}|=0.$$

In this study we introduce and examine the concepts of asymptotically deferred statistical equivalent and strong asymptotically deferred equivalent sequences of order α.

Key Words: Deferred statistical convergence, Statistical convergence, Asymptotically statistical equivalent sequences.

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On (p,q)-Analogue of Lupaş-Schurer Operators

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ABSTRACT

In 1912, Bernstein [5] defined the first Bernstein polynomial. Later, in 1987, Lupaş [6] pioneered the research of q-analogue of Bernstein operators. After that the applications of q-calculus have been studied by several researchers for thirty years. Very recently, Mursaleen, Ansari and Khan [3] initiated the research of (p,q)-analogue of Bernstein operators which reduce to q-Bernstein operators for p=1.

The purpose of this paper is to introduce Lupaş-Schurer operators based on (p,q)-integers. (p,q)-analogue of Lupaş-Schurer operators has an advantage to generate positive linear operators for all p > 0 and q > 0 whereas (p,q)-analogue of Bernstein-Schurer operators generates positive linear operators only if 0 < q < p ≤ 1.

We consider for each $p>0$, $q>0$ and for any $m\in\mathbb{N}$, $x\in[0,1]$ and $f\in C[0,l+1]$, fixed $l\in\mathbb{Z}^{+}\cup\{0\}$. We construct the (p,q)-analogue of Lupaş-Schurer operators by

$$L_{m,l}^{(p,q)}\left(f;x\right)=\sum_{k=0}^{m+l}\frac{f\left(\dfrac{p^{m+1-k}[k]_{p,q}}{[m]_{p,q}}\right)\begin{bmatrix}m+l\\k\end{bmatrix}_{p,q}p^{\frac{(m+l-k)(m+l-k-1)}{2}}q^{\frac{k(k-1)}{2}}x^{k}(1-x)^{m+l-k}}{\prod_{j=1}^{m+l}\{p^{j-1}(1-x)+q^{j-1}x\}}.$$

Note that, if we take p=q=1, (p,q)-Lupaş-Schurer operators reduce to be Schurer-Bernstein operators.

Then, we deal with the approximation properties for (p,q)-Lupaş-Schurer operators based on Korovkin type approximation theorem. We prove some auxiliary results which will be needed to establish the main results. Moreover, we estimate the rate of convergence by using modulus of continuity, with the help of functions of Lipschitz class and Peetre's K-functionals.

Key Words: (p,q)-integers, Lupaş operators, modulus of continuity, functions of Lipschitz class, Peetre's K-functionals.

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On Algebraic Structure of the Space of Quasilinear Operators

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ABSTRACT

It is a known fact in mathematics there has been a reluctance to deal with classes of sets and set-valued mappings. The mean reason of this is the classes of sets have no enough maturity in algebraic aspects. In 1986, Aseev [1] introduced a kind of the concepts of a quasilinear spaces both including a classical linear spaces and also nonlinear spaces of subsets and multivalued mappings. He launched a theory that we see it as the beginning of quasilinear algebra and quasilinear functional analysis. Hence, some problems about classes of subsets or set-valued mappings had been solved. This pioneer work has motivated a lot of authors to work the theory of quasilinear spaces [2-6].

As known, the theory of operator play a fundamental role in functional analysis and applications. Therefore, this actual and Aseev's work has motivated us to investigate quasilinear operators and the the space of bounded quasilinear operators between quasilinear spaces. In this paper, primarily we introduce the notions of quasilinear space, quasilinear operator and the space of bounded quasilinear operator which are given by Aseev. Secondly, we classify quasilinear spaces as consolidate and non-consolidate and we investigate algebraic structure of some set-valued function spaces which has a crucial role in set-differential equations and interval analysis.

Key Words: Quasilinear Spaces, Consolidate Quasilinear Spaces, Bounded Quasilinear Operators, The Space of Quasilinear Operators.

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On Approximation Theorems for (p,q)-Bernstein Operators

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ABSTRACT

In 1912, for a function $f(x)$ defined on the closed interval $[0,1]$, the expression

$$B_n(f;x)=\sum_{k=0}^{n} f\left(\frac{k}{n}\right)\binom{n}{k}x^k\left(1-x\right)^{n-k}$$

was called the Bernstein polynomial of order n of the function $f(x)$. Later, the various generalization of Bernstein polynomials $B_n(f;x)$ were investigated. In recent years, the development of q-calculus has allow to be made of new generalizations of approximation theory. Firstly, Lupas introduced the q-analogue of the Bernstein operators and investigated its approximation properties in 1987. After then, the various applications of q-Bernstein operators were handled by Philips. The approximation properties of q-generalization of other operators were studied. For instance, q-analogues of Bernstein– Kantorovich operators; q-Baskakov–Kantorovich operators; generalized integral Bernstein operators based on q-integers; q-Szasz–Mirakjan operators; q-Bleimann, Butzer and Hahn operators; q-analogue of Baskakov and Baskakov-Kantorovich operators; q-analogue of Szasz–Kantorovich operators; q-analogue of Stancu-Beta operators and q-Lagrange polynomials were defined and their approximation properties were investigated. Recently, Mursaleen et al. applied (p,q)-calculus approximation theory and introduced the (p,q)-analogue of Bernstein operator.

In this study, we introduce a new analogue of Bernstein operators and we call it as (p,q)-Bernstein operators which is a generalization of q-Bernstein operators. We also study approximation properties based on Korovkin's type approximation theorem of (p,q)-Bernstein operators and establish some direct theorems. We estimate the rate of approximation by modulus of continuity.

Key Words: Korovkin theorem, Bernstein operator, (p,q)-integers, modulus of continuity.

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On Characterization of Sobolev Spaces

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ABSTRACT

Sobolev spaces have their main motivation in the study of differential equations, both ordinary and partial ones. In many problems of mathematical physics and variational calculus, mostly those related to continuum mechanics or physics, it is not sufficient to deal with the classical solutions of differential equations. It is necessary to introduce the notion of weak derivatives and to work in the so called Sobolev spaces. From a practical standpoint, most useful feature of Sobolev spaces is their embedding characteristics, that is, their embedding properties are responsible for their usefulness.

Many researchers have suggested various definitions of Sobolev spaces and studied the well-known properties of Sobolev spaces. In a paper $[2]$ (see also $[2],[4],[5]$), Bourgain, Brezis, and Mironescu studied the limiting behaviour of the semi-norm

$$\left| f \right|_{W^{s,p}(\Omega)} = \left(\iint_{\Omega\Omega} \frac{\left| f(x) - f(y) \right|^p}{\left| x - y \right|^{N+sp}} dydx \right)^{\frac{1}{p}}$$

of the fractional Sobolev spaces, $W^{s,p}$ $\quad 0 < s < 1, \quad 1 < p < \infty,$ in which, when $s \uparrow 1$, then $\left| f \right|_{W^{s,p}(\Omega)} \to \infty$. This analysis has led them a new characterization of Sobolev spaces.

In $[1]$ and $[6]$ Nguyen extended some results and Leoni and Spector proved some open questions of the paper of Bourgain, Brezis, and Mironescu.

The purpose of this presentation is to discuss some new open questions, suggest some new insight and unify the papers which are studied in this area.

Key Words: Sobolev spaces, semi norm, radial mollifiers.

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On Convergence of Deferred Cesáro Means of Mellin-Fourier Series

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ABSTRACT

In this paper, we examined convergence of the Mellin-Fourier series of recurrent functions by using Deferred Nörlund means $(D_a^b N, p)$ and Deferred Riesz means $(D_a^b R, p)$. Also some important results were obtained.

A function $f\colon \mathbb{R}_+ \to \mathbb{C}$ is called recurrent if $f(e^{2\pi}x) = f(x), \forall x \in \mathbb{R}_+$. The function f will be called c-recurrent for $c \in \mathbb{R}$, if $e^{2\pi c} f(e^{2\pi}x) = f(x), \forall x \in \mathbb{R}_+$. Clearly that $c = 0$ equal to recurrent functions. We denote the function spaces under consideration by Y_c, where

$$Y_c = \left\{ f \in \mathbb{R}^1_{loc}(\mathbb{R}_+) : f \text{ is } c - \text{recurrent}, \|f\|_{Y_c} := \int_{e^{-\pi}}^{e^{\pi}} |f(u)| u^{c-1} du < \infty \right\}.$$

Mellin-Fourier series of $f \in Y_c$ is defined by

$$f(x) \sim \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} M^c(f;k) x^{-c-ik}, \quad x \in \mathbb{R}_+$$

where $M^c(f;k)$ is the finite Mellin transform of f at $k \in \mathbb{Z}$ defined as follows:

$$M^c(f;k) = \int_{e^{-\pi}}^{e^{\pi}} f(u) u^{c+ik-1} du.$$

Let $S_n^c(f;x)$ denote the partial sums of Mellin-Fourier series of f i.e.,

$$S_n^c(f;x) = \frac{1}{2\pi} \sum_{k=-n}^{n} M^c(f;k) x^{-c-ik}, \quad x \in \mathbb{R}_+.$$

Let $\sum_{n=0}^{\infty} u_n$ be given infinite series with the sequence of partial sums $\{s_n\}$. The Deferred Cesáro means of sequence $\{s_n\}$ is defined by [2],

$$D(a_n,b_n;s_n)=\frac{s_{a_n+1}+s_{a_n+2}+...+s_{b_n}}{b_n-a_n}$$

where (a_n) and (b_n) be sequences of nonnegative integers satisfying

$$a_n<b_n, \quad \lim_{n\to\infty} b_n=\infty. \tag{1.1}$$

In the notation of matrix transformation,

$$D(a_n,b_n;s_n)=\sum_{k=0}^{\infty} a_{nk}s_k$$

where

$$a_{nk}=\begin{cases}\dfrac{1}{b_n-a_n}, & a_n<k\le b_n\\ 0, & \text{otherwise}\end{cases}$$

This method is regular [2] under condition (1.1).

We now make the following definitions:

Let (p_n) be a sequence of positive real numbers and $S_n^c(f;x)$ the partial sums of Mellin-Fourier series of f. Therefore; the (D_a^bN,p)-means and (D_a^bR,p)-means of the Mellin-Fourier series of f are defined as

$$D_a^bN_n(f;x)=\frac{1}{P_0^{b_n-a_n-1}}\sum_{k=a_n+1}^{b_n} p_{b_n-k}S_k^c(f;x)$$

and

$$D_a^bR_n(f;x)=\frac{1}{P_{a_n+1}^{b_n}}\sum_{k=a_n+1}^{b_n} p_kS_k^c(f;x),$$

where

$$P_0^{b_n-a_n-1}=\sum_{k=0}^{b_n-a_n-1} p_k\neq 0, \quad P_{a_n+1}^{b_n}=\sum_{k=a_n+1}^{b_n} p_k\neq 0.$$

Two methods are known as Deferred Nörlund means (D_a^bN,p) and the Deferred Riesz means (D_a^bR,p), respectively. In the case $b_n=n$ and $a_n=0$, the methods (D_a^bN,p) and (D_a^bR,p) give us the classically known Nörlund and Riesz means, respectively. If we take $p_n=1$ for $\forall n\geq 0$, both of them correspond to Deferred Cesáro means

$$D_a^b(f;x)=\frac{1}{b_n-a_n}\sum_{k=a_n+1}^{b_n} S_k^c(f;x)$$

of $S_k^c(f;x)$. Besides this, if we take $b_n=n$, $a_n=0$ and $p_n=1$ for $\forall n\geq 0$ for this methods then we obtain Cesáro means

$$s_n^c(f;x)=\frac{1}{n+1}\sum_{k=1}^{n} S_k^c(f;x)$$

of $S_k^c(f;x)$. Moreover, $a_n=0$, $p_n=1$ for $\forall n\geq 0$ and (b_n) is a strictly increasing sequence of positive integers with, then we obtain Cesáro submethod which is obtained by deleting a set of rows from Cesáro matrix (see [1]).

Key Words: Recurrent Functions, Mellin Transform, Mellin Fourier Series, Deferred Nörlund Means, Deferred Riesz Means.

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On Deferred Nörlund Mean

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ABSTRACT

The limit notation plays very central role in many problems in mathematics. So, several summability methods have been defined to expand the set of usual convergent sequences. One of the most important summability methods is the Cesàro mean and weighted mean, which is its generalization.

In [1], Agnew defined deferred Cesàro mean and gave many inclusion results between deferred Cesàro mean and Cesàro mean.

In this talk, we will give a typical generalization of Deferred Nörlund mean by using any sequence of positive natural numbers $a = (a_n)$ and b=(b_n) such that $a_n < b_n$ and $\lim_{n\to\infty} b_n = +\infty$.

Deferred Nörlund mean of a sequence $x = (x_n)$ is defined as follows:

$$(DN_p^{a,b}x)_n := \frac{P_{a_n+1}x_1 + P_{a_n+2}x_2 + \cdots + P_{b_n}x_n}{P_{b_n} - P_{a_n}}$$

where $P_{b_n} := p_0 + p_1 + \cdots + p_{b_n}$ for positive natural numbers sequence $p = (p_n)$.

It is denoted by $\lim_{n\to\infty} x_n = l\ (DN_p^{a,b})$ if $\lim_{n\to\infty} (DN_p^{a,b}x)_n = l$.

Some cases of Deferred Nörlund Mean is well known in literature. For example; the case $a = (a_n) = (0,0,\ldots)$ and $b = (b_n) = (1,2,\ldots,n,\ldots)$ give us classic Nörlund mean of sequence $x = (x_n)$ and the case $p = (p_n) = (1,1,1,\ldots)$ is coincide with Deferred Cesàro Mean which is defined by Agnew in [1] as a generalization of Cesàro Mean for any $a = (a_n)$ and b=(b_n).

It is shown that Deferred Nörlund means is regular. Also, some inclusion result between Nörlund mean and Deferred Nörlund mean has been given under some certain restrictions on $a = (a_n)$ and b=(b_n).

Key Words: Cesàro Mean, Deferred Cesàro Mean, Nörlund Mean, Deferred Nörlund Mean, Convergence of sequences, Submethods.

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On Hermite-Hadamard Type Inequalities For (α, a, b) −Preinvex Functions via Riemann-Liouville Fractional Integrals

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ABSTRACT

The relationship between theory of convex functions and theory of inequalities has occured as a result of many researches investigation of these theories.

A very intersting result in this regard is due to Hermite and Hadamard independently that is Hermite-Hadamard’s inequality.

The $f: I \subset \mathbb{R} \to \mathbb{R}$ be a convex function defined on a interval I of real numbers $a, b \epsilon I$ and $a < b$, given by

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}.$$

The classical Hermite-Hadamard’s inequalities have attracted many researchers since 1893. Researchers investigated Hermite- Hadamard inequalities involving fractional integrals according to the associated fractional integral equalities and different types of convex functions. As a result many new and interesting generalizations of classical convex functions can be found in the literature. An important extension of conex functions was the introduction of preinvex function.

The $K = [u, u + \eta(v, u)]$ be a nonempty closed set in $\mathbb{R}^n$. Let $f: K \to \mathbb{R}$ be a continuous function and let $\eta(.,.): K \times K \to \mathbb{R}^n$ be a continuous bifunctions.

Now, we introduce the following definition.

Definition: A function $0 \neq T \subseteq [0,1]$ and $\alpha, a, b: T \to \mathbb{R}$ is said α, a, b −preinvex functions of second kind with respect to $\eta(.,.)$, we have

$$f(y + \alpha(t)\eta(x, y) \leq a(t)f(x) + b(t)f(y).$$

1.When $\alpha(t) = t,\ \eta(x,y) = x - y,\ a(t) = (1-t),\ b(t) = t,$ we obtained classic convex function.

2.When $\alpha(t) = t,\ a(t) = (1-t), b(t) = t,$ we obtained classic preinvex function.

3.When $\alpha(t) = t,\ a(t) = (1-t)^s, b(t) = t^s$, we obtained s-preinvex function.

4.When $\alpha(t) = t,\ a(t) = \left(\frac{1}{1-t}\right)^s, b(t) = \left(\frac{1}{t}\right)^s$, we obtained s-Godunova Levin preinvex function.

In this work, we derive a new fractional integral identity for differentiable functions. Using this we obtain our main results that are fractional Hermite-Hadamard type inequalities for differentiable $(\alpha;\ a;\ b)$ −preinvex functions.

We establish a new fractional integral inequalities for diferentiable function, then using this auxiliary result we derive some fractional Hermite-Hadamard type inequalities for differentiable $(\alpha;\ a;\ b)$ −preinvex functions. Moreover, we also consider their relevances for other related known results.

Key Words: Fractional Hermite-Hadamard inequalities, Preinvex functions, Riemann-Liouville Fractional Integral.

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On Inequalities Generalized Riemann-Liouville K-Fractional Integration

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ABSTRACT

Fractional calculus has been known since the 17th century. Recently, the interest in fractional analysis has been growing continuously because of its useful applications in many fields of sciences such as electromagnetic waves, visco-elastic systems, quantum evolution of complex systems, diffucion waves, physics, engineering, finance, social sciences mathematical biology and chaos theory. Furthermore, fractional calculus, improved not only in pure theoretical fields but also in diverse fields ranging from physical sciences and engineering to biological sciences and economics[1]. Anymore work in this direction has long been overdue.

In 1938, Ostrowski founded an interesting integral inequality associated with differentiable mappings. Then by making use of founded identity, some new Ostrowski type inequalities for Riemanne-Liouville fractional integral are founded. In 2009, Anastassiou gave the fractional version of Ostrowski inequality. Zeki gave a generalization of Anastassiou. There is not much working done in this way and needs to be discovered as fractional Ostrowski inequality is expectedas to have applications in many fields in the same direction as its counterpart.

In this paper, we generalized Montogomery identities for Riemanne-Liouville fractional integrals by using a new Peano kernel. We also use this Montogomery identities to establish some new Ostrowoski-type integral inequalities. Some inequalities via convex function are also discussed.

Definition 1: Generalized k-fractional integral operator of order $p \geq 0,$ $\alpha \geq 0$ and $k > 0$is defined as

$$J_{m,p}^{\frac{\alpha}{k}} g(x) = \frac{1}{\Gamma_k(\alpha)} \int_m^x (x^{p+1} - t^{p+1})^{\frac{\alpha}{k}-1} g(t) t^p dt, \ \ m < x, \tag{1}$$

$$J_{n,p}^{\frac{\alpha}{k}} g(x) = \frac{1}{\Gamma_k(\alpha)} \int_x^n (t^{p+1} - x^{p+1})^{\frac{\alpha}{k}-1} g(t) t^p dt, \ \ n > x. \tag{2}$$

This is integral Riemann-Liouville fractional integral operator also called.

Theorem 1: Let $p \geq 0, \alpha \geq 1$ and $k > 0$ for $m < n; g' \in L_1(m,n)$ and $g: [m,n] \to \mathbb{R}$ be differentiable $\forall x \in [m,n]$ if $|g'(x)| \leq M$ then the inequality is bellow exists:

$$\begin{aligned} &\left| \begin{matrix} (1-h)g(x) + \frac{h}{2}(n^{p+1} - x^{p+1})^{1-\frac{\alpha}{k}}(n^{p+1} - m^{p+1})^{\frac{\alpha}{k}-2} g(m) \\ -\frac{\Gamma_k(\alpha)}{n^{p+1} - m^{p+1}}(n^{p+1} - x^{p+1})^{1-\frac{\alpha}{k}} J_{m,p}^{\frac{\alpha}{k}}\big(g(n)\big) + J_{m,p}^{\frac{\alpha}{k}-1}\big(\Omega_2(x,n) g(n)\big) \end{matrix} \right| \\ &\qquad \leq \frac{M}{\frac{\alpha}{k}\left(\frac{\alpha}{k} - 1\right)} \left[\frac{2(n^{p+1} - x^{p+1})}{n^{p+1} - m^{p+1}} \left(\frac{\alpha}{k} - 1\right) \left(\frac{(n^{p+1} - m^{p+1})}{2} - x^{p+1} \right) \right. \\ &\qquad - \frac{2}{n^{p+1} - m^{p+1}}(n^{p+1} - x^{p+1})^2 + (n^{p+1} - m^{p+1})^{\frac{\alpha}{k}} \\ &\qquad \left. - \left(\frac{\alpha}{k} + 1\right) \frac{h}{2} (n^{p+1} - m^{p+1})^{\frac{\alpha}{k}} (n^{p+1} - x^{p+1})^{1-\frac{\alpha}{k}} \right]. \end{aligned} \tag{3}$$

Here Ω_2 is defined by

$$\Omega_2(x,t) = \begin{cases} \frac{1}{n^{p+1} - m^{p+1}}(n^{p+1} - x^{p+1})^{1-\frac{\alpha}{k}} \Gamma_k(\alpha) \left[t^{p+1} - \left(m^{p+1} + h\frac{n^{p+1} - m^{p+1}}{2} \right) \right], m \leq t \leq x \\ \frac{1}{n^{p+1} - m^{p+1}}(n^{p+1} - x^{p+1})^{1-\frac{\alpha}{k}} \Gamma_k(\alpha) \left[t^{p+1} - \left(n^{p+1} + h\frac{n^{p+1} - m^{p+1}}{2} \right) \right], x < t \leq n. \end{cases} \tag{4}$$

This (6) inequality is called Ostrowski integral inequality.

Key Words: Riemann-Liouville Fractional Integrations, Integral Inequalities, Ostrowski Inequalities.

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On Lacunary Statistical Convergence of Metric Valued Sequences

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ABSTRACT

The statistical convergence of real or complex valued sequences was first introduced by Fast [3], but the idea of statistical convergence goes back to Zygmund [8] in the first edition of his monograph published in Warsaw in 1935. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, Ergodic theory, Number theory, Measure theory, Trigonometric series, Turnpike theory and Banach spaces. In recent years, statistical convergence has become popular research area for many mathematicians [1, 2, 4, 5, 6, 7] etc.

In this study using a lacunary sequence, we introduce the concepts of lacunary d-statistical convergence sequences and lacunary strong d_p-Cesaro summable sequences in general metric spaces.

Definition 1. A metric-valued sequence $x = (x_n)$ is said to be lacunary d-statistical convergent to L if

$$\lim_{r\to\infty}(1/(h_r))|\{k\in I_r : d(x_k,L)\geq\varepsilon\}|=0$$

is true for every $\varepsilon > 0$.

Definition 2. A metric valued sequence $x = (x_n)$ is said to be lacunary strong d_p-Cesaro convergent to L, if there is a complex number L such that

$$\lim_{r\to\infty}(1/(h_r))\sum_{k\in I_r}[d((x_k,L)]^p=0.$$

Theorem Let θ be a lacunary sequence, then

i) If a sequence $x = (x_n)$ is lacunary strong d_p-Cesaro convergent to L, then it is lacunary d-statistical convergent to L.

ii) If a d-bounded sequence $x = (x_n)$ is lacunary d-statistical convergent to L, then it is lacunary strong d_p-Cesaro convergent to L.

Key Words: Metric valued sequences, Statistical convergence, lacunary sequence.

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On Solvability Of Some Nonlinear Fractional Interval Integral Equations

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ABSTRACT

As it is known, nonlinear integral equations and set-valued analysis constitute important branchs of nonlinear functional analysis. Particularly, integral equations are often used in characterization of several problems of engineering, mechanics, physics, economics, biology and so on, [4], [5]. On the other hand, set-valued maps provide a useful framework for control theory, optimization theory, game theory, robotics, chemical engineering and mathematical economics, [2], [8], [13]. For this reason $C([a,b],\Omega_c(\mathbb{R}))$ and $L_1([a,b],\Omega_c(\mathbb{R}))$ which are two classes of interval-valued maps have important places in set-valued analysis.

In recent years, some authors such as S. Arshad [1], V. Lupulescu [9], [10], Y. Shen [16], M.T. Malinowski [11], L. Stefanini and B. Bede [18], S. Salahshour and M. Khan [14] and references therein give various results about interval-valued differential and integral equations.

In this paper, firstly we deal with solvability of a first kind nonlinear interval integral equation of fractional order. Then we present a theorem giving sufficient conditions for existence of solution of a second kind nonlinear interval integral equation of fractional order in the space of continuous interval-valued functions on the interval $[a,b]$ by using Banach fixed point theorem. We give also some examples satisfying the conditions of our main theorems.

Key Words: Interval-valued function, Interval integral equations, Nonlinear integral equations, Banach fixed point theorem, Existence of solutions.

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On spectral separation of infinite matrices over some sequence spaces

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ABSTRACT

Spectral theory is one of the main branches of modern functional analysis and its applications. Roughly speaking, it is concerned with certain inverse operators, their general properties and their relations to the original operators. Such inverse operators arise quite naturally in connection with the problem of solving equations (systems of linear algebraic equations, differential equations, integral equations). For instance, the investigations of boundary value problems by Sturm and Liou- ville and Fredholm's famous theory of integral equations were important to the development of the field.

Let x = (x_n) be complex sequences. Cesaro operator $C_1 x = y = (y_n)$ is defined by $y_n = \frac{1}{n}\sum_{k=1}^{n} x_k$.

In 1965 Brown and his colleagues determined the spectrum and eigen values of Cesaro operator on the Hilbert space l_2 of square summable sequences.

In 1985, Reade identified the spectrum of the Cesaro operator over c_0.

Corollary [Reade, 1985] $C \in B(c_0)$ and $\|C\| = 1$.

Corollary [Reade, 1985] $C^* \in B(l^1)$ and $\|C^*\| = 1$.

Theorem [Reade, 1985] $C \in B(c_0)$ has no eigenvalues.

Theorem [Reade, 1985] $C \in B(c_0)$ has no eigenvalues.

Theorem [Reade, 1985] The eigenvalues $C^* \in B(l^1)$ are all λ satisfying $\left|\lambda - \frac{1}{2}\right| < \frac{1}{2}$, each having multiplycity 1.

Theorem [Reade, 1985] $\sigma(C, c_0) = \left\{\lambda : \left|\lambda - \frac{1}{2}\right| \le \frac{1}{2}\right\}$.

In the same year, Gonzalez identified the fine spectrum (in the sense of Goldberg) of the same operator over the l_p (1<p<∞) sequences space.

In 1985, Okutoyi identified the spectrum of the Cesaro operator over c.

Theorem [Okutoyi, 1986] $\sigma_p(C,c) = \{1\}$.

Theorem [Okutoyi, 1986] $\sigma_p(C^*,l^1) = \left\{\lambda: \left|\lambda - \frac{1}{2}\right| < \frac{1}{2}\right\} \cup \{1\}$.

Theorem [Okutoyi, 1986] $\sigma(C,c) = \left\{\lambda: \left|\lambda - \frac{1}{2}\right| \leq \frac{1}{2}\right\}$.

In 1985, Wenger identified the fine spectrum of the Cesaro operator over c.

Theorem [Wenger, 1975] If $Re\left(\frac{1}{\lambda}\right) < 1$, then $\lambda \epsilon \rho(C)$.

Theorem [Wenger, 1975] If $Re\left(\frac{1}{\lambda}\right) < 1, \lambda \neq \frac{1}{2}, \frac{1}{3}, \ldots$ then $\lambda \epsilon III_1 \sigma(C,c)$.

Theorem [Wenger, 1975] If $\lambda = \frac{1}{2}, \frac{1}{3}, \ldots$ then $\lambda \epsilon III_1 \sigma(C,c)$.

Theorem [Wenger, 1975] If $Re\left(\frac{1}{\lambda}\right) = 1, \lambda \neq 1$ then $\lambda \epsilon II_2 \sigma(C,c)$.

Theorem [Wenger, 1975] $1 \epsilon III_3 \sigma(C,c)$.

In 2011, Durna and colleagues first gave a non-discrete spectral decomposition (defect spectrum, approximation point spectrum, and compression spectrum) of a infinitive matrix.

Theorem [Amirov, Durna, Yildirim, 2011] a) $\sigma_{ap}(C,c) = \left\{\lambda: \left|\lambda - \frac{1}{2}\right| = \frac{1}{2}\right\}$,

b) $\sigma_\delta(C,c) = \left\{\lambda: \left|\lambda - \frac{1}{2}\right| \leq \frac{1}{2}\right\}$

c) $\sigma_{co}(C,c) = \left\{\lambda: \left|\lambda - \frac{1}{2}\right| < \frac{1}{2}\right\} \cup \{1\}$

In this study, these separations will be introduced and spectra and spectral separations of a certain operator will be given.

Key words: Spectrum, fine spectrum, subdivision of the spectrum, Cesaro operator

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On Subclasses Of Bi-Close-To-Convex Functions

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ABSTRACT

Let S denote the class of functions of the form

$$f\left(z\right)=z+\sum_{n=2}^{\infty}a_n z^n \qquad (1)$$

which are analytic and univalent in the open unit disk $U=\{z\in\mathbb{C}:|z|<1\}$. Let K and S^* denote the usual subclasses of whose members are close-to-convex and starlike in U, respectively. Gao and Zhou [1] discussed a class K_s of analytic functions related to the starlike functions, a function $f\in S$, is said to be in the class K_s if it satisfies the inequality:

$$Re\left(\frac{-z^2 f'(z)}{g(z)g(-z)}\right)>\gamma,\ (z\in U) \qquad (2)$$

where $g(z)=z+b_2z^2+b_3z^3+...\in S^*(\frac{1}{2})$.

They proved that if $G(z)$ defined by $g(z)$ then

$$G\left(z\right)=\frac{-g(z)g(-z)}{z}=z+\sum_{n=2}^{\infty}B_{2n-1}z^{2n-1} \qquad (3)$$

where for $n=2,3,...,$

$$B_{2n-1}=2b_{2n-1}-b_2b_{2n-2}+...+(-1)^n 2b_{n-1}b_{n+1}+(-1)^{n+1}b_n{}^2 \qquad (4)$$

then $G\in S^*$. Also, they showed that the class K_s is a subclass of the class K of close-to-convex functions.

It is known that every univalent function f has an inverse f^{-1} satisfying

$$f\left(f^{-1}\left(z\right)\right)=z \qquad \left(z\in \mathrm{U}\right)$$

and

$$f\left(f^{-1}\left(w\right)\right)=w \qquad \left(\left|w\right|<r_0\left(f\right);r_0\left(f\right)\geq\frac{1}{4}\right).$$

In fact, the inverse function is

$$F(w)=f^{-1}(w)=w-a_2w^2+\left(2a_2^2-a_3\right)w^3-\left(5a_2^3-5a_2a_3+a_4\right)w^4+\ldots \ .$$

A function $f\in\mathrm{A}$ is said to be bi-univalent if both f and f^{-1} are univalent in U. We denote by Σ the class of all bi-univalent functions in U stated by Taylor Maclaurin series expansion. Similarly, a function $f\in\mathrm{A}$ is said to be bi-close-to convex in U if both f and $F=f^{-1}$ are close-to-convex in U.

For a brief history of functions in the class Σ, see [1] (see also [3], [4] and [5]).

In this study, we give new subclasses of the bi-close-to-convex functions. Moreover, we obtain initial coefficient for the functions belonging these classes. These new classes will be able to described depending on this class for further studies[6].

Key Words: Bi-univalent, Close-to-convex, Starlike function,

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On Subordination Result Associated with a Subclass of Analytic Functions

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ABSTRACT

Let A denote the class of analytic functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

in the unit disc $U = \{z \in \mathbb{C};\ |z| < 1\}$ normalized by $f(0) = f'(0) - 1 = 0$. For two functions $f(z)$ and $g(z)$ given by $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$, their Hadamard product (or convolution) is defined by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n .$$

In [1], Zainab et al. introduced a class A_μ of functional fractional analytic functions $F_\mu(z)$ in the unit disc U such that $F_\mu(z) = \dfrac{z^\mu}{1 - z^\mu}$ where $\mu = \dfrac{n+m-1}{m}$, $n, m \in \mathbb{N}$. When $n = 1$ we have $\mu = 1$ and $F_\mu(z)$ has the power series

$$F_\mu(z) = z + \sum_{n=2}^{\infty} a_n z^{\mu n} \quad ; \quad \mu \geq 1;\ n \in \mathbb{N};\ z \in U \tag{2}$$

which normalized by $F_\mu(z)\big|_{z=0} = 0$ and $F'_\mu(z)\big|_{z=0} = 1$ for all $z \in U$.

Recall that a function $F_\mu \in A_\mu$ is called bounded turning if it satisfies the following inequality

$$\Re\left\{F'_\mu(z)\right\} > \psi \qquad (0 \le \psi < 1), \tag{3}$$

and a function $F_\mu \in A_\mu$ is starlike function in U if satisfies

$$\Re\left\{\frac{zF'_\mu(z)}{F_\mu(z)}\right\} > \psi \qquad (0 \le \psi < 1). \tag{4}$$

In addition, a function $F_\mu \in A_\mu$ is convex function in U if satisfies

$$\Re\left\{1 + \frac{zF''_\mu(z)}{F'_\mu(z)}\right\} > \psi \qquad (0 \le \psi < 1). \tag{5}$$

(see [2] and [3]). Then, if the function $F_\mu(z)$ given by (2) and $G_\mu(z) = z + \sum_{n=2}^{\infty} b_n z^{\mu n}$ are two functions in class A_μ, then the convolution (or Hadamard product) of two analytic functions is denoted by $F_\mu * G_\mu$ and is given by

$$F_\mu(z) * G_\mu(z) = z + \sum_{n=2}^{\infty} a_n b_n z^{\mu n}$$

and satisfy

$$\left[F_\mu(z) * G_\mu(z)\right]\Big|_{z=0} = 0 \text{ and } \left[\left(F_\mu(z) * G_\mu(z)\right)'\right]\Big|_{z=0} = 1.$$

Briefly, in this study we investigate functions belonging to the class of fractional analytic functions in the open unit disk. Furthermore, we derive several subordination results involving the Hadamard product (or convolution) of the associated these functions. A number of interesting consequences of some of these subordination results are also discussed. Also, this effort covers a number of consequences of subordination result.

Key Words: Subordination, analytic function, fractional analytic function.

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On the Approximation by Max-Min and Max-Product Operators

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ABSTRACT

In this paper, we study the approximation properties of max-min and max-product operators that are pseudo linear. These operators were first introduced and systematically investigated by Bede et. al. (see [1]). We should remark that since the concept of pseudo linearity is weaker than the usual linearity, the well-known Korovkin type approximation theory fails for these operators. However, we obtain a similar structure to ones provide by the classical approximation theory. The idea and the problem come from both the theory and the applications of fuzzy sets. In the present paper we discuss in detail the problem of how approximations can be defined based on different structures.

In the approximation we use not only the usual convergence but also some weaker convergence methods, such as, statistical convergence, arithmetic mean convergence, almost convergence (see [2] and [3]). For example, it is well-known that although a convergent sequence is statistically convergent to the same value, its converse is not always true. In the approximation, the rate of convergence is controlled by the modulus of continuity, which is one of the main tools in the approximation theory. At the end of this study, we give some applications to show that why we need such operators rather than the classical ones.

Key Words: Statistical convergence, max-product operators, max-min operators, modulus of continuity.

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On the New Banach Sequence Spaces Defined by Lucas Numbers

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ABSTRACT

The approach of constructing a new sequence space by the help of matrix domain of an infinite triangle matrix is interesting and it has been studied by a lot of authors. However, one of the topics being worked on since the geometry and analysis began to be mentioned together is geometric properties of sequence spaces. Because of the properties of the spaces $\ell(p)$ and ℓ_p, $\ell(p)$-type spaces have great numbers of practical applications. Moreover, it is natural to consider the geometric structure of these spaces since the space ℓ_p is reflexive and convex. In more recent times, there has been great interest in studying geometric properties of sequence spaces besides topological and some other common properties.

In the present paper, our main goal is to construct a new matrix and by using matrix domain of this matrix, to introduce a new sequence space and also examine some topological and geometrical properties.

In the light of this information, using Lucas sequence, we establish a new band matrix $\hat{E}=(\hat{L}_{nk})$ and define a new sequence space $\ell_p\left(\hat{E}\right), 1\leq p\leq\infty$ where L_k is the k th Lucas number. Besides, we show that this space is a BK-space and also linearly isomorphic to the space ℓ_p. In addition, we give some inclusion relations and investigate the geometrical properties such as Banach-Saks type p and the weak fixed point property.

Key Words: Difference sequence spaces, Lucas numbers, matrix domain.

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On the solutions of a class of nonlinear functional integral equations in space C[0,a]

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ABSTRACT

Nonlinear integral equations are important part of nonlinear analysis. It is caused by the fact that this theory is frequently applicable in other branches of mathematics and mathemathical physics, engineering, economics, biology as well in describing problems connected with real world, [5]. The measure of noncompactness and theory of integral equations are rapidly developing with the help of tools in functional analysis, topology and fixed-point theory. Many articles in the field of functional integral equations give different conditions for the existence of the solutions of some nonlinear functional integral equations. A. Aghajani and Y. Jalilian in [1], J. Banaś and K. Sadarangani in [3], Zeqing Liu et al. in [11] and so on are some of these.

The principal aim of this paper is to give sufficient conditions for solvability of a class of some nonlinear functional integral equations in the space of continuous functions defined on interval [0,a]. We should note that the equation given in this paper has rather general form and contains as particular cases a lot of nonlinear integral equations of Volterra type.The main tool used in our study is associated with the technique of measures of noncompactness. We give also some examples satisfying the conditions of our main theorem but not satisfying the conditions in [8].

Key Words: Nonlinear integral equations, Measure of noncompactness, Darbo fixed point theorem, Darbo condition.

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On the Spectrum and Fine Spectrum of the Upper Triangular Matrix $U(a_0, a_1, a_2; b_0, b_1, b_2)$ over the Sequence Space c_0

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ABSTRACT

Let X and Y be the Banach spaces, and $L: X \to Y$ also be a bounded linear operator. By $R(L)$, we denote the range of L, i.e.,

$$R(L) = \{y \in Y: y = Lx, x \in X\}.$$

By $B(X)$, we also denote the set of all bounded linear operators on X into itself. Let $L: D(L) \to Y$ be a linear operator, defined on $D(L) \subset X$, where $D(L)$ denote the domain of L and X is a complex normed linear space. For $L \in B(X)$ we associate a complex number λ with the operator $(\lambda I - L)$ denoted by L_λ defined on the same domain $D(L)$, where I is the identity operator. The inverse $(\lambda I - L)^{-1}$, denoted by L_λ^{-1} is known as the resolvent operator of L_λ.

A regular value of L is a complex number λ of L such that L_λ^{-1} exists, is bounded and, is defined on a set which is dense in X.

The resolvent set of L is the set of all such regular values a of L, denoted by $\rho(L,X)$. Its complement is given by $\mathbb{C} - \rho(L,X)$ in the complex plane $\mathbb{C}$ is called the spectrum of L, denoted by $\sigma(L,X)$. Thus the spectrum $\sigma(L,X)$ consist of those values of $\lambda \in \mathbb{C}$, for which L_λ is not invertible.

The spectrum $\sigma(L,X)$ is union of three disjoint sets as follows: The point (discrete) spectrum $\sigma_p(L,X)$ is the set such that L_λ^{-1} does not exist. We say that $\lambda \in \mathbb{C}$ belongs to the continuous spectrum $\sigma_c(L,X)$ of L if the resolvent operator L_λ^{-1} is defined on a dense subspace of X and is unbounded. Furthermore, we say that $\lambda \in \mathbb{C}$ belongs to the residual spectrum $\sigma_r(L,X)$ of L if the resolvent operator L_λ^{-1} exists, but its domain of definition is not dense in X; in this case L_λ^{-1} may be bounded

or unbounded. Together with the point spectrum, these two sub spectra form a disjoint subdivision

$$\sigma(L,X) = \sigma_p(L,X) \cup \sigma_c(L,X) \cup \sigma_r(L,X)$$

of the spectrum of L.

Several authors have determined spectra and fine spectra of different lower triangular and upper triangular matrices over different sequence space with non-zero diagonals has entries of some constant sequences or some convergent sequences.

In this study, we determine the spectra and fine spectra of the upper triangular matrix

$$U(a_0,a_1,a_2;b_0,b_1,b_2) = \begin{pmatrix} a_0 & b_0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & a_1 & b_1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & a_2 & b_2 & 0 & 0 & \cdots \\ 0 & 0 & 0 & a_0 & b_0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & a_1 & b_1 & \cdots \\ 0 & 0 & 0 & 0 & 0 & a_2 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

over the sequence space c_0.

Key Words: Spectrum, point spectrum, residual spectrum, continuous spectrum, fine spectrum.

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On The Trigonometric Approximation of Functions Belonging to weighted Lorentz spaces Using Cesaro Submethod

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ABSTRACT

One of the well known examples of Banach spaces is Lebesgue space. There are many results about the approximation theory obtained using different summability method in Lebesgue spaces [3,4,5,6]. This space may be generalized in different ways. One of the generalizations of this space is Lorentz space which is Banach space. Using a weight function satisfying Muckenhoupt condition weighted Lorentz space is obtained. Many researchers interested in the approximation problems of weighted Lorentz spaces in [7,8]. But these papers have no trigonometric approximation results obtained using Cesàro submethod in the weighted Lorentz spaces.

Let $(\lambda_n)_{n=1}^{\infty}$ be a strictly increasing sequence of positive integers. For a sequence (x_k) of the real or complex numbers, the Cesàro submethod C_λ is defined as

$$(C_\lambda x)_n := \frac{1}{\lambda_n}\sum_{k=1}^{\lambda_n} x_k, \text{ (n=1,2,...)}.$$

Particulary, when $\lambda_n = n$ we note that $(C_\lambda x)_n$ is the classical Cesàro method (C,1) of (x_k). Therefore, the Cesàro submethod C_λ yields a subsequence of the Cesàro method (C,1) in [1,2].

Our study has trigonometric approximation results obtained using Cesàro submethod in the weighted Lorentz spaces. In this space, the approximation properties of trigonometric polynomials are given using this method of their Fourier series. Especially, we investigated approximation properties of trigonometric polynomials in the weighted Lorentz spaces with Muckenhoupt weights using infinite

lower triangular regular matrix obtained by Cesàro submethod of Fourier series of functions in this space.

Key Words: Cesaro submethod, Weighted Lorentz spaces, Muckenhoupt weight, Trigonometric approximation.

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On λ-Statistical Convergence of Order α in Probability

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ABSTRACT

The statistical convergence of real or complex valued sequences was first introduced by Fast [4], but the idea of statistical convergence goes back to Zygmund [6] in the first edition of his monograph published in Warsaw in 1935. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, Ergodic theory, Number theory, Measure theory, Trigonometric series, Turnpike theory and Banach spaces. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also connected with subsets of the Stone-Čech compactification of the natural numbers. Moreover, statistical convergence is closely related to the concept of convergence in probability. Recently, statistical convergence has become popular research area for many mathematicians [1,2,3,5] etc.

In the paper [3]; Ghosal introduced and studied the concept of S_λ-convergence of a sequence of random variables in probability. In this study we examine others relations with related the concepts of S_λ-convergence of order α in probability and strong λ-Cesaro convergence of order α in probability.

Key Words: Statistical convergence of order α in probability, Cesaro summability of order α in probability, λ-statistical convergence.

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Riesz potential in the local Morrey-Lorentz spaces and some applications

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ABSTRACT

In a series of papers [2], [3] and [5] the local Morrey-Lorentz spaces $M^{loc}_{p,q;\lambda}(R^n)$ have been introduced and the basic properties of these spaces have been given, and the boundedness of the Hilbert transform H, and the Hardy-Littlewood maximal operator M, the Calderon-Zygmund operators T on the local Morrey-Lorentz spaces $M^{loc}_{p,q;\lambda}(R^n)$ has been extensively studied, respectively.

In this talk, we give the boundedness of the Riesz potential I_α defined by

$$I_\alpha f(x) = \int_{R^n} \frac{f(x-y)}{|y|^{n-\alpha}} dy, \quad 0<\alpha<n,$$

in the local Morrey-Lorentz spaces $M^{loc}_{p,q;\lambda}(R^n)$. We find the necessary and sufficient conditions for the boundedness of the Riesz potential I_α in the local Morrey-Lorentz spaces $M^{loc}_{p,q;\lambda}(R^n)$.

The following theorem is the main result of our paper, in which we get the boundedness of the Riesz potential in the local Morrey-Lorentz spaces $M^{loc}_{p,q;\lambda}(R^n)$.

Theorem. Let $0<\alpha<n$, $0\le\lambda<1$, $1\le r\le s\le\infty$, $1\le q\le\infty$, $\frac{r}{r+\lambda}\le p\le\left(\frac{\lambda}{r}+\frac{\alpha}{n}\right)^{-1}$ and $f\in M^{loc}_{p,r;\lambda}(R^n)$. Then the Riesz potential $I_\alpha f$ exists almost everywhere. Furthermore,

(i) If $\frac{r}{r+\lambda} < p < \left(\frac{\lambda}{r} + \frac{\alpha}{n}\right)^{-1}$, then the condition $\frac{1}{p} - \frac{1}{q} = \lambda\left(\frac{1}{r} - \frac{1}{s}\right) + \frac{\lambda}{n}$ is necessary and sufficient for the boundedness of the operator I_α from the spaces $M_{p,r;\lambda}^{loc}(R^n)$ to $M_{q,s;\lambda}^{loc}(R^n)$.

(ii) If $p = \frac{r}{r+\lambda}$, then the condition $1 - \frac{1}{q} = \frac{\alpha}{n} - \frac{\lambda}{s}$ is necessary and sufficient for the boundedness of the operator I_α from the spaces $M_{p,r;\lambda}^{loc}(R^n)$ to $WM_{q,s;\lambda}^{loc}(R^n)$.

This result we apply to the boundedness of particular operators such as the fractional maximal operator, fractional Marcinkiewicz operator and fractional powers of some analytic semigroups on the local Morrey-Lorentz spaces $M_{p,q;\lambda}^{loc}(R^n)$.

Key Words: Local Morrey-Lorentz space, Riesz potential, Hardy operator.

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Second Hankel Determinant Problem for Starlike and Convex Functions Related to Fibonacci Numbers

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ABSTRACT

Geometric function theory is that branch of complex analysis, which deals and studies the geometric properties of the analytic functions. That is, geometric function theory is an area of mathematics characterized by an intriguing marriage between geometry and analysis. Its origins date from the 19th century but new applications arise continually. Univalent functions are one of the important subjects in geometric function theory. The coefficient estimates for univalent functions are used in solution of several problems. One of the important subjects of complex analysis is also Hankel determinant. Hankel determinant is frequently used in many areas. Hankel determinant holds an important place in univalent function theory. Hankel determinant problem is focused on finding upper bound of $H_2(2)$ and $H_3(1)$.

The Fibonacci numbers are the numbers in the integer sequence 1, 1, 2, 3, 5, 8, 13, 21, …, called the Fibonacci sequence, and characterized by the fact that every number after the first two is the sum of the two preceding ones. In this presentation, we will show an association with Fibonacci numbers of an analytic function of the class which we are working on under a special condition.

In this presentation, we consider second Hankel determinant problem on some interesting subclasses of analytic functions related to Fibonacci numbers. These subclasses were investigated and studied by Sokol in 1999 and by Dziok, Raina and Sokol in 2011, respectively.

The second Hankel determinant problem is $|H_2(2)| = |a_2a_4 - a_3^2|$. The upper bounds for second Hankel determinant of some special classes of functions were

found by several researchers. However, they still study on this problem for different subclasses.

We obtain the upper bounds for second Hankel determinant for analytic functions in some classes. Also, we present certain new results for these classes of functions.

Key Words: Hankel determinant , starlike, convex, shell-like curve, Fibonacci numbers.

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Simultaneous Approximation of Generalized Bieberbach Polynomials

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ABSTRACT

Let $G \subset \mathbb{C}$ be simply connected region whose boundary $\Gamma := \partial G$ is a rectifiable Jordan curve and $z_0 \in G$ be an arbitrary fixed point. Let also $w = \varphi_0(z)$ be the conformal mapping of G onto the disk $D_{r_0} := \{w : |w| < r_0\}$ satisfying $\varphi_0(z_0) = 0$, $\varphi_0'(z_0) = 1$. Let us set

$$\varphi_p(z) := \int_{z_0}^{z} [\varphi_0(\zeta)]^{2/p} d\zeta$$

for $0 < p < \infty$ and let $\pi_{n,p}(z)$ be a generalized Bieberbach polynomials for the pair (G, z_0), which minimizes the integral

$$\|\varphi_p' - P_n'\| := \left(\iint_G |\varphi_p'(z) - P_n'(z)|^p d\sigma_z \right)^{\frac{1}{p}}$$

in the class of all polynomials of degree at most $\leq n$ with $P_n(z_0) = 0, P_n'(z_0) = 1$. If $p > 1$, then the polynomial $\pi_{n,p}(z)$ is determined uniquely. It is also well known that the polynomial $\pi_{n,p}$ uniformly convergence to the function φ_p in any Carathedory region [1, p.63]. In case of $p = 2$, let us emphasize that $\pi_{n,2}$ coincides with the well-known $n-$ th Bieberbach polynomial π_n for the pair (G, z_0).

In the paper [2] S.N. Mergelyan also noted without an estimate the convergence of any derivatives of the Bieberbach polynomials π_n to the phenome is also true for derivatives of conformal mapping function φ_0 so called the simultaneous

approximation. First results relating on simultaneously approximation of Bieberbach polynomials π_n to the function φ_0 by our opinion are obtained Suetin [3] and Israfilov [4].

In this presentation we deal with the simultaneous approximation problem of generalized Bieberbach polynomials to the function φ_p in the L^p- and $C-$norms for some regions of the complex plane. It is shown that approximation rate of $\pi_{n,p}^{(k)}$ to the $\varphi_p^{(k)}$ depends on not only geometric and analytic properties of Γ but also depends on k.

Key Words: Conformal mapping, Bieberbach polynomial, Simultaneous approximation.

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Some Approximation Properties Of Szász-Kantorovich-Chlodowsky Operators Based On $q-$Integer

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ABSTRACT

Approximation theory has an important role in mathematical research because of its great potential for applications. Korovkin gave his famous approximation theorem in 1950, since then the study of the linear methods of approximation given by sequences of positive and linear operators became a deep-rooted part of approximation theory. Considering it, various operators as Bernstein, Kantorovich Szász etc. and their generalizations are being studied. In last two decades, the approximation of function by using the linear positive operators introduced via $q-$calculus is currently become active research. After the paper of Phillips [4] who generalized the classical Bernstein polynomials base on $q-$integer, many generalizations of well-known positive linear operators, based on $q-$integer were introduce and studied several authors. The application of $q-$calculus has played an important role in the area of approximation theory, number theory and theoretical physics.

In this paper, we introduce to q-Szász-Kantorovich-Chlodowsky operators as fallows.

$$K_n(f,q;x)=\frac{[n+1]_q^2}{[n]_q\,b_n}\sum_{k=0}^{\infty}q^{-k}\,\frac{[n]_q^k\,x^k}{b_n^k\,[k]_q!}e_q^{\left(-\frac{[n]_q\,x}{b_n}\right)}\int_{\frac{[k]_q[n]_q\,b_n}{[n+1]_q^2}}^{\frac{[k+1]_q[n]_q\,b_n}{[n+1]_q^2}} f(t)\,d_qt$$

Where $q \in (0,1)$, $x \in [0, b_n]$, $\lim_{n\to\infty} b_n = \infty$, $\lim_{n\to\infty} \frac{b_n^2}{[n]_q} = 0$

Some approximation properties of these operators are explored, we estimate the central moments up to order 4,we give some weighted approximation theorems and a Voronovskaja type theorem, and we also examine the rate of convergence in weighted space for the constructed operators by means of modulus of continuity.

Key Words: q -Szász operators, Rate of Convergence, Modulus of Continuity.

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Some Common Fixed Point Results For Contractive Mappings In Ordered G_p-Metric Spaces

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ABSTRACT

In a numerous class of studies, the classical concept of a metric space has been generalized in different directions by partly changing the conditions of the metric. Among these generalizations, we can consider the partial metric spaces and G-metric spaces. The notion of partial metric space was defined by Matthews [1] in 1994 as a generalization of metric spaces. The partial metric differ from the metric in the sense of self-distance of the points which is not necessarily to be zero, but it satisfies the property of symmetric and modified version of triangle inequality. In addition to these ideas, in 2005, Mustafa and Sims [2] introduced a new class of generalized metric spaces which is known *G*-metric space. Recently, based on the notions mentioned above, Zand and Nezhad [3] defined the concept of Gp-metric space as a generalization and unification of both partial metric spaces and G-metric spaces. In this present article, the sufficient conditions for the existence and uniqueness of fixed points and common fixed points of single and double mappings satisfying various contractive conditions within the partially ordered G_p-complete G_p-metric spaces have been obtained. Also, some examples supporting the results obtained have been given. The theorems obtained generalize some fixed point results existing in the literature.

Key Words: Fixed point, partially ordered G_p-complete G_p-metric spaces, Banach pairs.

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Some Convergence, Stability and Data dependency Results for a Picard-S Iteration Method of Quasi-strictly Contractive Operators

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ABSTRACT

Many problems arising from various branches of science can be modelled as $Tx = x$ with an appropriate map T. One can encounter situations where the solution of this equation can't be obtained analytically. In a case, fixed point iteration methods play a very important role to locate the fixed point of T.

Let T be a self-map of a nonempty closed convex subset C of a normed space X and $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$, $\{\gamma_n\}_{n=0}^{\infty} \subset [0,1]$ are real sequences satisfying certain control condition(s).

For arbitrary chosen w_0, $x_0 \in C$, construct two iterative sequences $\{w_n\}_{n=0}^{\infty}$ and $\{x_n\}_{n=0}^{\infty}$ by

$$\begin{cases} w_{n+1} = (1-\alpha_n)w_n + \alpha_n T\overline{w}_n, \\ \overline{w}_n = (1-\beta_n)w_n + \beta_n T r_n, \\ r_n = (1-\gamma_n)w_n + \gamma_n T w_n, \text{ for all } n \in \mathbb{N}, \end{cases} \tag{1}$$

and

$$\begin{cases} x_{n+1} = Ty_n, \\ y_n = (1-\alpha_n)Tx_n + \alpha_n T z_n, \\ z_n = (1-\beta_n)x_n + \beta_n T x_n, \text{ for all } n \in \mathbb{N}, \end{cases} \tag{2}$$

where the iteration methods defined by (1) and (2) are called Noor [1] and Picard-S [2] iteration methods respectively.
The Noor iteration method reduces to:

(i) Picard iteration method [3] if $\alpha_n = 1$, $\beta_n = \gamma_n = 0$, $\alpha_n = 1$ for all $n \in \mathbb{N}$;

(ii) Mann iteration method [3] if $\beta_n = \gamma_n = 0$ for all $n \in \mathbb{N}$;

(iii) Ishikawa iteration method [4] if $\gamma_n = 0$ for all $n \in \mathbb{N}$;

(iv) Normal-S iteration method [5] if $\alpha_n = 1$, $\gamma_n = 0$ for all $n \in \mathbb{N}$.

However, the Picard-S iteration method is independent of all the Noor, Ishikawa, Mann, Picard and normal-S iteration methods.

The above-mentioned iteration methods has been intensively investigated in view of convergence, rate of convergence, stability and data dependency in the literature the different classes of mappings including the class of contraction mappings satisfying:

$$\|Tx - Ty\| \le \delta\|x - y\|,\ \delta \in [0,1),\ \forall x, y \in X.$$

In 2010, Bosede and Rhoades [6] proved some stability results for the Picard and Mann iteration methods of quasi-strictly contractive operators satisfying the following condition

$$\|x^* - Ty\| \le \delta\|x^* - y\|,\ \delta \in [0,1),\ \forall y \in X$$

where x^* is fixed point of T. It can be easily shown that the class of quasi-strictly contractive operators properly includes the class of contraction operators.

In this presentation, we study some qualitative features like convergence, stability and data dependency for Picard-S iteration method of a quasi-strictly contractive operator under weaker conditions imposed on parametric sequences in the mentioned method. We compare the rate of convergences among the Mann, Ishikawa, Noor, normal-S, and Picard-S iteration methods for the quasi-strictly contractive operators. Results reveal that the Picard-S iteration method converges fastest to the fixed point of quasi-strictly contractive operators. Some numerical examples are given to validate the results obtained herein. Our results substantially improve many other results available in the literature.

Key Words: Iteration methods, quasi-strictly conractive operators, convergence, rate of convergence, stability, data dependency.

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Some Fixed Point Theorems For $F-$ Contractive Mappings In $G-$Metric Spaces

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ABSTRACT

Recently, Wardowski [5] introduced the notion of F-contraction mapping and investigated the existence of fixed points for such mappings. The results of Wardowski extend and unify several fixed point results in the literature including the Banach contraction principle and some other known contractive conditions can be obtained as special cases. In 2006, Mustafa and Sims [6] introduced a new structure called G-metric space as a generalization of the usual metric spaces. Afterwards Mustafa, Sims and other authors introduced and developed several fixed point theorems for mappings satisfying different contactive conditions in G-metric spaces, also extend known theorems in metric spaces to G-metric spaces. In this work, we prove some fixed point theorems with helping compatible maps for type 1 and type 2 $F-$contraction in complete $G-$ metric spaces.

Let (X,G) $G-$complete metric spaces and $f,g: X \to X$ compatible mappings. Furthermore

$$f(x) \subseteq g(x) \tag{1}$$

f or g is continuous (2)

A mapping $f,g: X \to X$ is said to be type 1 $F-$ contraction on (X,G) if there exists a number $\tau > 0$ such that for all $x,y,z \in X$ satisfying $G(fx,fy,fz) > 0$, the following holds;

$$\tau + F\big(G(fx,fy,fz)\big) \leq F\big(G(gx,gy,gz)\big). \tag{3}$$

Moreover $f,g: X \to X$ is said to be type 2 $F-$ contraction on (X,G) if there exists a number $\tau > 0$ such that for all $x,y,z \in X$ and $\beta \in \left[0,\frac{1}{3}\right]$ satisfying $G(fx,fy,fz) > 0$, the following holds;

$$\tau + F\big(G(fx,fy,fz)\big) \leq F(\beta[G(gx,fx,fx) + G(gy,fy,fy) + G(gz,fz,fz)]). \qquad (4)$$

Then f and g have a unique common fixed point.

Key Words: fixed point, $G-$ metric spaces, $F-$ contractive mapping.

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Some New Results About Porosity Convergence and Porosity Cluster Points

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ABSTRACT

Notion of porosity for sets was given by Denjoy [3] and Khintchine [5] under different terminologies in 1920 and 1924 respectively. Then, in 1967 independently from these studies porosity appeared Dolzenko's work on the concept of cluster sets [4].

Let $A \subset \mathbb{R}^+ = [0,\infty)$, then the right upper porosity of A at the point 0 is defined as

$$p^+(A) := \limsup_{h \to 0^+} \frac{\lambda(A,h)}{h}$$

where $\lambda(A,h)$ denotes the length of the largest open subinterval of $(0,h)$ that contains no point of A [6]. By using the right upper porosity of a set at the point 0, definition of the right upper porosity for the subsets of natural numbers at infinity was given by authors and Dovgoshey in [1] as follows:

Let $\mu : \mathbb{N} \to \mathbb{R}^+$ be a strictly decreasing function such that $\lim_{n \to \infty} \mu(n) = 0$, and let E be a subset of $\mathbb{N}$

$$\bar{p}_\mu(E) := \limsup_{n \to \infty} \frac{\lambda_\mu(E,n)}{\mu(n)}$$

where $\lambda_\mu(E,n) := \sup\{|\mu(n^{(1)}) - \mu(n^{(2)})| : n \le n^{(1)} < n^{(2)}, (n^{(1)}, n^{(2)}) \cap E = \emptyset\}$.

A set E is called

- porous at infinity if $\bar{p}_\mu(E) > 0$.
- nonporous at infinity if $\bar{p}_\mu(E) = 0$.
- strongly porous at infinity if $\bar{p}_\mu(E) = 1$.

By using this new concept $\bar{p}_\mu$-convergence was defined in [2] as follows:

Let $x = (x_n)$ be a real valued sequence. $x = (x_n)$ is $\bar{p}_\mu$-convergent to l if for every $\varepsilon > 0$

$$\bar{p}_\mu(A_\varepsilon) > 0,$$

where $A_\varepsilon := \{n : |x_n - l| > \varepsilon\}$ and it is denoted by $x_n \to l(\bar{p}_\mu)$.

A number β is said to be a $\bar{p}_\mu$-cluster point of $x = (x_n)$ if for every $\varepsilon > 0$, the set $\{n : \ |x_n - \beta| < \varepsilon\}$

is nonporous. i.e., $\boldsymbol{\bar{p}_\mu(\{n : \ |x_n - \beta| < \varepsilon \}) = 0}$.

The set of all $\boldsymbol{\bar{p}_\mu}$-cluster point of a given sequence $\boldsymbol{x = (x_n)}$ is denoted by $\boldsymbol{\Gamma_{\bar{p}_\mu}(x)}$.

In this study, we will investigate porosity cluster points for real valued sequences and we will give some new results about this concept.

Key Words: Porosity, porosity convergence, porosity cluster point.

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Some New Results on Inner Product Quasilinear Spaces

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ABSTRACT

Aseev in [1] introduced the theory of quasilinear spaces which is generalization of classical linear spaces. He used the partial order relation when he defined the quasilinear spaces and so he can give consistent counterparts of results in linear spaces. Further, he also described the convergence of sequences and norm in quasilinear space. We see from the definition of quasilinear space which given in [1], the inverse of some elements of in quasilinear space may not be available. In [4], these elements are called as singular elements of quasilinear space. At the same time the others which have an inverse are referred to as regular elements. Then, [6], she noticed that the base of each singular elements of a combination of regular elements of the quasilinear space. Therefore, she defined the concept of the floor of an element in quasilineer space in [6] which is very convenient for some analysis of quasilinear spaces. This work has motivated us to introduce some results about the floors of inner product quasilinear spaces.

In this article, we research on the properties of the floor of an element taken from an inner product quasilinear space. We prove some theorems related to this new concept. Further, we try to explore some new results in quasilinear functional analysis. Also, some examples have been given which provide an important information about the properties of floor of an inner product quasilinear space.

Key Words: Quasilinear Space, Inner Product Quasilinear Space, Solid-floored Quasilinear Space.

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Some Operator Inequalities Involving Positive Linear Maps

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ABSTRACT

Inequalities are powerful tool in mathematics. It is the aim of this presentation to prove new operator inequalities. Whenever we handle an inequality concerning complex numbers, a natural question arise whether it is true for bounded linear operators on a Hilbert space. There are many papers on operator versions of some classical scalar inequalities. For example operator versions of Kantorovich inequalities, Wielandt inequalities and arithmetic-geometric mean inequalities etc. Throughout this presentation, we reserve M, m for real numbers and I for the identity operator. Other capital letters denote general elements of the C^* algebra $B(H)$ (with unit) of all bounded linear operators acting on a Hilbert space $(H, <\cdot,\cdot>)$. Denote by $\|\bullet\|$ the operator norm. An operator A is said to be positive if $\langle Ax, x\rangle \geq 0$ for all $x \in H$. $A \geq B$ means $A - B \geq 0$. A linear map is called $\varphi(H) \to \varphi(K)$ positive if $\varphi(A) \geq 0$ whenever $A \geq 0$. It is said to be unital if $\varphi(I) = I$. It is well known that for two positive operators A, B: $A \geq B \Rightarrow A^p \geq B^p$ for $0 \leq p \leq 1$ but for $1 \leq p$ this is not true. Let $0 < m \leq A \leq M$ and φ be positive linear map. Marshall and Olkin proved the following operator Kantorovich inequality.

$$\varphi\left(A^{-1}\right) \leq \frac{(M+m)^2}{4Mm}\varphi\left(A\right)^{-1}$$

$\frac{(M+m)^2}{4Mm}$ is known as Kantorovich constant. Lin [1] proved that this operator inequality can be squared

$$\varphi^2\left(A^{-1}\right) \le \left(\frac{\left(M+m\right)^2}{4Mm}\right)^2 \varphi\left(A\right)^{-2}$$

Based on the paper of Lin, many authors investigated operator inequalities whether can be squared or not. Also they obtained results for $1 \le p$. Especially operator versions of arithmetic-geometric means are handled. Mond and Pecaric [2] proved the following operator inequality

$$\varphi\left(A^2\right) \le \frac{\left(M+m\right)^2}{4Mm} \varphi\left(A\right)^2 \tag{1}$$

It is interesting to ask whether t^p $\left(p \ge 1\right)$ for the inequality (1) and related inequalities. This is a main motivation for the presented paper.

Key Words: Positive linear maps, operator inequalities, bounded linear maps, Kantorovich inequality

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Some Properties of Generalized Cheney-Sharma Operators

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ABSTRACT

The classical Bernstein operators $B_n : C[0,1] \to C[0,1]$ are defined by

$$B_n(f,x)=\sum_{k=0}^{n} f\left(\frac{k}{n}\right)\binom{n}{k}x^k(1-x)^{n-k}.$$

As is well known, Bernstein operators suggest to mathematicians the construction of a great variaty of other approximation operators. In 1932, Chlodovsky [4], introduced the classical Bernstein-Chlodovsky polynomials as a generalization of Bernstein polynomials on unbounded set, have the form

$$C_n(f,x)=\sum_{k=0}^{n} f\left(\frac{k}{n}b_n\right)\binom{n}{k}\left(\frac{x}{b_n}\right)^k\left(1-\frac{x}{b_n}\right)^{n-k}$$

where $0 \le x \le b_n$ and $\{b_n\}$is a positive sequences with the properties $\lim_{n\to\infty} b_n = \infty, \lim_{n\to\infty}\frac{b_n}{n} = 0.$

By using Abel-Jensen equalities (see [1, p. 322 and p. 326)] Cheney-Sharma [3] introduced two Bernstein type operators for $f \in C[0,1], x \in [0,1]$ and proved that if $nb_n \to \infty$ as $n \to \infty$, then for f∈ C[0,1], these operators uniformly convergence to *f* on [0,1]. Korovkin type approximation theory is concerned with the convergence of the sequences of positive linear operators to the identity operatör. Recently, many authors have studied the Korovkin type approximation properties of particular sequences of operators. In this paper, we deal with the Korovkin type approximation properties of Cheney-Sharma Chlodowsky operators We study convergence of these operators in a weighted space of functions on a positive semi-axis. We also show that these operators preserve properties of the modulus of continuity function when *f* is a modulus of continuity function and Lipschitz condition of a given Lipschitz continuous function *f*. Furthermore, we give a result for the new operators when *f* is a convex functions.

Key Words: Chlodovsky operators, Lipschitz continuous function, modulus of continuity function.

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Some Properties of the Bernstein Type Operator

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ABSTRACT

S. N. Bernstein, introduced the Bernstein polynomials to give a constructive proof of the Weierstrass approximation theorem in 1912. Such polynomials have been studied intensively and their connection with different branches of analysis, such as convex and numerical analysis, total positivity and the theory of monotone operators have been investigated. Basic facts on Bernstein polynomials and their generalizations investigated by Aral, Lorentz, Nowak, Phillips, Ostrovska, Stancu. In the present paper, we introduce a new positive linear operator $T_\lambda(f,x)$ and give some approximation properties of this operator and the order of approximation after that we give the Voronovskaya-type theorem. We also find the eigenvalues of the restriction of our operator and special case of exponential operators to the space of polynomials of degree at most N. Earlier May [2] considered the case when p is a polynomial of degree at most 2 and showed that the operators of Sz´asz, Baskakov, Gauss-Weierstrass, Bernstein (polynomials), and Post-Widder, correspond to the cases $p(t) = t$, $t(1+t)$, $t(1-t)$, 1, t^2; respectively. The general operator with $p(t) = 1+t^2$ was later found in [3]. The exponential operators were introduced by Ismail and May in [3] in 1978. Later Morris [1] introduced the Natural Exponential Families of distribution with quadratic variance. These are the same as W's in C. P. May's original paper [2]. For reference, we define the IM operator associated with $p(t) = 1+t^2$.

Key Words: Rate of convergence, Voronovskaya-type theorem, Eigenstructure of operator, Exponential operators

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Some Results for Max-Product Operators via Power Series Method

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ABSTRACT

The classical Korovkin theorem states the uniform convergence of a sequence of positive linear operators in C[a,b], the space of all continuous real valued functions defined on [a,b], by providing the convergence only on three well known test functions. There are also trigonometric versions of this theorem with different test functions and abstract Korovkin type results. These type of results let us say the convergence with minimum calculations and also have important applications in the polynomial approximation theory, in various areas of functional analysis, in numerical solutions of differential and integral equations. Recently it has been asked: Do all the approximation operators need to be linear? It has been shown that the linear structure is not the only one which allows us to construct particular approximation operators. Then following this idea, these type of approximation results have been extended with the use of statistical convergence and summation process by max-product operators. In this paper, we obtain an approximation theorem by max-product operators with the use of power series method which is more effective than ordinary convergence and includes both Abel and Borel methods. We also estimate the error in this approximation. As a concluding remark, we present an example which makes impossible to approximate the function f by means of a sequence of operators with the given theorems before.

Key Words: Power series method, Max-Product operators, Approximation theory.

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Some Time Scales Weighted Čebyšev and Ostrowski-Grüss Type Integral Inequalities

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ABSTRACT

In this study, we derive some Čebyšev type integral inequalities involving functions whose first derivatives belong to $L_p(a,b)\,(1\le p\le\infty)$ and weighted Ostrowski-Grüss type integral inequalities on time scales and then unify corresponding discrete and continuous versions. Futhermore, some particular integral inequalities on time scales are given as special cases.

In 1938, Ostrowski derived a formula to estimate the absolute deviation of a differentiable function from its integral mean, the so-called Ostrowski inequality, which can also be shown by using Montgomery identity. Using the Montgomery identity on time scales, Bohner and Matthews established Ostrowski inequality on time scales. In 1988, S. Hilger introduced the time scales theory to unify continuous and discrete analysis.

If $f,g:[a,b]\to R$ are integrable on $[a,b]$ are integrable on $[a,b]$, then we consider the $T(f,g)$ Čebyšev functional

$$T(f,g)=\frac{1}{b-a}\int_a^b f(t)g(t)dt-\left(\frac{1}{b-a}\int_a^b f(t)dt\right)\left(\frac{1}{b-a}\int_a^b g(t)dt\right).$$

In 1882, P.L.Čebyšev proved that, if $f',g'\in L_\infty[a,b]$, then

$$\left|\frac{1}{b-a}\int_a^b f(t)g(t)dt-\left(\frac{1}{b-a}\int_a^b f(t)dt\right)\left(\frac{1}{b-a}\int_a^b g(t)dt\right)\right|\le\frac{1}{12}(b-a)^2\|f'\|_\infty\|g\|_\infty.$$

Key Words: Ostrowski-Grüss type inequality, Čebyšev type inequality, Ostrowski type inequality, Time scales.

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Spectral Properties Of A Selfadjoint Quantum Difference Equation With Matrix Coefficient

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ABSTRACT

Quantum calculus which known as the calculus without limits is a connection between mathematics and physics. It arose interest due to high demand of mathematics that models quantum competing. *q*-analogues of some well-known definitions and theorems of ordinary calculus have been given by Kac and Cheung.
Some useful generalizations and important results were given for dynamic equations on arbitrary time scales, which contain q-difference equations as a special case by Bohner, Guseinov and Peterson.

In recent years, *q*-difference equations has become an important area of research. Because such equations have an interest role due to their applications in several mathematical areas such as number theory, orthogonal polynomials, mathematical control theory, basic hyper-geometric functions and other disciplines including mechanic, theory of relativity, biology, economics. Since it has huge applications, several problems of *q*-difference equations have been treated by various authors. But there is not much more studies about spectral analysis of these equations. Adıvar and Bohner have investigated the spectral analysis of non-selfadjoint *q*-difference equations in 2006. Huseynov and Bairamov have studied on an eigenvalue problem for quadratic pencils of *q*-difference equations and its applications. Also, Aygar and Bohner have studied spectral analysis of same equations in different aspects.

In this study, a selfadjoint second order matrix quantum difference equation and its spectral properties are discussed. We get the Jost solution of this equation and investigate the analytical properties, asymptotic behaviour of this solution. Also

we define the operator L generated with same *q*-difference expression and a boundary condition y(1)=0. We find that the operator L has a finite number of simple eigenvalues.

Key Words: Eigenvalue, *q*-difference equation, spectral analysis, Jost solution.

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Strongly Convergence Of Multipliers On Banach Algebras

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ABSTRACT

Let A be a commutative semisimple Banach algebra and let Σ_A be the Gelfand space of A equipped with the w^*-topology. By a, where $a(\zeta)=\zeta(a)$, $(\zeta \in \Sigma_A)$, we denote the Gelfand transform of $a \in A$. A linear mapping $T: A \to A$ is called a multiplier of A if $T(ab)=(Ta)b\;(=a(Tb))$ holds for all $a,b \in A$. For each multiplier T on A, there is a uniquely determined bounded continuous function T on Σ_A such that

$$(Ta)(\zeta)=T(\zeta)a(\zeta) \quad \forall a \in A, \forall \zeta \in \Sigma_A.$$

The function T is often called the Helgason-Wang representation of T [2, 4]. Recall that a commutative Banach algebra A is said to be regular if given a closed subset S of Σ_A and $\zeta \in \Sigma_A \setminus S$, there exists an $a \in A$ such that $a(\zeta) \neq 0$ and $a(S)=\{0\}$. For a regular semisimple Banach algebra A and for a closed subset S of Σ_A, there are two distinguished closed ideals in A with hull equal to S namely $I_S := \{a \in A : a(S)=\{0\}\}$ is the largest closed ideal whose hull is S and $J_S := \overline{J_S^o}$ is the smallest closed ideal whose hull is S, where $J_S^o := \{a \in A_{00} : \operatorname{supp} a \cap S = \varnothing\}$ and $A_{00} := \{a \in A : \operatorname{supp} a \text{ is compact}\}$. The set S is said to be a *set of synthesis for* A *if* $I_S = J_S$ [3, Sect. 8.3]. Recall that a multiplier T on A is said to be power bounded if $\sup_{n\geq 0}\|T^n\| < \infty$. If T is a power bounded multiplier on A, then

$I_T := \left\{ a \in A : \lim_{n\to\infty} \left\| T^n a \right\| = 0 \right\}$ is a closed ideal of A associated with T. If A is regular and semisimple, then $\mathrm{hull}(I_T) = \varepsilon_T$, *where* $\varepsilon_T := \left\{ \zeta \in \Sigma_A : \left| T(\zeta) \right| = 1 \right\}$. (see, [1, Theorem 2.6] and [5, Proposition 2.1]).

We have the following.

Theorem Let A be a commutative semisimple regular Banach algebra and let T be a power bounded multiplier on A. If ε_T is a set of synthesis for A, then the sequence $\left\{ T^n a \right\}_{n\in\mathbb{N}}$ converges for every $a \in A$ if and only if ε_T is clopen (closed and open) and $T(\varepsilon_T) = \{1\}$

Key Words: Banach algebra, multiplier, strongly convergence

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Subdivision of the Spectra for the Upper Triangular Matrix $U(a_0, a_1, a_2; b_0, b_1, b_2)$ over the Sequence Space c_0

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ABSTRACT

Spectral theory is an important part of functional analysis. It has numerous applications in many parts of mathematics and physics including matrix theory, function theory, complex analysis, differential and integral equations, control theory and quantum physics. For example, in quantum mechanics, it may determine atomic energy levels and thus, the frequency of a laser or the spectral signature of a star.

In recent years, spectral theory has witnessed an explosive development. There are many types of spectra, both for one or several commuting operators, with important applications, for example the approximate point spectrum, Taylor spectrum, local spectrum, essential spectrum, etc.

In the last year, several authors have investigated spectral divisions of generalized differance matrices. For example, Akhmedov and El-Shabrawy, have studied the spectrum and fine spectrum of the generalized lower triangle double-band matrix Δ_v over the sequence spaces c_0, c and ℓ_p, where $1 < p < \infty$. In [5], Das has calculated the spectrum and fine spectrum of the upper triangular matrix $U(r_1, r_2; s_1, s_2)$ over the sequence space c_0.

The above-mentioned articles, concerned with the decomposition of spectrum which defined by Goldberg. However, in [3] Durna and Yildirim have investigated subdivision of the spectra for factorable matrices on c and ℓ_p . Basar, Durna and Yildirim have investigated subdivisions of the spectra for genarilized difference operator over certain sequence spaces and in [4] Durna, have studied subdivision of the spectra for the generalized upper triangular double-band matrices Δ^{uv} over the sequence spaces c_0 and c.

In this study we determine the approximate point spectrum, defect spectrum and compression spectrum of the upper triangular matrix

$$U(a_0, a_1, a_2; b_0, b_1, b_2) = \begin{pmatrix} a_0 & b_0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & a_1 & b_1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & a_2 & b_2 & 0 & 0 & \cdots \\ 0 & 0 & 0 & a_0 & b_0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & a_1 & b_1 & \cdots \\ 0 & 0 & 0 & 0 & 0 & a_2 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

over the sequence space c_0.

Key Words: approximate point spectrum, defect spectrum, compression spectrum.

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Subdivisions of the spectra for compact Cesaro type operator on l^r $(1 < r < \infty)$

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ABSTRACT

Compact linear operators are very important in applications. For instance, they play a central role in the theory of integral equations and in various problems of mathematical physics. Their properties closely resemble those of operators on finite dimensional spaces.

Definition (Compact linear operator) : [Kreyzing, 1978] Let X and Y be normed spaces. If L is linear and if for every bounded subset K of X, $\overline{L(K)}$ is compact, an operator $L: \mathrm{X} \to \mathrm{Y}$ is called a compact linear operator.

Theorem : [Kreyzing, 1978] Let X and Y be normed spaces and $L: \mathrm{X} \to \mathrm{Y}$ a linear operator. Then L is compact if and only if it maps every bounded sequence (x_n) in X onto a sequence (Lx_n) in Y which has a convergent subsequence.

Theorem : [Kreyzing, 1978] Let X and Y be normed spaces and $L: \mathrm{X} \to \mathrm{Y}$ a linear operator. Then:

(a) If L is bounded and $\dim\big(L(X)\big) < \infty$, the operator L is compact.

(b) If $\dim(X) < \infty$, the operator L is compact.

Theorem : [Kreyzing, 1978] Let (L_n) be a sequence of compact linear operators from a normed space X into a Banach space Y. If (L_n) is uniformly operator convergent, then the limit operator L is compact.

Let x = (x_n) be complex sequences. p-Cesaro operator $C_p x = y = (y_n)$ is defined by $y_n = \frac{1}{n^p} \sum_{k=1}^{n} x_k$.

In 1989 Rhaly determined the spectrum and eigenvalues of Cesaro operator on the Hilbert space l_2 of square summable sequences.

Theorem : [Rhaly, 1989] $\sigma_p\left(C_p, l^2\right) = \left\{\frac{1}{m^p} : \mathrm{m} = 1,2,3, \dots \right\}$.

Theorem : [Rhaly, 1989] $\sigma\left(C_p, l^2\right) = \left\{\frac{1}{m^p} : \mathrm{m} = 1,2,3, \dots \right\} \cup \{0\}$.

In 1992, Coşkun identified the spectrum of the p-Cesaro operator on l^r for $1 < r < \infty$.

Theorem : [Coşkun, 1992] a) $C_p \in B(l^r)$ for $1 < r < \infty$.

b) $C_p \in B(l^r)$ is is compact for $1 < r < \infty$.

c) $\sigma_p\left(C_p, l^r\right) = \left\{\frac{1}{m^p} : \mathrm{m} = 1,2,3, \dots \right\}$ for $1 < r < \infty$.

d) $\sigma\left(C_p, l^2\right) = \left\{\frac{1}{m^p} : \mathrm{m} = 1,2,3, \dots \right\} \cup \{0\}$ for $1 < r < \infty$.

In 2011, Durna and colleagues first gave a non-discrete spectral decomposition (defect spectrum, approximation point spectrum, and compression spectrum) of a infinitive matrix.

In this work, we will determine the spectrum of a compact Cesaro type operator on l^r $(1 < r < \infty)$.

Key Word : Cesaro operator, spectrum, fine spectrum, the subdivision of the spectrum.

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Sublinear operators with rough kernel generated by Calderón-Zygmund operators and their commutators on vanishing generalized Morrey spaces

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ABSTRACT

In this talk, we shall dwell on the boundedness of the sublinear operators with rough kernels satisfying size condition generated by Calderón-Zygmund operators with rough kernel and their commutators on vanishing generalized Morrey spaces. For this, at first, basic spaces such as Morrey space, vanishing Morrey space and vanishing generalized Morrey space have been defined and later the relationships between these spaces have been considered. Moreover, some basic concepts and theorems related to these spaces have been given. Later, the concepts of sublinear operators with rough kernel generated by Calderón-Zygmund operators with rough kernel and their commutators, BMO (bounded mean oscillation space) have been defined and some properties associated with these operators have been considered. The rest of the conversation is devoted to main results. Also, in order to prove the main results we need some useful lemmas. But, these lemmas for both operators and commutators by with rough kernels will be given without any explanation in this paper, respectively. At last, main results have been written, respectively.

In the first part of the conversation, the boundedness of a sublinear operator with rough kernel generated by Calderón-Zygmund operators is given on vanishing generalized Morrey space. In the second part of the conversation, the boundedness of the commutator operators with rough kernel formed by BMO functions and Calderón-Zygmund operators with rough kernel is discussed on vanishing generalized Morrey space. Also, associate results for both operators and commutators have been given on vanishing generalized Morrey space, respectively.

Key Words: Sublinear operator; Calderón-Zygmund operator; rough kernel; vanishing generalized Morrey spaces; commutator; BMO

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Summability Process on the Approximation by Nonlinear Integral Operators of Convolution Type

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ABSTRACT

In this study we obtain and improve some error estimates, convergence results and rate of approximation for functions belonging to the space of functions with bounded variation by means of a sequence of nonlinear integral operators of convolution type, which was first systematically investigated in [1]. In the approximation we use a semi-norm on the space and consider a general convergence method, the so-called summability process, introduced by Bell (see [2]). Due to the nonlinearity of the kernels of our integral operators we assume some suitable and reasonable conditions on the kernels. Furthermore, when computing rates of approximation we use some suitable Lipschitz classes of functions. At the end of the paper we give some applications obeying such an approximation method but not the usual one. Some graphical illustrations are also displayed.

We should note that the application of summability methods has been made possible in various fields of mathematics. For instance, summability methods are applied in function theory in connection with the analytic continuation of holomorphic functions; in applied analysis for the generation of iteration methods for the solution of a linear system of equations; in approximation theory for convergence of a Fourier series. In the present paper we use such methods in the approximation by nonlinear integral operators in order to overcome the lack of convergence and improve the classical theory.

Key Words: Summability process, nonlinear integral operators, convolution type integral operators, bounded variation.

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The Boundedness of Hilbert Transform in the Local Morrey-Lorentz Spaces

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ABSTRACT

In this talk, we give the boundedness of the Hilbert transform H and the maximal Hilbert operatör in the local Morrey–Lorentz spaces $M^{loc}_{p,q;\lambda}(R^n)$, under conditions $q\,/\,(q+\lambda) \le p \le q/\lambda, 1 \le q \le \infty$. We prove that the Hilbert transform H is bounded in the local Morrey–Lorentz spaces $M^{loc}_{p,q;\lambda}(R^n)$ under the condition $q\,/\,(q+\lambda) < p < q/\lambda, 1 \le q < \infty$. In the limiting case $p = q/(q+\lambda), 1 < q < \infty$, we prove that the Hilbert transform H is bounded from the local Morrey–Lorentz space $M^{loc}_{p,q;\lambda}(R^n)$ to the weak local Morrey–Lorentz space $WM^{loc}_{p,q;\lambda}(R^n)$. Also we show that for the limiting case $p = q/\lambda, 0 < q \le \infty$, the modified Hilbert transform $\tilde{H}$ is bounded from the local Morrey–Lorentz space $M^{loc}_{p,q;\lambda}(R^n)$ to the bounded mean oscillation BMO space. Furthermore, we give the boundedness of the maximal Hilbert operator in the local Morrey-Lorentz spaces $M^{loc}_{p,q;\lambda}(R^n)$, under conditions $q\,/\,(q+\lambda) \le p \le q/\lambda, 1 \le q \le \infty$. We prove that the maximal Hilbert transform is bounded in the local Morrey–Lorentz spaces $M^{loc}_{p,q;\lambda}(R^n)$ under the condition $q\,/\,(q+\lambda) < p < q/\lambda, 1 \le q < \infty$. In the limiting case $p = q/(q+\lambda), 1 < q < \infty$, we prove that the maximal Hilbert transform is bounded from the local Morrey–Lorentz space $M^{loc}_{p,q;\lambda}(R^n)$ to the weak local Morrey–Lorentz space $WM^{loc}_{p,q;\lambda}(R^n)$.

The proofs of our theorems are based on O'Neil inequality and the boundedness of the weighted Hardy operators in the Morrey spaces. For this purpose, we prove the boundedness of the Hardy operators in the weak Morrey spaces.

Key Words: Local Morrey-Lorentz spaces, Hilbert transform, modified Hilbert transform.

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The Boundedness of Some Classical Operators and Their Commutators on the Vanishing Generalized Morrey Spaces

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ABSTRACT

The classical Morrey spaces were introduced by Morrey [1] in 1938. Vanishing Morrey spaces which are a new version of Morrey spaces and were introduced by Vitanza [4] in 1990. This is a subspace of functions in Morrey spaces satisfying a special condition. Ragusa [2] studied the boundedness of some classical operators in vanishing Morrey spaces. The vanishing generalized Morrey spaces were introduced and studied by Samko in [3]. There is not enough study in these spaces which makes it interesting to study.

In our paper, we studied some classical operators and also their commutators and searched for their boundedness on Vanishing Morrey spaces. In all the theorems, the boundedness conditions are given as Zygmund type integral inequalities and without monotonicity on (ϕ, ψ).

Firstly we introduce some classical operators such as Riesz potential operator, Maximal operator, oscillatory integral operator, fractional oscillatory integral operator and their commutators and give their boundedness conditions on the generalized Vanishing Morrey spaces.

Later the vanishing generalized weighted Morrey spaces are defined and the boundedness conditions of Zygmund type are given for oscillatory integral operator, fractional oscillatory integral operator and for their commutators on these spaces.

Finally, we prove the boundedness of fractional maximal operator and fractional integral operator and their commutators on vanishing generalized weighted Morrey spaces.

Key Words: Vanishing Morrey spaces, Oscillatory integral operator, Commutator

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The Midpoint Type Inequalities for φ- Convex Functions via Fractional Integral Operators

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ABSTRACT

Fractional calculus was born in 1695. In the past three hundred years, fractional calculus developed in diverse fields from physical sciences and engineering to biological sciences and economics. Fractional Hermite-Hadamard inequalities involving all kinds of fractional integrals have attracted by many researches.

Let $f: I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex mapping defined on the interval I of real numbers and $\mathrm{a}, \mathrm{b} \in I$ with $a < b$. The following double inequality:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2} \tag{1.1}$$

is known in the literature as Hermite-Hadamard inequality for convex mappings. Note that some of the classical inequalities for means can be derived from (1.1) for appropriate particular selections of the mapping f. Both inequalities hold in the reversed direction if f is concave.

In this study, using a general class of fractional integral operators, we establish new fractional integral inequalities of Hermite-Hadamard type for $\varphi-$ convex functions. The main results are used to derive Hermite-Hadamard type inequalities involving the familiar Riemann-Liouville fractional integral operators for $\varphi-$ convex functions.

Definition: A function $f: I \to \mathbb{R}$, is said to be convex if for every $x, y \in I$ and $t \in [0,1]$, given by

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y).$$

We say that f is concave if $(-f)$ is convex.

Definition: Let $\varphi : [a,b] \subset \mathbb{R} \to [a,b]$. A function $f : [a,b] \to \mathbb{R}$ is said to be φ-convex on $[a,b]$ if, for every $x, y \in [a,b]$ and $t \in [0,1]$, the following inequality holds:

$$f(t\varphi\,(x) + (1-t)\,\varphi\,(y)) \leq t\varphi(t)\,f(x) + (1-t)\varphi(1-t)f(y).$$

If we choose $\varphi\,(t) = 1$, then we get classic convex function.

Our aim in this paper is to continue our work on the Hermite-Hadamard type inequalities involving the familiar Riemann-Liouville fractional integral operators for $\varphi -$ convex functions. We state and prove some refinements on the left side of the inequality (1.15) in [7].

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Key Words: Hermite-Hadamard's inequalities, Riemann-Liouville fractional integral, Integral inequalities, φ-convex functions.

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Third Hankel Determinant Problem for Starlike and Convex Functions Related to Fibonacci Numbers

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ABSTRACT

One of the important subjects of complex analysis is also Hankel determinant. Hankel determinant is frequently used in many areas. Hankel determinant holds an important place in univalent function theory. The Hankel determinant problem is focused on finding upper bound of $H_2(2)$ and $H_3(1)$.

The Fibonacci numbers are the numbers in the integer sequence 1, 1, 2, 3, 5, 8, 13, 21, …, called the Fibonacci sequence, and characterized by the fact that every number after the first two is the sum of the two preceding ones. In this presentation, we will show an association with Fibonacci numbers of an analytic function of the class which we are working on under a special condition.

This presentation is concerned with the problems related to the estimation of third Hankel determinant for some special classes of analytic functions. We obtained the sharp upper bound of $H_3(1)$ for the functions belonging to these special classes.

In this presentation, we consider third Hankel determinant problem on some subclasses of analytic functions connected with Fibonacci numbers.

The third Hankel determinant problem is

$$H_3(1)=a_3\left(a_2a_4-a_3^2\right)-a_4\left(a_2a_3-a_4\right)+a_5\left(a_3-a_2^2\right).$$

The upper bounds for third Hankel determinant of some special classes of functions were found by several researchers. However, they still study on this problem for different subclasses.

As a result, we obtain the upper bounds for third Hankel determinant for analytic functions in some special subclasses related to Fibonacci numbers. Also, we present certain new results for these classes of functions.

Key Words: Third Hankel determinant, starlike function, convex function, Fibonacci numbers.

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Λ-Statistical Convergence Of Order α For Double Sequences

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ABSTRACT

The idea of statistical convergence goes back to the first edition of monograph of Zygmund [8]. This notion was first defined for real sequences by Steinhaus [7] and Fast [3]. Over the years, and under different names, statistical convergence was discussed in the theory of Fourier analysis, ergodic theory, fuzzy sequence and number theory.

The idea of statistical convergence was later extended to double sequences by Mursaleen and Edely [6]. For more details and related concepts, we refer to Çolak [2].

In [5], Mursaleen introduced the notion of λ– statistical convergence and show how it is related with (V,λ) –summability.

The order of statistical convergence of a sequence was previously given by Gadjiev and Orhan [6] and after then statistical convergence of order α and strong $p-$ Cesàro summability of order α studied by Çolak [2].

Let f be a real valued function defined on $[0,\infty)$ having the following properties:

$i)$ $f(x)=0$ if and only if $x=0$,

$ii)$ $f(x+y)\leq f(x)+f(y)$ for $x,y\geq 0,$

$iii)$ f is increasing,

$iv)$ f is continuous from the right at $0.$

Such an f is called a modulus; it is clear that a modulus is a continuous function on $[0,\infty)$.

Aizpuru *et al*. [1] introduced the notions of density and f –statistical convergence for double sequence of complex or real numbers, where f– is an unbounded modulus.

In this study, we explain the notions of $f_{\lambda\alpha}$ – density and $f_{\lambda\alpha}$ –statistical convergence for complex or real double sequence for $0<\alpha\leq 1$.

Furthermore, some connections between $f_{\lambda\alpha}$ –statistical convergence and strong $f_{\lambda\alpha}$ –Cesaro summability of order α are given.

Key Words: Modulus function, density, statistical convergence.

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Δ-Convergence and Uniform Distribution in Lacunary Sense

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ABSTRACT

In 1916 formal definition of uniform distribution with modulo 1 (u.d. mod 1) was given by Weyl in [1]. Let $\tilde{x}=(x_n)$ be a sequence of non-negative real numbers and (a_n) be congruence (mod 1) of $\tilde{x}$. If

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N} f(a_n)=\int_0^1 f(x)d(x)$$

holds for every real valued function f defined on $[0,1]$, then the sequence $\tilde{x}=(x_n)$ is called (u.d. mod 1)(see more in [1-2]).

Let $\tilde{x}=(x_n)$ be a sequence of non-negative real numbers and $\Delta:=\{0=z_0<z_1<z_2<...\}$ be a subdivision of the interval $[0,\infty)$. For all $n\in\mathbb{N}$, there exists a unique $k\in\mathbb{N}$ such that $z_{k-1}\le x_n<z_k$ holds. Δ-integer and Δ-fractional part of the sequence $\tilde{x}=(x_n)$ are defined by

$$[x_n]_\Delta := z_{k-1}; \quad \{x_n\}_\Delta := \frac{x_n - z_{k-1}}{z_k - z_{k-1}}$$

respectively [2-3-4]. It is clear that $0\le\{x_n\}_\Delta<1$ holds for all $n\in\mathbb{N}$. Some applications can be find in [5], [6], [7].

We have defined Δ-convergence of a given sequence as follows:

A sequence $\tilde{x}=(x_n)$ of non-negative real numbers is said to be Δ-convergent if $\{x_n\}_\Delta$ is convergent (in usual sense).

It is shown that every convergent sequence is Δ-convergent to the Δ-fractional part of usual limit but the converse is not true, in general.

Besides, some basic properties of Δ-convergence as well as the second part of this talk by using any lacunary sequences as a partition of non-negative real numbers, lacunary uniform distribution is defined and some inclusion result between uniform distribution modulo 1 and lacunary uniform distribution has been given.

Key Words: Convergence of sequence, Uniform distribution of sequence, Lacunary convergence.

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APPLIED MATHEMATICS

A Comparative Study of Tests for Significance of Ridge Regression Coefficients

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ABSTRACT

The Least Squares (LS) method is the most common method to estimate the regression coefficients but some assumptions are required for the LS method. One of the most important assumptions is that there is no relationship among explanatory variables. However, it is difficult to achieve this assumption in practice and the case when there is a relationship among explanatory variables is known as multicollinearity problem. Therefore, the most used method to overcome this problem is the ridge regression where the ridge estimators based on optimum k values have smaller mean square errors (MSEs) than the variances of the LS estimator.

In the literature, there are many papers regarding different optimum k values such as Saleh (2006), Alkhamisi et. al. (2006), Alkhamisi and Shukur, (2008), Muniz and Kibria (2009), Gökpınar and Ebegil (2012), etc. However, most of these researchers generally focused on the optimum k values to obtain ridge estimators.

A very common problem in applied statistics is to test the equality of significance of regression coefficients. Halawa and Basuiouni (2000) investigated non-exact tests based on ridge regression by using two different biasing parameters (k) proposed by Hoerl and Kennard (1970) and Hoerl et al. (1975). In this paper, other popular k values used in the ridge regression were investigated by using non-exact tests proposed by Halawa and Basuiouni (2000) for testing significance of regression coefficients. Then, the tests were compared in terms of type I error rates and powers by using Monte Carlo simulation and it was summarized that which k values give better results than the others in given cases.

Key Words: Ridge regression, biasing parameters, hypothesis testing.

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A General Nonlinear Discrete-Time Population Model involving Allee Effect and Stability Analysis

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ABSTRACT

The study of the stability in population models with and without Allee effect have been a very important topic in many areas of ecology and biology. This effect is firstly introduced by Allee in 1931 as negative density dependence when the growth rate of the population decreases in low population density. The main causes of the Allee effect are the difficulty in finding mates, imbreeding depression, social dysfunction at small population sizes, predator avoidance and food exploitation.

In late years, many authors have studied the stability of different population models with and without Allee effect. A positive equilibrium solution of the models which is exposed to an Allee effect may become either one of destabilization or of stabilization. That is to say, the local stability of a positive equilibrium solution can be changed from the stable case to an unstable case or vice versa. It is also possible that even if the model is stable at an equilibrium solution, to arrival time its equilibrium point may take a much longer. This case has been expressed to in the statement that the `Allee effect decreases the local stability of the equilibrium point' .

The aim of this study is to investigate and compare the local stability of equilibrium solutions with and without Allee effect by considering a more general state of the model studied by Gumus and Kose [1]. So we consider a general nonlinear discrete-time population model. In conclusion, Allee effect at different times increase the local stability of equilibrium point of model. The numerical simulations confirm the analytical results.

Key Words: Stability Analysis, Population Model, Allee Effect, Equilibrium Point.

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A Geometric Approach for The One-Dimensional Hyperbolic Telegraph Equation With Variable Coefficients

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ABSTRACT

Group-preserving scheme (GPS) can preserve the internal symmetry group of the considered system. Although we do not know previously the symmetry group of nonlinear differential equations systems, Liu has embedded them into the augmented dynamical systems, which deal with the evolution not only of state variables but also the magnitude of the state variables vector. That is, for a *k*-dimensional ordinary differential equations system we can embed it into the *k*+1-dimensional augmented dynamical system. The GPS, as a geometric method, is formulated in the Minkowski space, whereas the traditional numerical methods (Non-geometric methods) are all formulated directly in the usual Euclidean space. Avoiding the spurious solutions and ghost fixed points is one of the benefits of utilizing the augmented Minkowski space as a Lie group.

In the present paper, we introduce a simple and accurate geometric numerical scheme for solving the non-homogenous one-dimensional telegraph equation (TE) with variable coefficients. Different variable coefficients of this equation leads to the conditional or unconditional stabilities of the proposed method. Power and accuracy of proposed approach has been confirmed through some numerical experiments of TE. In this work, we examine the performance of the MOL-GPS method to solve the TE. All numerical computations have been done in *Matlab R2015b.* Employing the combination of GPS and semi-discretization on TE with variable coefficients can preserve the geometric structures of this equation. This preserving property concludes the reliable approximations with minimized errors in the absence of spurious solutions. Stability analysis of the MOL-GPS method is successfully investigated.

Key Words: Telegraph equation, Variable coefficient, Method of Line, Group preserving scheme, Stability analysis.

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A new modification to homotopy perturbation method for solving Schlömilch's integral equations

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ABSTRACT

The linear and nonlinear Schlömilch's integral equations are considered to be important and useful equations in atmospheric and terrestrial physics. The equations and their solutions have been used for some ionospheric problems. They can also be considered as Fredholm integral equations of the first kind. This relation allows one to apply the techniques that are available for solving Fredholm integral equations of the first kind to Schlömilch's integral equations of various kinds. An important such technique is the homotopy perturbation method. This method and its variations have been applied to solve many application-based problems emerging from engineering, physics, and mathematics. In this study we modify the homotopy perturbation method by introducing a new function and define a new homotopy to solve Schlömilch's integral equations. As a result of this modification, we obtain solutions for various kinds of Schlömilch's integral equations, including the linear, nonlinear, and generalized Schlömilch's integral equations. We also establish the relationship between solutions obtained from the proposed method and the well-known gamma function. Illustrative examples are provided to show the simplicity and applicability of the proposed algorithm. For the sake of comparison, we finally test the proposed method on some problems that were solved by using different techniques available in the literature.

Key Words: Schlömilch's integral equations; Fredholm integral equations, Nonlinear integral equations

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A note on parameter derivatives of some Koornwinder polynomials in two variables

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ABSTRACT

In the last two decades, many authors [1-5] have studied the representations for the parameter derivatives of the classical orthogonal polynomials and various special functions which have many applications in applied mathematics, mathematical and theoretical physics and many branches of mathematics. For instance, the representations of parametric derivatives have been obtained by Wulkow [8] for discrete Laguerre polynomials, by Froehlich [3] for Jacobi polynomials, by Koepf [4] for generalized Laguerre polynomials and Gegenbauer polynomials, by Koepf and Schmersau [5] for all the continuous and discrete classical orthogonal polynomials. Also, the derivatives of the Legendre function of the first kind and associated Legendre polynomials of the first kind are studied by many authors. In this paper, we give parametric derivative representations for the orthogonal polynomials in two variables on the triangle, on the unit disc, on the parabolic biangle and on the square. Also, parametric derivatives of some new examples of Koornwinder polynomials which are called Laguerre–Jacobi Koornwinder polynomials and Laguerre–Laguerre Koornwinder polynomials [6,7] are discussed. Furthermore, orthogonality properties for the derivatives of orthogonal polynomials on the triangle, on the unit disc, on the parabolic biangle and on the square and also orthogonality relations for the derivatives of Laguerre–Jacobi Koornwinder and Laguerre–Laguerre Koornwinder polynomials are studied.

Key Words: Orthogonal polynomials, Jacobi polynomials, Laguerre polynomials, Koornwinder polynomials, parameter derivatives.

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A note on Solutions of the Benjamin-Bona-Mahony Equation

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ABSTRACT

In this presentation, we introduce a different sine-Gordon expansion method by providing analytical solutions of the Benjamin-Bona-Mahony equation. We target to find some new solutions by using the hyperbolic function structures. This equation, namely, Benjamin-Bona-Mahony equation has a wide range of applications in modelling long surface gravity waves of small amplitude. Furthermore, we illustrate the 2- and 3-dimensional graphics of all analytical solutions derived in this talk. On the other hand, by considering the finite difference method and operators, we achieve discretize equation. Then we take into account one of the analytical solutions to be the Benjamin-Bona-Mahony equation with the new initial condition. We analyze that the finite difference method is stable when Fourier-Von Neumann technique is used. We also analyze the accuracy of the finite difference method with terms of the errors L_2 and L_∞. By using the finite difference method, the numerical solutions of the Benjamin-Bona-Mahony equation are obtained and some comparisons for the numerical results and the exact solutions that are obtained in this talk are given. Besides, we support this comparison with the graphics, figures and tables. We perform all the computations and the graphics plot in this study with the help of Wolfram Mathematica 9.

Key Words: Sine-Gordon expansion method, finite difference method, Benjamin-Bona-Mahony equation.

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A Note On Sturm-Liouville Problems And Completeness Of Their Eigenfunctions

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ABSTRACT

The theory of boundary-value problems for Sturm-Liouville equations is one of the most actual and extensively developing fields in theoretical and applied mathematics. Sturm-Liouville differential equation is a class of differential equation often encountered in solving PDE's using the method of separation of variables. Its solutions define many of the well-known special functions, such as Bessel functions, Legendre polynomials, Chebyshev polynomials, or the various Hypergeometric functions arising in engineering and science applications. The solutions of many problems in mathematical physics are involved in investigation of a spectral problem i.e., the investigation of the spectrum and the expansion of an arbitrary function in terms of eigenfunctions of a differential operator. The issue of expansion in eigenfunctions is a classical one going back at least to Fourier. In the recent years there is a growing interest in discontinious Sturm-Liouville problems with the supplementary transmission conditions at the interior singular points. In this study we study certain spectral aspects for the Sturm–Liouville problem with finite number interior singularities. It is the purpose of this paper to extend and generalize such important spectral properties as Rayleigh quotient, eigenfunction expansion, Rayleigh-Ritz formula(minimization principle), and Parseval’s equality and Carleman equality for Sturm-Liouville problems with interior singularities. The “Rayleigh

quotient" is the basis of an important approximation method that is used in solid mechanics as well as in quantum mechanics.

Key Words: Sturm -Liouville problems, expansions theorem, Parseval's equality.

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A Note on the Convergence of the Perturbation Iteration Method

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ABSTRACT

In this research, we intend to examine the convergence of the perturbation iteration algorithms for solving linear and nonlinear differential equations by proposing a different approach of the newly developed perturbation iteration technique. We have used some important theorems which are derived with the help of Banach's fixed point theorem to achieve our goals. It is proven that under certain assumptions PIM series solution is convergent to the exact solution of the problem. We also state the importance of selections of the initial functions to verify the expected convergence. An illustration is given which demonstrates that it is possible to make the PIM solution convergent by changing the initial guess function. We have emphasized that there are no general theorems to choose initial functions for most common methods. Therefore, one needs to start an appropriate function which satisfies the initial or boundary conditions. Other examples are also solved to reveal more opportunities to extend the region of convergence of the obtained solutions.

Finally, we can say that this paper presents a detailed study on the convergence of the newly developed perturbation iteration technique and it will be encouraging for further studies.

Key Words: Perturbation iteration method, nonlinear differential equation, convergence, series expansion.

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A Numerical Solution of the Advection-Diffusion Equation by Using Extended Cubic B-Spline Functions

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ABSTRACT

It is well-known that many real life problems in physics and engineering can be modelled by the advection-diffusion equation which describe phenomena with weak nonlinearity and dispersion waves. Since the solutions of the advection-diffusion equation include the sharp behaviour with the some selection of parameters, the numerical methods is of interest due to modelling the steep solutions. Commonly the use of spline functions in lots of these methods is seen. Some of these numerical methods are the quasi-Lagrangian cubic spline method, the characteristic methods integrated with splines, the cubic B-spline Galerkin method, the quadratic B-spline subdomain collocation method, the spline approximation with the help of upwind collocation nodes, the exponential spline interpolation in characteristic based scheme, the cubic spline interpolation for the advection component and the Crank-Nicolson scheme for the diffusion component, the meshless method based on thin-plate spline radial basis functions, the least-square B-spline finite element method, the standard finite difference method, the cubic B-spline collocation method, the extended cubic B-spline collocation method, the quadratic/cubic B-spline Taylor-Galerkin methods, the cubic/quadratic B-spline least-squares finite element techniques, the cubic B-spline differential quadrature method, the quadratic Galerkin finite elements method.

The B-spline functions are generalized by adding higher order terms to the piece-wise parts of the B-spline functions having one free parameter into the extended B-spline functions by keeping the continuity. The shape of the extended B-spline can be changed by using the different free parameters. The effect of the

additional term and free parameters for the extended B-splines have been shown by solving some standard boundary value problems [4,5]. Extended B-spline functions are not as widespread as the B-spline functions to form the numerical methods for finding the numerical solutions of the partial differential equations. Recently the method of the collocation based on the extended cubic B-splines has been described for solving one-dimensional heat equation with a nonlocal initial condition in the study [3]. The study of Dag and his coworkers has appeared to solve the modified regularized long-wave equation using the method of the collocation accompanied the extended B-spline functions [2]. The collocation method based on the extended B-spline functions as trial functions is set up to find numerical solutions of the advection-diffusion equation numerically in the study [1]. In parallel with these studies, we used the extended cubic B-spline Galerkin method that were not implemented before. Thus advection-diffusion equation is fully integrated with combination of the extended cubic B-spline Galerkin method for space discretization and Crank-Nicolson method for time discretization.

Key Words: Extended cubic B-spline, Galerkin finite element method, advection-diffusion equation.

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A Numerical Solution of the Kuramoto-Sivashinsky Equation by Collocation Method using Quintic Trigonometric B-spline

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ABSTRACT

It is well known that nonlinear Kuramoto-Sivashinsky (K-S) equation governs many physical phenomena in various fields such as waves and plasma physics, flame propagation and reaction-diffusion combustion dynamics, free surface film-flows and two face flows in cylindrical or plain geometries [1-3].

Many of the problem encountered in the area of science are modelled with the differential equations. Since the model problems, especially nonlinear problems, cannot be solved analytically, the numerical methods are used to reveal the model. Thus numerical methods have been developed to solve the differential equations under the initial and boundary conditions.

K-S equation has the following form

$$U_t + UU_x + \varepsilon U_{xx} + \nu U_{xxxx} = 0, \qquad (1)$$

where ε and ν are arbitrary constants which corresponding the growth of linear stability and surface tension, respectively. The Eq. (1) includes terms of nonlinear advection UU_x, linear growth U_{xx} and high order dissipation U_{xxxx}. When ν is zero, the equation gets reduced to Burgers' equation.

Splines of various degrees have been used to construct efficient numerical methods for finding approximate solutions of differential equations. When the B-spline functions are used for the finite element methods to solve differential equations, economical and easy computer codes can be developed. The trigonometric B-spline functions are alternative to the polynomial B-spline functions. The trigonometric B-splines have been used to fit curve and to approximate the

surfaces. But few studies in which the differential equations have solved with the collocation method incorporated the trigonometric B-splines exist.

In this study, we have proposed an algorithm for numerical solution of the K-S equation, which is a finite element approach by using collocation method over finite element with quintic trigonometric B-spline interpolation functions. For the numerical procedure, time derivative is discretized in the Crank-Nicolson scheme. Solution and its principal derivatives over the subintervals are approximated by the combination of the quintic trigonometric B-splines and unknown element parameters. The resulting matrix system is solved with the matlab packet program after the boundary conditions are applied. In order to show the accuracy of the algorithm and make a comparison of numerical solution with exact one, we have studied the standard test problems.

Key Words: Kuramoto-Sivashinsky Equation, Collocation, Quintic Trigonometric B-spline.

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A Solution of Schrödinger Equation with Laplace-Beltrami Operator

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ABSTRACT

In the quantum mechanics, it is well-known that microscopic systems are analyzed by means of wave function $\Psi(t,r)\neq 0$ which is obtained by solving Schrödinger Equation $i\hbar\frac{\partial\Psi(t,r)}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\Psi(t,r)+V(r)\Psi(t,r)$ where $V(r)$ is potential function which describes potential energy of system considered and $m, \hbar, i$ are mass, planck constant and square root of -1 respectively [2]. It is well-known that information about values of position, momentum and others can not be obtained exactly because there is no exact solution for these values unlike Newton's mechanics. By using the wave function one can obtain some information which is called expectation values about the system. So we try to obtain expectation values which is expectative values when make measurement for the system. In this submission, we use Laplace-Beltrami operator $L=\nabla\lambda\nabla$ instead of Laplace operator in the Schrödinger Equation. We choose function λ as $\lambda(r)=1+\varepsilon p(r)$ in order to test consistency of wave functions as $\varepsilon\to 0$. Here function $p(r)$ is a positive function. We give an example, the infinity square well, to show results we obtained. In the example we seek energy values and eigen functions correspondingly and give some graphics for some values of parameters described. Then we compute expectation values for the position and the momentum and compare these with values obtained when Laplace operator is used. Finally we show that uncertainty principle is hold.

Key Words: Schrödinger Equation, Laplace-Beltrami operator, Potential.

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A Splitting Technique Based on the Cubic B-Spline Collocation Method for RLW Equation

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ABSTRACT

The branch of differential equations, especially partial differential equations, is an important mathematical field that has been growing rapidly from the past to the future. In addition, in the nature, many complex mechanical and engineering systems can be characterized by differential equations. Although some of them can be solved analytically, most of them can only be solved numerically. Thus, numerical solutions of differential equations have become important day by day. There are many different methods and techniques used for numerical solutions of differential equations. In the present study, one them is going to be used and tested on different problems.

In this study, the numerical solutions of the one-dimensional Regularised Long Wave (RLW) equation have been obtained using the Strang splitting technique. For this purpose, cubic B-spline base functions are used with the finite element collocation method. Moreover, single solitary wave motion, two solitary wave collision and undular bore problems are investigated with the newly presented method. Then the effectiveness of the method have been investigated on three test problems for different values of parameters in each test problem. In order to test the accuracy and efficiency of the presented method, the invariants, the error norms L_2 and L_∞ have been calculated and presented in tabular form along with others. The new results have been compared with those of some of the previous studies in the literature.

Key Words: Collocation Method, RLW Equation, Strang Splitting, Solitary waves.

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A Study On Finite Spectrum Of Sturm-Liouville Problems With $\delta-$ Interactions

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ABSTRACT

Let's consider the following equation

$$-(py')' + qy = \lambda wy.$$

A standard and well known result in Sturm-Liouville theory states that the spectrum of a regular or singular, self-adjoint Sturm-Liouville problem above is unbounded and therefore infinite. This result is generally established under the assumption that the leading coefficient p and the weight function w are both positive. However, Atkinson (see, [1], [2] and [3]) has greatly relaxed these positivity conditions. Although he assumes that p and w are non-negative on the entire underlying interval, both $1/p$ and w are allowed not only to have zeros but also to be identically zero on subintervals.

The goal of this presentation is to analyse the finite spectrum of Sturm-Liouville problems with $\delta-$ interactions. Such a problem consists of a equation which is on the interval $J=(a,x_1)\cup(a,x_2)\cup\ldots\cup(x_n,b)$, $x_1,x_2,\ldots,x_n\in(a,b)$, with $-\infty<a<b<\infty$. That equation comes from the time independent one dimensional Schrödinger equation. Indeed, a Sturm-Liouville problem with $\delta-$ interactions is equivalent to a Sturm-Liouville problem which has n transmission conditions. On account of this, we show that for any positive numbers m_j $(j=0,1,\ldots,n)$, we can construct a Sturm-Liouville problem with $\delta-$ interactions, which has exactly at most $d+1$ eigenvalues. Where d is the sum of m_j's and m_j's are related to number of the partition of the intervals between two successive interaction points. At the end of this presentation, an example is given to illustrate the this procedure.

Key Words: Sturm-Liouville problem, Sturm-Liouville problem with $\delta -$ interactions, transmission condition.

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A Study On The Rational Difference Equation Systems Order Four

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ABSTRACT

In recent times the difference equations and systems have gained great importance. Recent applications of these equations have emerged in many scientific fields such as biology, ecology, physics, engineering and economics. In particular, rational difference equations and systems are of great importance in practice. Therefore, it is very valuable to examine the behavior of the solutions of the rational differential equation system and to discuss the stability of the equilibrium points. In recent years, many researchers have researched the solutions of nonlinear difference equations and the global behavior of their two or three-dimensional systems.

El-Owaidy et al. (2005) investigated global behavior of the difference equation

$$x_{n+1} = \frac{\alpha x_{n-1}}{\beta + \gamma x_{n-2}^{p}}, \; n \in \mathrm{N}_0 \tag{1}$$

with non-negative parameters and non-negative initial conditions.

Gumus and Soykan (2016) studied the dynamical behavior of the positive solutions for a system of rational difference equations of the following form

$$u_{n+1} = \frac{\alpha u_{n-1}}{\beta + \gamma v_{n-2}^{p}}, \; v_{n+1} = \frac{\alpha_1 v_{n-1}}{\beta_1 + \gamma_1 u_{n-2}^{p}}, \; n \in \mathrm{N}_0 \tag{2}$$

where the parameters and the initial conditions are positive real numbers.

In this study, we investigate the global behavior of the positive solutions of the system of difference equations

$$u_{n+1} = \frac{a u_{n-1}}{b + c v_{n-3}^{p}}, \; v_{n+1} = \frac{d v_{n-1}}{e + f u_{n-3}^{q}}, \; n \in \mathrm{N}_0 \tag{3}$$

where the initial conditions are non-negative real numbers and the parameters are positive real numbers, by extending some results in the literature. System (3) is a naturel extention of Eq.(1) and System (2). Note that system (3) can be written as

$$x_{n+1} = \frac{\alpha x_{n-1}}{1+y_{n-3}^{p}},\ y_{n+1} = \frac{\beta y_{n-1}}{1+x_{n-3}^{q}},\ n \in \mathrm{N}_0 \tag{4}$$

by the change of variables $u_n = \left(\frac{e}{f}\right)^{1/q} x_n$, $v_n = \left(\frac{b}{c}\right)^{1/p} y_n$ with $\alpha = \frac{a}{b}$ and $\beta = \frac{d}{e}$. So, we will consider system (4) instead of system (3) in this study.

Key Words: System of difference equations, Stability, Global behavior.

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A Test Statistic For The Population Mean Based On Median Ranked Set Sampling

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ABSTRACT

Ranked Set Sampling (RSS), first introduced by McIntyre (1952), is a well-known sampling technique used in fields such as environment, ecology, and agriculture. The measurement of the sampling units according to the variable of interest may usually be very difficult or costly in these fields in terms of money, time, and other factors. In this case, RSS is a more effective design than the Simple Random Sampling (SRS) for the estimation of the population parameters. Some modified RSS designs were stated for different distribution types to increase the efficiency of RSS. Median RSS (MRSS) is one of the most widely used modified RSS designs (Muttlak(1997)). MRSS is preferred to be utilized not only to reduce the errors in ranking but also to increase the efficiency of the estimator especially for symmetric unimodal distributions, such as normal distribution.

In statistical inferences such as confidence interval and hypothesis tests on population parameters, RSS can also be utilized. First papers about hypothesis tests based on RSS were given by Mutlak and Abu Dayyeh (1998), Shen and Pan (2002), and Shen (1994). Then, Tseng and Wu (2007) obtained a formula for critical values that depend on relative efficiency values for the means of normal and exponential distributions for the median RSS and classical RSS designs. In recent years, there

are some studies on confidence interval and hypothesis test of parameters in RSS (Özdemir et. al. 2016, Albatineh et. al. 2014).

In this study, using MRSS, a test statistic was investigated for the hypothesis test on the population mean. This procedure is based on the assumption of normality with more than one cycle. Monte Carlo simulation was conducted to test the performance of this testing procedure with unknown variances against simple random sampling according to their type I error and power of test. The obtained outcomes reveal that this proposed test performs quite well.

Key Words: Median Ranked Set Sampling, Hypothesis Test, Monte Carlo Simulation.

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A Trace Formula for Discontinuous Eigenvalue Problem

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ABSTRACT

We deal with a Sturm-Liouville problem which has transmission conditions at one point of discontinuity and contains an eigenparameter in a boundary condition. We obtain the asymptotic formulas for the eigenvalues and the regularized trace formula for this discontinuous problem. The regularized trace for the classical Sturm-Liouville problem was first calculated by Gelfand and Levitan [1]. Then, Levitan suggested one more method for computing the traces of operator: by matching the expressions for the characteristic determinant via the solution of an appropriate problem and via the corresponding infinite product, he found and compared the coefficients of the asymptotic expansions of these expressions thus obtaining trace formulas [2]. To derive trace formula for our problem we will use some techniques of these works with some modifications. The regularized traces of differential operator are used in the solution of inverse problems with respect to two spectrums. Therefore, the examination of regularized traces has great importance in the area of inverse spectral problems. Also, it has important applications in numerical calculation of eigenvalues, theory of solitons and theory of integrable system. There are a few works about the regularized trace of discontinuous boundary value problems. So that our investigation will be a good example for these fields.

Key Words: Regularized trace, Gelfand-Levitan trace formula, eigenvalue problem.

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An Accurate Technique for Nonlinear Systems of Higher–Order Boundary Value Problems

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ABSTRACT

Reproducing kernel method (RKM), which computes the numerical solution, is of great attention to many branches of applied sciences. Many studies have been dedicated to the application of RKM to a wide class of differential equations. The accuracy and power of the method were considered by too researches to investigate many applications. Geng and Cui implemented the RKM to handle second-order boundary value problems. Yao and Wang et al. considered a class of singular boundary value problems by the RKM. Zhou et al. applied the RKM successfully to solve second-order boundary value problems. The RKM was applied to nonlinear infinite-delay differential equations. Wang and Chao, Zhou and Cui independently employed the RKM to variable coefficient partial differential equations. Geng and Cui, Du and Cui considered the forced Duffing equation with integral boundary conditions by joining the HPM and RKM. Lv and Cui introduced a novel procedure to solve linear fifth-order boundary value problems. Cui and Du acquired the analytical solution for nonlinear Volterra–Fredholm integral equations by RKM. Wu and Li implemented an iterative reproducing kernel method to get the analytical approximate solution of a nonlinear oscillator with discontinuities. Recently, the method was implemented to fractional PDEs and multi-point boundary value problems. Also, there

are some other methods in the literature which are utilized to extract the approximate and analytical solutions of differential equations.

In this work, we implement the reproducing kernel method to nonlinear systems of higher order boundary value problems. We prove the applicability and efficiency of the technique by some specific examples. Results present that the reproducing kernel method is very impressive.

Key Words: Reproducing kernel method, Series solutions, Nonlinear systems of high order boundary value problems.

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An Adaptive Gaussian Filter For Edge-Preserving Image Smoothing

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ABSTRACT

Filtering is well-studied issue in image processing area. However recent years bore testimony to appearance of new image filtering techniques which have proposed new perspectives about this intensively studied problem. Especially, these methods are able to detect image textures and its edges [1-3].

The Gaussian filtering is one of the most widely used blurring algorithms. The main objective of the method is to make edge clearer while smoothing the image. It is practiced by applying a convolution kernel to every pixel of an image. Gaussian filter smoothed out nosie, but at the same time the original values of signals are also distorted [4]. 2D Gaussian function is expressed as :

$$G(x,y)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{1}$$

Where σ^2 is the variance of Gaussian filter. Sigma (σ) determines the amount of smoothing and Gaussian kernel coefficients depend on the value of sigma. As the sigma becomes larger the more variance allowed around mean and as the sigma becomes smaller the less variance allowed around mean. Thus large filter variance is efficient to smooth an image, but at the same time it corrupts edges and important structure of the image.

In this paper, we developed fully adaptive Gaussian filter which is capable of adjusting filter variance and kernel size according to local characteristics of an image. To accomplish the purpose, Region Growing segmentation algorithm is applied to estimate whether the pixel point is on the image region or the image edge. The segmentation algorithm is also used to enlarge the mask size if the neighbors of the

target pixel are in the same region. In the study, the convolution kernel might enlarge up to 7x7 and sigma value is adjusted between 0.3 and 2.5. If the pixel is decided to belong to an edge, 3 by 3 convolution kernel and 0.3 sigma value are used.

Our experimental results have shown that the proposed approach always gives higher peak signal-to-noise ratio (PSNR) as compared to conventional Gaussian filter and other Gaussian noise removal filter [5,6] in the literature. The study emphasized the properties of the proposed method that effectively reduces the noise while preserving image boundaries and their information via an adaptive variance value and kernel size obtained through region growing based segmentation.

Key Words: image smoothing, Gaussian filter, edge detection, region growing segmentation

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An Application of Chaotic Dynamical System on $(\Sigma_2/_\sim, d_1)$

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ABSTRACT

Our aim in this study is to obtain a metric on the quotient set $\Sigma_2/_\sim$, which is equivalent to usual metric on [0,1]. Therefore, we first give an equivalence relation on the sequence space Σ_2 which is the set of all infinite sequences of 0's and 1's. Then, we define a distance function, d_1, on $\Sigma_2/_\sim$ and we show that this function is a metric on the quotient space with related to sequence space. We also obtain an isometry map between the metric space $(\Sigma_2/_\sim, d_1)$ and $([0,1], d_{eucl})$.

The metric d_1 which is defined on the quotient set $\Sigma_2/_\sim$ can provide some conveniences. We can convert some specific dynamical systems on $([0,1], d_{eucl})$ to dynamical systems on $(\Sigma_2/_\sim, d_1)$. Tent map on [0, 1], the doubling map on [0,1) and shift map on the sequence space in [1-4] are the well-known examples of chaotic dynamical systems. In [2, 3] there are different ways to show that these maps are chaotic dynamical system. For example, in [2] it was proved that tent map is a chaotic dynamical system by using itineraries. At the end of this study, we give an application of the metric d_1 and we show that tent map on the quotient set $\Sigma_2/_\sim$ is chaotic in a different way.

Key Words: Metric spaces, symbolic dynamics, chaotic dynamical systems.

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An Approximate Technique For Variable Order Fractional Equations

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ABSTRACT

So far, fractional calculus has been frequently used in many fields of engineering and science due to its advantages on the process of memory and hereditary properties of materials. The last studies have proved that the fractional order equations describe more realistic models when they are compared with the integer order equivalents. Variable order fractional equations have also great importance on modelling for complex systems; for instance, many physical processes exhibit anomalous diffusion behaviour with respect to time or space variables, therefore they cannot be well defined by constant order fractional equations. Consequently, there is need to use variable order fractional order equations in many cases as mathematical models. On the other hand, solving these equations analytically are difficult matter and that requires numerical or techniques which are stable and have higher order accuracy.

In this study, we use a numerical integration technique for Riemann-Liouville fractional derivative operator same as in [1] and investigate a numerical approach to the following initial value problem:

$$\begin{cases} {}_{RL}D_{0,t}^{\alpha(t)} y(t) = f(t, y(t)), \quad 0 \le t \le T \\ y(0) = 0, \end{cases}$$

where Riemann-Liouville fractional derivative is defined as,

$${}_{RL}D_{0,t}^{\alpha(t)} y(t) = \frac{1}{\Gamma(1-\alpha(t))} \frac{d}{dt} \int_0^t (t-s)^{-\alpha(t)} y(s) ds.$$

To construct a second order approximation to the above derivative, shifted Grünwald approximation is used [2]. As a result, the numerical scheme can be written as,

$$\begin{cases} \tau^{-\alpha_k} \sum_{j=0}^{k} w_j^{\alpha_k} y^{k-j} = f(t_k, y^k), \quad 0 \le k \le N \\ \qquad y^0 = 0, \end{cases}$$

where $\tau = \dfrac{T}{N}$ is the time is step, N is an integer. For $k = 0,\ldots,N,\ t_k = k\tau, \alpha(t_k) = \alpha_k$.

We use, $w_0^{\alpha_k} = \dfrac{2+\alpha_k}{2} g_0^{\alpha_k}, w_j^{\alpha_k} = \dfrac{2+\alpha_k}{2} g_j^{\alpha_k} - \dfrac{\alpha_k}{2} g_{j-1}^{\alpha_k},\ k \ge 1$ and $g_k^{\alpha(t)} = (-1)^k \binom{\alpha(t)}{k}$.

Finally, some numerical examples have been given to illustrate the efficiency and the reliabilty of the proposed method.

Key Words: Variational order ordinary fractional equations, Riemann-Liouville derivative, numerical integration.

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An h-Adaptive Approach for Element Free Galerkin Method with Applications for Inner or Boundary Layer Problems

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ABSTRACT

In this study, we consider typical one dimensional time dependent Partial Differential Equations that develop inner or boundary layers as time evolves starting with rather smooth states. Such problems require use of adaptive grids in the neighbourhood of regions where layers develop. Such grid refinement is known as h-adaptivity. There are various strategies for h adaptivity [1],[2]. In this study we propose a refinement strategy based on finite difference approximation of the second derivative of unknown, considering the estimates that indicate that larger errors occur around the points where the second derivate has large magnitude[3].

For spatial discretization we use Element Free Galerkin Method[4] which is a kind of meshless method. In this method, shape functions are obtained by Moving Least Square Method[5]. The choice of Element Free Galerkin method is motivated by our desire not to deal with mesh generation for higher dimensional problems. For temporal discretization we use appropriate routines from MATLAB ODE suite[6].

We test the performance of adaptive strategy on three typical problems that lead to inner or boundary layers as time evolves. The results are compared with the exact analytical solutions. We observe that the suggested algorithm is very successful for capturing layers and performing required refinement.

Key Words: Element Free Galerkin Method, Moving Least Square Method, Finite Difference Method, h-adaptivity, Boundary Layers.

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An Improvement to the Perturbation Iteration Technique for Solving Klein-Gordon Equations

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ABSTRACT

In this work, we introduce a new efficient technique, namely optimal perturbation iteration method (OPIM) as an alternative to existing methods in solving nonlinear partial differential equations. It has been successfully applied to obtain the solution of the nonlinear Klein-Gordon equation. Examples show that the proposed technique is effective and powerful mathematical tool to cope with these types of nonlinear problems. In OPIM, it is essentially important to find the parameters P_0, P_1,… . and this makes it time consuming, especially for large *n*. After 3-rd and 4-th iterations, we confront with high cost of calculations that occupy a vast space of computer's memory. However, we have seen that this method converges rapidly at lower order of approximations for the problems discussed in applications. On the other hand, since the method is often tedious to use by hand, one has to use a symbolic computer program to obtain approximate solutions. In this study, Mathematica 9.0 has been used to perform the complex calculations in applications. It is worth noting that one can get better results with more powerful computers or processors. Finally, we can say that the proposed method is very powerful, efficient, reliable and accurate in comparison with many competitive numerical and analytical techniques.

Key Words: Optimal perturbation iteration method, nonlinear Klein–Gordon equation, quantum mechanics, convergence.

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An Inverse Problem for a Fractional Diffusion Equation

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ABSTRACT

Inverse problems for differential equations have gained a considerable importance especially due to the recent advances in the fields of remote sensing and nondestructive evaluation. In this work, we discuss an inverse problem of determining a coefficient in a half-order fractional diffusion equation from an additional data. These equations arise in modelling anomalous diffusion phenomena in a heterogeneous aquifer. We first reduce the coefficient inverse problem to a source problem. Our aim is to investigate the stability of the solution of this problem under some assumptions on the regularity of the solutions and coefficients and the key is Carleman estimate which is a weighted L2-estimate. However, there is no Carleman estimate for the fractional diffusion equation. Therefore, first by using the definition of derivatives in the Caputo and Riemann-Liouville sense, we obtain a fourth order partial differential equation. The conditional stability of Hölder type is studied for a function with compact support. This work is based on the results by Xu et al. (2011) and Yamamoto and Zhang (2012). In the last two decades, many kinds of Carleman estimates have been obtained by several authors. They have been used for proving observability inequalities in controllability theory and uniqueness and stability results for some inverse problems.

Key Words: Inverse problem, fractional diffusion equation, Carleman estimate.

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An Investigation of a Boundary Layer Separation Near a Re-entrant Corner

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ABSTRACT

In this study, a boundary layer separation near a re-entrant corner is investigated using both analytical and numerical solutions for a steady viscous incompressible two-dimensional flow. For this purpose, least-square numerical methods associated with a treatment of the singularities is used. The resulting solution obtained by Joseph and Sturges' method is used to match with the asymptotic solution of the Stokes equation taking the first terms expansion into account along the circumference of the arc. To demonstrate the validity of the developed numerical method and its applicability, we consider the left bottom corner of the cavity which is one lid driven by the uniform motion

The topology of the boundary layer separation according to the first order term of the asymptotic solution of the stream function is then found from the wall shear stress. By using parameter in the stream function, the upstream boundary layer separation and the downstream boundary layer separation can be determined. The developed theory is then used to find the boundary layer separation around the re-entrant corner in the L-shaped cavity. Therefore, the coefficients of the stream function is obtained with the help of the least square matching method

Key Words: Flow structures, Stokes flow, Matching, Least-squares approximations

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Analysis of Electrical Machinery with Interior Variables

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ABSTRACT

Today, it is very popular to produce power with electric machines and to control this power produced. For this reason, works have been done on this subject for many fields. In electrical engineering, internal structures of some power generating machines need to be accounted according to the applied drive systems while investigating the electric machines. In this paper, an asynchronous motor is analyzed according to the applied energy and the variables in the internal structure of the machine are examined. Firstly, Engine control signals are generated to be adapted to the drive system. Then, this designed drive system is operated to drive an asynchronous motor. After these operations, the current and voltage are calculated for the internal structure of the machine. These calculations are progressively presented with electrical current and voltages having different sizes in a new approach. So, this approach relies on the axis transformation and the relation of one of the variables on this axis to the other. Finally, the operation performance of this system, which is described mathematically, is given with the simulation results. The observed results show that they have values accepted in international standards. According to this, the system described by the mathematical method provides the contribution to this field.

Key Words: Asynchronous motor, designed drive system, axis transformation, internal structure.

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Analysis of Random Zeeman Heartbeat Model by Modified Differential Transformation Method

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ABSTRACT

In this paper, the differential transformation method is used to examine the random Zeeman Heartbeat Model. Some of the parameters and the initial conditions of the model are taken as random variables with Beta and Normal distributions, respectively. The approximate analytical solution of the random Zeeman Model is obtained and used to the expectation and variance of the model components. Model formulation is given in the beginning with an introduction of the equation systems and its parameters and components. Modified Differential Transformation Method is includes the use of Laplace transforms and Padé approximants to increase the accuracy and the convergence rate of the truncated series solution obtained by Differential Transformation Method. The results for the random model and the random behaviour of its components are given with figures and tables in the conclusion. The results from the random models including Beta and Normally distributed random effects are compared and an analysis of these results are given along with the deterministic results of the model to conclude the study with an overall investigation of the random behaviour of the Zeeman Heartbeat Model. The approximations for the moment formulas obtained by Modified Differential Transformation Method are also compared with simulation results to examine the consistency of the approximate analytical results obtained for the model.

Keywords: Zeeman Heartbeat Model, Random Differential Equation, Expected Value, Variance, Laplace Transform, Padé approximant.

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Application of Differential Transformation Method for Solving Some Random Partial Differential Equations

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ABSTRACT

In this study, the solutions of random partial differential equations are examined. The parameters and the initial conditions of the partial differential equations are added random effect terms with Beta and Gaussian (Normal) Distributions, respectively. A few examples are given to illustrate the efficiency of the solutions obtained with the Differential Transformation Method (DTM). Functions for the expected values and the variances of the approximate analytical solutions of the random equations are obtained. Differential Transformation Method is applied to examine the solutions of these partial differential equations and MAPLE software is used for the finding the solutions and drawing the figures. The results for the partial differential equations with both Beta and Gaussian distributed random effects are analysed separately to investigate the differences in these two distributions. Random characteristics of the equations are compared for both cases along with the results of the deterministic partial differential equations. The efficiency of the method under random effects is investigated by comparing the formulas for the expected values and variances with results from the simulations of the random equations. MATLAB software is used to simulate the results of the random partial differential equations and various other characteristics such as the standard deviations, confidence intervals and etc. are obtained from the simulation results.

Keywords: Random Partial Differential Equation, Random Effect, Expected Value, Differential Transformation Method, Normal Distribution.

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Application of the Fractional Calculus to the Radial Schrödinger Equation given by the Makarov Potential

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ABSTRACT

The fractional calculus theory enables a set of axioms and methods to generalize the coordinate and corresponding derivative notions from integer k to arbitrary order ρ, $\{x^k, \partial^k/\partial x^k\} \to \{x^\rho, \partial^\rho/\partial x^\rho\}$ in a good light. Fractional differential equations are applied in a widespread manner in robot technology, PID control systems, Schrödinger equation, heat transfer, relativity theory, economy, filtration, controller design, mechanics, optics, modelling and so on.

Fractional calculus and its generalizations are used for the solutions of some classes of differential equations and fractional differential equations. In this paper, our aim is to solve the radial Schrödinger equation given by the Makarov potential by the help of fractional calculus theorems.

The Makarov potential is an analytically solvable problem in physics and can be used to describe ring-shaped molecules such as benzene and interactions between deformed pairs of nuclei. In spherical coordinates (r, θ, φ), the Makarov potential is given by

$$V(r,\theta) = -\frac{\alpha}{r} + \frac{\beta}{r^2 \sin^2\theta} + \frac{\gamma \cos\theta}{r^2 \sin^2\theta} \quad (\alpha > 0), \tag{1)}$$

where the first term of Equ. (1) is the Coulomb potential, the second and the third are the short range ring-shape terms. The Schrödinger equation for the Makarov potential is defined as

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r,\theta) - \mathcal{E}\right]\psi(r,\theta,\varphi) = 0. \tag{2)}$$

In analogy to the practice for usual spherical potential, we can write

$$\psi(r,\theta,\varphi)=\frac{R(r)}{r}Y(\theta)\phi(\varphi), \qquad 3)$$

and, radial part of the Schrödinger equation is

$$\left[\frac{d^2}{dr^2}+\frac{2m}{\hbar^2}\left(\mathcal{E}+\frac{\alpha}{r}\right)-\frac{\ell(\ell+1)}{r^2}\right]R(r)=0, \qquad 4)$$

and finally, we obtain the hypergeometric forms of the solutions of the Equ. (4).

Key Words: Fractional calculus, generalized Leibniz rule, radial Schrödinger equation.

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Blow up of solutions for Viscoelastic wave equations with arbitrary positive initial energy

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ABSTRACT

In this work deals with the initial boundary value problem for the viscoelastic wave equations with damping terms in a bounded domain. Viscoelastic wave equation is evolution equations. Evolution equations, namely partial differential equations with time t as one of the independent variables, arise not only from many fields of mathematics, but also from other branches of science such as physics, mechanics and material science.

In the physical point of view, this type of the problems arises usually in viscoelasticity. It has been considered first by Defermos [1], in 1970, where the general was discussed. This type problem have been considered by many authors and several results concerning local and global existence, blow up and asymptotic behaviour have been established. Special case of the our problem studied many authors (see [2, 4, 5, 6] and and references therein).

In this work, we consider viscoelastic wave equation and will establish a blow up result our problem with arbitrary positive initial energy under suitable assumptions on the function g_i and the initial data. This result extends earlier one [6], in which only some a positive initial energy considered. The main tool in proving blow up result is the "concavity method" [3] where the basic idea of this method is to construct a positive function F(t) of the solution by the energy inequality and show that $F^{-c}(t)$ is a concave function of t.

Key Words: Blow up, viscoelastic wave equation, damping term.

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Comparision Of The Tests For The Equality Of Two Population Means Based On Simple Random Sampling And Ranked Set Sampling With Auxiliary Variable

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ABSTRACT

McIntyre (1952) introduced Ranked Set Sampling (RSS) which is a sampling design and could be used instead of Simple Random Sampling (SRS) design for the estimation of population mean. RSS is widely used in some fields such as environment, ecology, agriculture and medicine whose measurement of the sampling units according to variable of interest is quite difficult or expensive in terms of cost, time and other factors. Takahasi and Wokimoto (1968) showed that the sample mean obtained by RSS is an unbiased estimator for the population mean. Moreover, the variance of this estimator is smaller than the variance of the sample mean, which is obtained from SRS with the same sample size. Dell and Clutter (1972) indicated that the RSS estimator is also an unbiased estimator for the population mean under the presence of ranking error. Mutlak and Abu Dayyeh (1998), Shen and Pan (2002) and Shen (1994) are the first authors who studied hypothesis tests using RSS. There have been several studies on confidence interval and hypothesis test of parameters in RSS recently, (Özdemir et. al. 2016, Albatineh et. al. 2014).

The hypothesis testing for the difference of means of two populations was investigated under RSS for normal distributions with unknown variances in this study. Using the Monte Carlo method, critical values were obtained for different sample sizes. Type I errors and powers of the proposed test based on RSS were compared to those of SRS under normality. The performance of the test was also examined based on RSS under non-normality. Therefore, the student t distribution was considered with different degrees of freedom combinations meeting equal and unequal variance assumptions. The performance of the RSS was also investigated under an imperfect ranking process. For this purpose, an auxiliary variable was employed to rank the interested variable. As a result, the RSS design is found to perform better than the SRS design in all the compared cases.

Key Words: Ranked Set Sampling, Ranking Error, Hypothesis Test, Monte Carlo Simulation.

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Comparison on Solving a Class of Nonlinear Systems of Partial Differential Equations

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ABSTRACT

A large number of problems in science and engineering such as problems posed in solid state physics, fluid mechanics, chemical physics, plasma physics, optics and etc, are modeled as nonlinear partial differential equations (PDEs) or systems of nonlinear PDEs. Nonlinear systems of PDEs have attracted much attention in studying evolution equations. Many authors have studied the analytical and approximate solutions of nonlinear systems of PDEs by using various techniques. We apply the reproducing kernel method to a class of nonlinear systems of partial differential equations.

Reproducing kernel space is a special Hilbert space. In recent years, there are many papers on the solution of the nonlinear problems with reproducing kernel method. The concept of reproducing kernel can be traced back to the paper of Zaremba in 1908. It was proposed for discussing the boundary value problems of the harmonic functions. This is the first reproducing kernel corresponding to function family introduced in special case and with the reproducibility proved. In the early development stage of the reproducing kernel theory, most of the works were applied by Bergman. Bergman put forward the corresponding kernels of the harmonic functions with one or several variables, and the corresponding kernel of the analytic

function in squared metric, and applied them in the research of the boundary value problem of the elliptic partial differential equation. This is the first stage in the development history of reproducing kernel.

The reproducing method has successfully been applied to several nonlinear problems, such as nonlinear system of boundary value problems, nonlinear initial value problems, singular nonlinear two-point periodic boundary value problems and singularly perturbed turning point problems. We have reached meaningful results by reproducing kernel method in this work. These results have been depicted by figures. This method is very impressive method for solving nonlinear systems of partial differential equations.

Key Words: Reproducing kernel space, reproducing kernel method, series solutions, nonlinear systems of partial differential equations.

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Computation of inverse of matrix with iterative decreasing dimension method in floating point arithmetic

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ABSTRACT

Calculation of matrix inversion is a classic problem in linear algebra. Let consider the system $AY = A[Y_1 Y_2 ... Y_n] = [AY_1 AY_2 ... AY_n] = [I_1 I_2 ... I_n]$ where A is an $n \times n$ regular matrix, Y is inverse matrix of the matrix A and $I_l, l = 1(1)n$ are column vectors of unit matrix I to be solved for.

In (Keskin and Aydın 2007), an iterative decreasing dimension method (IDDM) is given for system $Ax = f$ where A is an $n \times n$ regular matrix, x and f are n-vectors. According to this method, the solution of the system $AY = I$ is

$$Y = A^{-1} = [Y_1 Y_2 ... Y_n]; Y_l = \sum_{i=1}^{n} \left(\prod_{j=1}^{i-1} R^{(j)} \right) Y_{lö}^{(i)}, l = 1(1)n.$$

Here $Y_{lö}^{(i)}$, $R^{(k)}$ and $\prod_{j=1}^{i-1} R^{(j)}$ symbols are used same as in the relevant article (Çıbıkdiken and Aydın 2008).

When a computer is used for the computations of the inverse of the matrix A, errors occur naturally. Because the computers make the calculations with computer numbers. The computer numbers set (or format set) is defined by a set of **F** = **F**(γ, p_-, p_+, k), where $p_- \in$ **Z**$^-$, k, $p_+ \in$ **Z**$^+$ and γ- base. The set **F** is characterized by the characteristics $\varepsilon_0 = \gamma^{p_- - 1}$, $\varepsilon_1 = \gamma^{1-k}$, $\varepsilon_\infty = \gamma^{p_+}(1 - 1/\gamma^k)$ (Godunov et all 1993, Akın and Bulgak 1998).

The operator *fl* (fl : $\mathbf{D} \to \mathbf{F}$) converts the real numbers to floating point numbers with rounding or chopping errors, where $\mathbf{D} = [-\varepsilon_\infty, \varepsilon_\infty] \cap \mathbf{R}$. According to Wilkinson model the operator *fl* is defined as

$$\boldsymbol{z} \in \mathbf{D} \Rightarrow fl(\boldsymbol{z}) = \boldsymbol{z}(1+\alpha),\ |\alpha| \le u,$$

where $u = \frac{\varepsilon_1}{2}$ – rounding and $u = \varepsilon_1$ – chopping (Wilkinson 1963, Goldberg 1991, Golub and Van Loan 1996, Shampine et all 1997, Bjoerck and Dahlquist 2008).

In this study, the effects of floating point arithmetic in the computation of inverse of a matrix A with IDDM are investigated. The obtained results are supported by numerical examples.

Key Words: Iterative decreasing dimension method, calculation of matrix inversion, floating point arithmetic, error analysis.

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Computing of Some Importance Measures for Coherent Systems

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ABSTRACT

One of the system's efficiency measures is its survival probability as time goes by so called system reliability. In terms of system reliability some components are more important than other components for the systems. Thus, several methods have been developed to measure the importance of components that affect system reliability. The importance measures are also used to rank the components in order to ensure that the system works efficiently or to improve its performance or design. The first of these methods is Birnbaum reliability importance. Birnbaum Importance Measure (BIM) of a component is independent of the reliability of the component itself. BIM is the rate of increase of the system reliability with respect to increase of the component reliability. Some of the other importance measures whose common properties are derived from BIM are Structural Importance Measure, Bayesian Reliability Importance and Barlow-Proschan Importance. In this study, the definitions and formulas of these importance measures are given first and the reliability of coherent systems in serial and parallel structures is computed. Then for determining the important components for these systems Birnbaum, Structural, Bayesian and Critically Reliability Importance of components are examined. The numerical results of these methods are compared with the help of various graph and tables.

Key Words: Birnbaum reliability importance, Structural importance, Bayesian reliability importance, Barlow-Proschan Importance.

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Conformable Differential Transform Method for Fractional Differential Equations

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ABSTRACT

Fractional differential equations have been widely investigated and still being investigated due to their numerous essential applications in many fields such as, physics, chemistry, biology, astronomy, economics, mathematics, optics, fluid mechanics, architecture, engineering and other applied sciences. Many scientists and researchers have paid to attention the solutions of linear and nonlinear fractional differential equations. There are some fractional derivative definitions to use obtain solutions of linear and nonlinear fractional differential equations. There are most commonly used definitions the Rieman–Liouville and Caputo sense. Riemann-Liouville and Caputo fractional derivatives of order α are defined as respectively:

(i) $$D_a^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_a^x \frac{f(t)}{(x-t)^{n-\alpha-1}} dt, \ \alpha \in (n-1, n],$$

(ii) $$D_a^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_a^x \frac{f^{(n)}(t)}{(x-t)^{n-\alpha-1}} dt, \ \alpha \in (n-1, n].$$

The conformable derivative, one of the new defined derivative as:

(iii) $$(T_{\alpha} f)(x) = \lim_{\varepsilon \to 0} \frac{f^{(\lceil \alpha \rceil - 1)}(x + \varepsilon x^{(\lceil \alpha \rceil - \alpha)}) - f^{(\lceil \alpha \rceil - 1)}(x)}{\varepsilon}, \ x > 0, \ \alpha \in (n-1, n].$$

In this presentation, we give conformable differential transform method (CDTM). This method is a new version of well-known differential transformation method (DTM) based on conformable derivative to solve linear and nonlinear fractional ordinary differential equations. Firstly, we present some basic definitions, theorems and properties for proposal methods to solve linear and nonlinear ordinary partial differential equations. And then to better understand, the presented method is supported by some numerical applications. The applications have been analysed and illustrated in the tables and graphics. The finding values are showed that this method is very powerful and easy applicable mathematical tool for fractional ordinary differential equations. The numerical results are obtained by using Maple programming.

Key Words: Conformable derivative, fractional derivative, conformable differential transform method (CDTM), Fractional ordinary differential equations.

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Conformable Reduced Differential Transform Method for Fractional Partial Differential Equations

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ABSTRACT

Linear and nonlinear fractional differential equations have been widely investigated and still being investigated due to their numerous essential applications in many fields such as, physics, chemistry, biology, astronomy, economics, mathematics, optics, fluid mechanics, architecture, engineering and other applied sciences. Many scientists and researchers have paid to attention the solutions of linear and nonlinear fractional differential equations. There are some fractional derivative definitions to use obtain solutions of linear and nonlinear fractional differential equations. There are most commonly used definitions the Rieman–Liouville and Caputo sense. Riemann-Liouville and Caputo fractional derivatives of order α are defined as respectively:

(iv) $$ {}_{a}D^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_a^x \frac{f(t)}{(x-t)^{n-\alpha-1}} dt, \ \alpha \in (n-1, n], $$

(v) $$ {}_{a}D^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_a^x \frac{f^{(n)}(t)}{(x-t)^{n-\alpha-1}} dt, \ \alpha \in (n-1, n]. $$

The conformable derivative, one of the new defined derivative as:

(vi) $$ (T_\alpha f)(x) = \lim_{\varepsilon \to 0} \frac{f^{(\lceil \alpha \rceil - 1)}(x + \varepsilon x^{(\lceil \alpha \rceil - \alpha)}) - f^{(\lceil \alpha \rceil - 1)}(x)}{\varepsilon}, \ x > 0, \ \alpha \in (n-1, n]. $$

In this presentation, we give conformable reduced differential transform method (CRDTM). This method is a new version of well-known reduced differential transformation method (RDTM) based on conformable derivative to solve linear and nonlinear fractional partial differential equations. Firstly, we present some basic definitions, theorems and properties for proposal methods to solve linear and nonlinear fractional partial differential equations. And then to better understand, the presented newly defined method is supported by some numerical examples. The applications have been analysed and illustrated in the tables and graphics. The finding values are showed that this method is very powerful and easy applicable mathematical tool for fractional ordinary differential equations. The numerical results are obtained by using Maple programming.

Key Words: Conformable derivative, fractional derivative, conformable reduced differential transform method (CRDTM), Fractional partial differential equations.

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Creating a Web Interface for Step Size Strategies

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ABSTRACT

Differential equations are used to model mathematically many physical problem and events in nature. For this reason, it is important to be able to solve the differential equations. Consider the Cauchy problem

$$x' = f\big(t, x(t)\big), \quad x(t_0) = x_0 \tag{1}$$

and suppose that the solution of Cauchy problem (1) exists and unique. The solution of Cauchy problem (1) may not be expressed with elementary functions or its calculation may be very difficult. In such cases, the numerical methods are used, which help for find approximate solution. While using numerical methods, the using of numerical solution instead of analytical solution at each step causes error. Also, the calculation errors (rounding and chopping errors) are inevitable in numerical integration. These errors accumulates and may cause that the numerical solution diverges from analytical solution. So, it is important to choose the appropriate step size to keep under control the error accumulation.

There are some studies about variable step size in literature (Barrio ve ark. 2005,
Çelik Kızılkan 2004, Çelik Kızılkan 2009, Çelik Kızılkan and Aydın 2012, Jorba and Zou 2005, Golberg 2007, Ritschel 2013, Söderlind 2005). In these studies, it is observed that the close results to analytical solution were obtained with fewer iterations.

All numerical methods are designed to be implemented on computers. In this study, we have written the computer programs for given variable step size strategies in (Çelik Kızılkan 2004, Çelik Kızılkan 2009) with Python programming language. Because, Python is a more easily learned programing language with compared the other many programming language. Programs written in Python run fast. It is rather easy both write a program and read a program which written by another one since python has simple syntax. Therefore, Python has been a preferred language in worldwide. Recently, it has been commonly used in our country.

Also, in this study, with help of the written programs, we have created a Python-based interactive web interface. Thus, numerical integration of given a problem are easily calculated by using given variable step size strategies in (Çelik Kızılkan 2004, Çelik Kızılkan 2009).

Key Words: Cauchy problems, variable step size strategies, numeric integration.

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Critical Oscillation Constant for Half-Linear Euler Type Differential Equations With Different Periodic Coefficients

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ABSTRACT

An equation of the form

$$\left(r(t)\Phi(x')\right)' + c(t)\Phi(x) = 0, \Phi(s) = |s|^{p-1}s, p > 1, \tag{1}$$

İs called half-linear differential equation, where *r,c* are continuous functions and $r(t) > 0$.

In this work we are interested in the conditional oscillation of half-linear differential equations with different periodic coeffiicients. We say that the equation

$$\left(r(t)\Phi(x')\right)' + \gamma c(t)\Phi(x) = 0, \tag{2}$$

with positive coeffiicients is conditionally oscillatory if there exists a constant γ_0 such that Eq.(2) is oscillatory for all $\gamma > \gamma_0$ and nonoscillatory for all $\gamma < \gamma_0$. The constant γ_0 is called an oscillation constant of Eq.(2).

$$\left(\Phi(x')\right)' + \frac{\gamma}{t^p}\Phi(x) = 0, \tag{3}$$

and the half-linear Riemann-Weber differential equation of the form

$$\left(\Phi(x')\right)' + \frac{1}{t^p}\left(\gamma + \frac{\mu)}{log^2 t}\right)\Phi(x) = 0, \tag{4}$$

was considered and it was shown that Eq.(3) is nonoscillatory if and only if $\gamma \leq \gamma_p = \left(\frac{p-1}{p}\right)^p$ and Eq.(4), with $\gamma = \gamma_p$ is nonoscillatory if $\mu < \mu_p = \frac{1}{2}\left(\frac{p-1}{p}\right)^{p-1}$ and scillatory if $\mu > \mu_p$.

In [8], the half-linear differential equation of the form

$$\left(r(t)\Phi(x')\right)' + \frac{\gamma c(t)}{t^p}\Phi(x) = 0 \tag{5}$$

was considered for *r,c* being $\alpha -$ periodic, positive functions and it was shown that Eq.(5) is oscillatory if $\gamma > K$ and nonoscillatory if $\gamma < K$ where K is is given by

$$K = q^{-p}\left(\frac{1}{\alpha}\int_0^{\alpha}\frac{1}{r^{q-1}(t)}dt\right)^{1-p}\left(\frac{1}{\alpha}\int_0^{\alpha}c(t)dt\right)^{-1} \text{ and } \frac{1}{p}+\frac{1}{q}=1.$$

In [8], Eq.(5) and the half-linear differential equation of the form

$$\left(r(t)\Phi(x')\right)' + \frac{1}{t^p}\left[\gamma c(t) + \frac{\mu\, d(t)}{log^2 t}\right]\Phi(x) = 0, \qquad (6)$$

was considered positive, , $\alpha-$ periodic functions *r,c* and *d* which are defined on [0,∞) and it was shown that Eq.(5) is oscillatory if and only if $\gamma \leq \gamma_{rc}$, where γ_{rc} is given by

$$\gamma_{rc} = \frac{\alpha^p \gamma_p}{\left(\int_0^{\alpha} r^{1-q}(t)dt\right)^{p-1}\int_0^{\alpha} c(t)dt}$$

and in the limiting case $\gamma = \gamma_{rc}$, Eq.(6) is nonoscillatory if $\mu < \mu_{rd}$ and it is oscillatory if $\mu > \mu_{rd}$, where μ_{rd} is given by

$$\mu_{rd} = \frac{\alpha^p \mu_p}{\left(\int_0^{\alpha} r^{1-q}(t)dt\right)^{p-1}\int_0^{\alpha} d(t)dt}\ .$$

In this work our goal is to find the explicit oscillation constant for Eq.(6) with periodic coefficients having different periods. We point out that the main motivation of our research comes from the paper [6], where the oscillation constant was computed for Eq.(6) with the periodic coefficients having the same $\alpha-$period. But in that paper the oscillation constant wasn't obtained for the periodic functions having different periods and consequently for the case when the least common multiple of these periodic coefficients is not defined. Thus in this paper we investigate the oscillation constant for Eq.(5) with periodic coefficients having different periods.

Key Words: Half-linear differential equation, Prüfer transformation, Critical oscillation constant.

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Dependence Structure Analysis with Copula Garch Method and for Data Set Suitable Copula Selection

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ABSTRACT

Multivariate GARCH (MGARCH) models are forecasted under normality. In this study, it is used non-elliptically distributed data set which are generated Weilbull distribution. Also, we use copula-based GARCH (Copula-GARCH). The aim of the paper is to model GARCH for non-normal distributions using copulas. It is used two-step Copula-GARCH model to analyse the dependence structure of data sets. In the first step, we show data using univariate GARCH model to get standard residuals and construct marginal distributions. In this section GARCH (p,q) and GARCH (1,1) methods are introduced. We prefer GARCH (1,1) method for data set. In the second step, for dependence structures of the data sets, it is calculated Kendall Tau and Spearman Rho values which are nonparametric. Based on this method, parameters of copula are obtained. A clear advantage of the copula-based model is that it allows for maximum-likelihood estimation using all available data. The main aim of the method is to find the parameters that make the likelihood functions get its maximum value. With the help of the maximum-likelihood estimation method, for copula families, it is obtained likelihood values. This values, Akaike information criteria (AIC) and Schwartz information criteria (SIC) are used to determine which copula supplies the suitability for the data set.

Key Words: Copula Function, GARCH method, Kendall Tau.

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Dıscreteness Of The Spectrum Of One Boundary-Value Problem Wıth Interıor Sıngularıtıes

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ABSTRACT

The Sturm-Liouville problems with interior singularities arise from diverse physical models have been studied in various formulations by many authors (see, for example [5,6] and references cited therein). In recent years there has been growing interest of Sturm-Liouville type problems with supplementary interaction conditions (so-called transmission conditions) (see, for example [1-4] and references cited therein). This kind of boundary-value problems appears in solving several classes of partial differential equations, particularly, in solving heat and mass transfer problems, in diffraction problems, in vibrating string problems when the string loaded additionally with point masses and in various type of physical transfer problems. Such properties as discreteness of the spectrum, coerciveness with respect to spectral parameter, Abel basis property of a system of root functions, minimization principle of eigenvalues and etc. have been investigated in [5,7] and corresponding references cited therein. In this study we shall investigate a new type boundary-value problem, which consist of the Sturm-Liouville equation on finite number disjoint intervals together with boundary and transmission conditions. We give an operator-theoretic formulation such a way that the boundary value problem under consideration can be realized as an eigenvalue problem for suitable differential operator. Particularly, by modifying the classical Lebesgue space (in fact, the usual inner product of this space is adopted to the transmission conditions) it is established

such important properties as discreteness of the spectrum, compactness of the resolvent operator, asymptotic of the eigenvalues, coercive solvability of corresponding non-homogeneous boundary-value-transmission problem and etc.

Key Words: Sturm - Liouville problems, discreteness, spectrum, compactness, resolvent operator, interior singularity.

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Difference Scheme Method for the Numerical Solution of Telegraph Partial Differential Equation

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ABSTRACT

In this work, we presented the following telegraph partial differential equation

$$\begin{cases} u_{tt}(t,x)+u_t(t,x)+\ u(t,x)=u_{xx}(t,x)+u_x(t,x)+f(t,x), & 0\le t\le T \\ u(t,0)=u(t,L)=0\ ,u(0,x)=\varphi(x)\ ,u_t(0,x)=\Psi(x), & 0\le x\le L. \end{cases}$$

Although exact solution of this partial differential equation is known it is important to test reliability of difference scheme method. The Stability estimates for this telegraph partial differential equation are given. The first and second order difference schemes are formed for the abstract form of the above given equation by using initial conditions. Theorem on matrix stability is established for these difference schemes. The first and second order of accuracy difference schemes to approximate solution of this problem are stated. For the approximate solution of this initial-boundary value problem, we consider the set $w_{\tau,h}=[0,T]_\tau\times[0,L]_h$ of a family of grid points depending on the small parameters $\tau=\frac{T}{N}\ (N>0)$ and $h=\frac{L}{M}\ (N>0)$. Gauss elimination method is applied for solving this difference schemes in the case of telegraph partial differential equations. Exact solutions obtained by Laplace transform method is compared with obtained approximation solutions. The theoretical terms for the solution of these difference schemes are supported by the results of numerical experiments. The numerical solutions which found by Matlab program has nice results in terms of accuracy. Illustrative examples are included to demonstrate the validity and applicability of the presented technique. As a result, difference scheme method is important for above mentioned equation.

Key Words: Telegraph equation, Initial value problem, Difference scheme, Stability.

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Discrete Fractional Solutions of a Weber Equation

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ABSTRACT

In this article, we will give theorem for the discrete fractional solutions of the homogeneous Weber's equation by using the nabla discrete fractional calculus operator.

In recent years, fractional differential equations have been of great interest. It is caused both by the intensive development of the theory of fractional calculus itself and by the applications of such constructions in various sciences such as physics, mechanics, chemistry and engineering [1--3].

Recently, there appeared a number of papers on the discrete fractional calculus, which has helped to build up some of the basic theory of this area. For example, Atici and Eloe introduced the discrete Laplace transform method for a family of finite fractional difference equations in [4]. Atici and Eloe [5] defined the initial value problems in the discrete fractional calculus. Atici and Eloe [6] studied properties of discrete fractional calculus with the nabla operator. They developed exponential laws and the product rule to the forward fractional calculus. Atici and Sengul [7] developed the Leibniz rule and summation by parts formula in discrete fractional calculus. Nishimoto studied on the applications of N- fractional calculus to Weber's equation and Gauss' hypergeometric equations [8].

Theorem. Let $\psi \in F = \{\psi : 0 \neq |\psi_\eta| < \infty,\ \eta \in \mathbb{R}\}$, then the homogeneous Weber equation

$$L[\psi, z; \lambda] = \psi_2 + \psi(\lambda - z^2) = 0$$

has particular solution of the form,

$$\psi = He^{-z^2/2}\left(e^{z^2}\right)_{-\left[1+q^{-1}\left(\frac{\lambda-1}{2}\right)\right]},$$

where $\psi_k = \frac{d^2\psi}{dz^k}$ $(k = 0,1,2)$, $\psi_0 = \psi = \psi(z)$, H is a arbitrary constant and λ is a given constant.

Key Words: Weber equation, discrete fractional calculus, nabla operator.

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Effect Of Peak-To-Baselıne Ratıo On Phase Dıfferences Of Two Coupled Hodgkın Huxley Neurons

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ABSTRACT

One of the most complex dynamical systems is human brain. There are some 10^{11} neurons which of each can be connected to 10^4 other neurons exist [1] and the exact working mechanism is still unknown. In nature, variety of different systems in physics, chemistry and biology also exhibit periodic activity and they can be mathematically modeled as nonlinear oscillators [2]. The phase response curve (PRC) is one of the significant tools to investigate neuronal Dynamics [3]. Phase response curve (PRC) examines how weak perturbation effect spike time of neurons. Peak-to-baseline ratio is one of the most important specification of type II PRC neurons and it gives a brief explanation of PRC in terms of numerical sense. In this study, Hodgkin Huxley (HH) model neurons coupled via gap junction under three different applied currents are investigated in terms of PRCs, peak-to-baseline ratio and required time interval of minimum phase difference. Although the used three HH model neurons have same type of excitability and PRCs, the shapes and maximum and minimum peaks are varied. The close relationship between peak-to- baseline ratio and the required time interval of minimum phase difference of coupled neurons are found. To sum up, peak-to-baseline ratio gives invaluable information about phase differences of two coupled HH neurons.

Key Words: Peak-to-baseline ratio,Phase Response Curve, HH Model

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Examining phase response curve of nerve cell by using three different methods

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ABSTRACT

Rhythmic motion is observed in variety of different field including physical, chemical and biological system [1]. Neural system, that consists of billions of neurons are also exhibit periodic motion. Phase Response Curves; act like bridge between, a single neuron and neural network; briefly measure change in period of oscillation by giving perturbation at different points of oscillation. Phase Response Curve (PRC) found from measurement of electrical activities of neurons by experimental methods or theoretically derived from three different methods that are Direct Method, Lineer Adjoint Method and Adapted Direct Method [2] orderly. As far as we known from literature, these three different theoretical methods have been not used all together. The main purpose of this study, deriving PRC theoretical by three different methods and compare their weak and strong sides. At the beginning of article types of excitabilities of neural models, types of PRC and peak to baseline ratio are mentioned. After then, these three different methods explained deeply. PRC shapes obtained from theoretical methods compare in terms of peak to baseline ratio, simulation time and applicability. Our result clearly indicate that required simulation time for Adapted Direct method more that 4, 190 times faster than Lineer Adjoint and Direct methods respectively. The derived shape of PRC by using Adapted Direct method and Linear Adjoint method are similar to each other but the shape of PRC for Direct method approximately ten times smaller than the others.

Key Words: Phase response curve, Direct method, Linear adjoint method, Adapted direct method, peak to baseline ratio

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Existence and Uniqueness Results for a Dirichlet Problem in Orlicz-Sobolev Spaces

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ABSTRACT

In this study, we are interested in the existence and uniqueness of solutions to a quasilinear elliptic equation with Dirichlet boundary condition of the following form

$$\begin{cases} div\left(a\left(|\nabla u|\right)\nabla u\right) = f\left(u\right) + h\left(x\right) & in\ \Omega, \\ u = 0 & on\ \partial\Omega, \end{cases} \qquad \text{(P)}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, f is a continuous function, $h \in L^{p/(p-1)}(\Omega)$ and $\varphi(t) := a(|t|)t$ is an increasing homeomorphisms from $\mathbb{R}$ onto $\mathbb{R}$. We want to remark that in problem (P) if we let $a(t) := |t|^{p-2}$ problem (P) turns into the well-known p-Laplace equation and in case of $p = p(x)$, i.e., $a(t) := |t|^{p(x)-2}$, problem (P) becomes the $p(x)$-Laplace equation, the generalization of p-Laplace equation. Since the problem contains a nonhomogeneous function φ in the differential operator, problem is settled in the Orlicz-Sobolev spaces. The main tools used here are Variational Method and Critical Point Theory. We show that the energy functional corresponding to problem (P) is strictly convex and differentiable. Therefore, it must have at most one critical point in the Orlicz-Sobolev space. Then we also obtain that the functional is continuous and coercive, and hence, it has a global minimum point, which is in turn a weak solution to problem (P). Furthermore, since the functional is strictly convex and differentiable, it must have only one critical point, that is, problem (P) has a unique solution.

We want to remark that the importance of problem (P) is that it generalizes the well-known p-Laplace equation and $p(x)$-Laplace equation. The study of variational problems in the classical Sobolev and Orlicz-Sobolev spaces is an interesting topic of research due to its significant role in many fields of mathematics, such as approximation theory, partial differential equations, calculus of variations, non-linear potential theory, the theory of quasiconformal mappings, non-Newtonian fluids, image processing, differential geometry, geometric function theory, and probability theory. Moreover, problem dealt this presentation possess more complicated nonlinearities, for example, it is inhomogeneous, so in the discussions, some special techniques will be needed. However, the inhomogeneous nonlinearities have important physical background. Therefore, problem (P) may represent a variety of mathematical models corresponding to certain phenomenon.

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Explicit Solutions of the Whittaker Equation by Means of Discrete Fractional Calculus Operator

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ABSTRACT

In this work, we acquire new discrete fractional solutions of the Whittaker differential equation by using discrete fractional nabla calculus operator. We consider homogeneous and non-homogeneous Whittaker differential equation.

In recent years, fractional differential equations have been of great interest. It is caused both by the intensive development of the theory of fractional calculus itself and by the applications of such constructions in various sciences such as physics, mechanics, chemistry and engineering [1--3].

Recently, there appeared a number of papers on the discrete fractional calculus, which has helped to build up some of the basic theory of this area. For example, Atici and Eloe introduced the discrete Laplace transform method for a family of finite fractional difference equations in [4]. Atici and Eloe [5] defined the initial value problems in the discrete fractional calculus. Atici and Eloe [6] studied properties of discrete fractional calculus with the nabla operator. They developed exponential laws and the product rule to the forward fractional calculus. Atici and Sengul [7] developed the Leibniz rule and summation by parts formula in discrete fractional calculus. Bastos and Torres [8] presented the more general discrete fractional operator and this operator was defined by the delta and nabla fractional sums.

Key Words: Discrete fractional calculus; Whittaker equation; nabla operator

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Formulation of Peridynamic Orthotropic Plates with Transverse Shear Deformation

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ABSTRACT

Composite materials are commonly used in defence and aerospace applications. In typical applications, carbon or glass fabrics are combined with honeycomb cores to take advantage of their relatively high in plane shear strength. Depending on the structure the thickness of the honeycomb core can vary between 6-35 and 25.4. Due to their high thickness, transverse shear deformation should not be neglected in structural analyses of such aircraft components.

Classical Continuum Mechanics approach is commonly applied to both isotropic and orthotropic structures in the industry. However, in classical continuum mechanics, crack initiation and propagation requires an external criteria such as critical energy release rate. Peridynamic theory that is proposed by Dr. Silling is capable of simulating crack growth in both isotropic and orthotropic media [1] without referring to an external criteria. Peridynamic Theory (PD) is a nonlocal version of Classical Continuum Mechanics. In PD, equations of motion is formulated using integro-differential equations.

In this paper, a Peridynamic orthotropic plate formulation with transverse shear deformation will be derived. PD equations of motion will be derived by applying principle of virtual work. The proposed formulation will be validated by applying pure bending loading to an orthotropic plate.

Key Words: Peridynamic Theory, Composite Materials, Transverse Shear Deformation.

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Fractional-Order Modeling of Bacterial Resistance to Multiple Antibiotics

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ABSTRACT

In this study, it is described the general forms of fractional-order differential equations and asymtotic stability of their system's equilibria. In addition that, the stability analysis of equilibrium points of the local bacterial infection model which is fractional-order differential equation system, is made.

The proposed model in this study is fractional-order form of model suggested in [1]. In this respect, the population sizes of sensitive and resistant bacteria to multiple antibiotics at time t is denoted by $S(t)$and $R(t)$, respectively. In addition that, the concentration of the i-th antibiotic, $i = 1,2,\dots,n$ is showed by $C_i(t)$. Therefore, it is obtained the following system of $(n+2)$ fractional-order differential equation:

$$\begin{aligned} D^\alpha S(t) &= S\left(\beta_s\left(1-\frac{S+R}{K}\right)-\left[\sum_{i=1}^{n}(\overline{q_i}+\overline{\alpha_i})\,C_i\right]-\mu_s\right) \\ D^\alpha R(t) &= \beta_r R\left(1-\frac{S+R}{K}\right)+S\left[\sum_{i=1}^{n}\overline{q_i}\,C_i\right]-\mu_r R \\ D^\alpha C_i(t) &= \Lambda_i-\mu_i C_i, \qquad i=1,2,\dots,n \end{aligned} \tag{1}$$

where $\alpha \in (0,1]$. The parameters used in the model (1) are as follows: it is presumed that bacteria follow a logistic growth with carrying capacity K. The parameter β_S and β_r are the birth rate of susceptible and resistant bacteria, respectively. Specific mutations emerging resistance to chemical control often include an inherent fitness cost which may be outcomed through reduced reproductive capacity and/or competitive ability. Thus, it is

$\beta_S > \beta_R \quad (2)$

The sensitive and resistant bacteria to multiple antibiotics have per capita natural death rates $\mu_{B_1}$and μ_{B_2}, respectively. During the administration of the i-

thantibiotic, a number of resistant bacteria to it can be showed up due to mutations of exposed sensitive bacteria to such antibiotic, it is modeled this situation by the term $\overline{q_i}C_iS$ where $\overline{q_i}$ is the mutation rate of sensitive bacteria due to exposure to i-th antibiotic. Sensitive bacteria also die due to the action of the antibiotics, and it is assumed that this situation in model is by the term $\overline{\alpha_i}C_iS$, where $\overline{\alpha_i}$ is the death rate of sensitive bacteria due to exposure to i-th antibiotic. Finally, the i-th antibiotic concentration is supplied at a constant rateΛ_i, and is taken up at a constant per capita rateμ_i.

These interact between bacteria and antibiotic have depicted a generalised model of a local bacterial infection, such as wound infection or tuberculosis.

The qualitative analysis results of model are supported via numerical simulations drawn by datas obtained from literature for mycobacterium tuberculosis and the antibiotics isoniazid (INH), rifampicin (RIF), streptomycin (SRT) and pyrazinamide (PRZ) used against this bacterial infection.

Key Words: Fractional-order differential equation system, Mathematical model, Stability analysis, Equilibrium points, Multiple antibiotics

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Galerkin Method for the Numerical Solution of the Modified Equal Width Wave (MEW) Equation

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ABSTRACT

The modified equal width wave (MEW) equation has the form

$$u_t + \varepsilon u^2 u_x - \mu u_{xxt} = 0, \tag{1}$$

with the boundary conditions

$$\begin{aligned} &u(a,t) = u(b,t) = 0, \\ &u_x(a,t) = u_x(b,t) = 0, t \in [0,T] \end{aligned} \tag{2}$$

and initial condition

$$u(x,0) = f(x),\ a \le x \le b \tag{3}$$

in a restricted solution domain over a space/time interval $[a,b]\times[0,T]$ [1] . This equation is a nonlinear partial differential equation and represents a model for nonlinear dispersive waves. A few analytical solutions of the MEW equation together with some initial and boundary conditions are available. So many authors have investigated the numerical solution of the MEW equation [2-4].

In this study, the MEW equation is solved numerically by Galerkin finite element method, based on quadratic trigonometric B-spline for the space discretization and Crank Nicholson method for time discretization. Galerkin method based on different degrees B-spline finite elements have been widely used to find numerical solutions of the MEW or similar equation, but there are a few studies in which trigonometric B-splines are used to construct numerical methods for partial differential equations using Galerkin method. Proposed method is investigated on the problems of propagation of single solitary wave and interaction of two solitary waves

for MEW equation. To see the accuracy and efficiency of the method, error norm L_∞ for the first test problem is computed and results are compared with previous published studies. The three conservation quantities of the motion are calculated to accurate numerical scheme for both of the test problems. According to the numerical results, It can be seen that the numerical scheme leads to accurate and efficient results.

Key Words: Trigonometric B-spline, Modified Equal Width Wave equation, Galerkin method

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Global nonexistence of the solutions for a Petrovsky equation with fractional damping

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ABSTRACT

In this work we prove global nonexistence result for solutions of the Petrovsky equation with fractional order damping terms. Petrovsky equation is evolution equations. Evolution equations, namely partial differential equations with time t as one of the independent variables, arise not only from many fields of mathematics, but also from other branches of science such as physics, mechanics and material science.

This type problem have been considered by many authors and several results concerning local and global existence, blow up and asymptotic behaviour have been established. Special case of the our problem studied many authors (see [1, 2, 3] and and references therein).

The Petrovsky equation with classical nonlinear weak damping terms (u_t^p) studied by Messaoudi [3]. He studied local and global existence and blow up of the solutions. Later, Pişkin and Polat [5] showed exponential decay, for p=1 and the polynomial decay, for p>1 were established by using Nakao's inequality. When $0<a<1$, the term $(-D)^a u_t$ is said to be fractional damping or structural damping, and it plays a dissipative role, which is weaker than the strong damping but stronger than the weak damping.

We will prove the global nonexistence of the solution with negative initial energy by using the technique of [1, 2] with a modification in the energy functional due to the different nature of the problems.

Key Words: Global nonexistence, Petrovsky equation, Fractional damping.

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Goodness of fit tests for Weibull distribution

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ABSTRACT

The Weibull distribution is widely used in modelling positively skewed data coming from the areas of reliability and life testing. So it is important to test whether the data set comes from Weibull distribution. For this purpose, goodness of fit tests are used for checking whether the data set is compatible with the Weibull distribution. Goodness of fit tests are designed to measure the compatibility of a random sample with a theoretical probability distribution function. Several goodness of fit tests are available in the literature such as those of Kolmogorov-Smirnov, Anderson Darling, Cramer von Mises, Watson, Zhang and Esteban test statistics. These test statistics are generally measure, in different ways the distance between a continuous distribution function $F(x)$ and the empirical distribution function $F_n(x)$. In this study, some goodness of fit tests are investigated for Weibull distribution with estimated parameters. Extensive tables of goodness of fit critical values for the Weibull distribution are developed thorough simulation for the Zhang, Modified Anderson Darling, Esteban and Mann test statistics. Moreover, type1 error rates and the power of the test statistics are compared by a Monte Carlo simulation study. These test statistics are compared under gamma, log-normal, inverse Gaussian, generalized Weibull, skew normal, and truncated normal and chi square distributions. Finally, three real data sets are analyzed at the end of the study for application purposes.

Key Words: Weibull distribution, Goodness of fit test, Critical values, power comparison, Monte Carlo simulation.

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Growth of solutions for nonlinear coupled wave equations with damping terms

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ABSTRACT

Our aim in this work is to give the exponential growth of solutions for coupled nonlinear wave equations with damping terms by the using the energy method.

Viscoelastic wave equation is evolution equations. Evolution equations, namely partial differential equations with time t as one of the independent variables, arise not only from many fields of mathematics, but also from other branches of science such as physics, mechanics and material science.

In the physical point of view, this type of the problems arises usually in viscoelasticity. It has been considered first by Defermos [1], in 1970, where the general was discussed. This type problem have been considered by many authors and several results concerning local and global existence, blow up and asymptotic behaviour have been established. Special case of the our problem studied many authors (see [2, 3, 5] and and references therein).

We will prove that the energy associated to the system is unbounded. In fact, it will be proved that the energy will grow up as an exponential function as time goes to infinity, provided that the initial data are large enough. The key ingredient in the proff is a method used in Vitillaro [7] and developed in Houari [3] for a system of wave equations, with necessary modificaiton imposed by the nature of our problem.

Key Words: Growth, nonlinear coupled wave equations, nonlinear damping terms.

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Hılbert Space Method For Sturm- Lıouvılle Problems Wıth Interface Condıtıons

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ABSTRACT

The Sturm-Liouville problem is important because of the solutions to a homogeneous BVP with homogeneous BCs produce a set of orthogonal functions. Such functions can be used to represent functions in Fourier series expansions. The completeness of classical systems of eigenfunction expansions was originally related to mechanical problems and boundary value problems for differential operators. Later the study of eigenfunctions expansions has gained an independent and abstract status. The expansion has an integral operator form whose kernel is a spectral function, the representation of which is the Green function of the operator.

In this study we introduce a new approach to the two-interval Sturm-Liouville eigenfunction expansions, based essentially on the method of integral equations and Green's function method in suitable Hilbert space. Note that in physics many problems arise in the form of boundary value problems involving second order ordinary differential equations with supplementary transmission conditions. This derivation based on that in [5] however, is more thorough than that in most elementary physics texts; while most parameters such as density and other thermal properties are treated as constant in such treatments, the following allows fundamental properties of the bar to vary as a function of the bar's length, which will lead to a Sturm-Liouville problem of a more general nature.

Key Words: Hilbert space method, Sturm-Liouville problems, transmission conditions.

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High Order Accurate Numerical Solution of the Generalized Burgers-Fisher Equation

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ABSTRACT

In this work, we investigate numerically solution of the generalized Burgers-Fisher equation using fully implicit exponential finite difference method.

The generalized Burgers-Fisher equation has significant applications in various fields of applied mathematics and physics such as fluid dynamics, shock wave formation, turbulence, heat conduction, traffic flow, gas dynamics, sound waves in viscous medium, and some other fields of applied science[1]. We would like to point out that the development of the present methodology was inspired of exponential finite difference method was originally developed by Bhattacharya. Firstly, Bhattacharya defined the explicit exponential finite difference method to solve heat equation[2]. After, Bhattacharya applied the method for the solution of Burgers equation[3]. Bhattacharya's approach has been followed extensively in order to provide different exponential computational techniques to approximate the solutions of various partial differential equations[4-8].

The numerical solutions obtained by fully implicit exponential finite difference method for the generalized Burgers-Fisher equation compared with exact solutions and with other methods available in the literature. We can say definitely from the comparisons the results obtained by the fully implicit exponential finite difference schemes have better than results obtained from the other considered numerical schemes. The results show that the present method is efficient and reliable.

Keywords. The generalized Burgers-Fisher equation, Finite difference method, Exponential finite difference method, Fully implicit exponential finite difference method.

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Hyers-Ulam Rassias Stability of a Functional Differential Equation

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ABSTRACT

Equation stability is an important subject in the applications. In general terms, we may say that the main issue in the stability of functional equations is to answer the question of when the solutions of an equation, differing slightly from a given one, must be close to a solution of the given equation. Considering this point view, the Hyers-Ulam Rassias stability of differential equations is fundemantal. Recently, many authors have used different methods (integral factor, fixed point method) to show that the Hyers-Ulam Rassias and Hyers-Ulam stability of differential equations. In this study, we use Banach's contraction principle to show that the Hyers-Ulam Rassias stability of a first order functional differential equation with constant delay of the form

$$y'(t) + f(t, y(t-\tau)) = 0$$

in which f is continuous function and τ is a nonnegative real constant. By taking advantage of this result, we investigate Hyers-Ulam Rassias stability of a second order functional differential equation with constant delay of the form

$$y''(t) + f(t, y(t-\tau))y'(t-\tau) + h(t, y(t-\tau)) = 0,$$

where f, h are continuous functions and τ is a nonnegative real constant. An example is given to illustrate the theoretical analysis in this work. The results to be obtained here provide a contribution to the subject in the literature and may be useful for researchers working on the qualitative behaviors of the functional differential equations.

Keywords: Hyers-Ulam Rassias stability, Banach contraction principle, second order.

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Inequalities with Fractional Operators

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ABSTRACT

Leibniz and L'Hospital started the theory of fractional calculus in 1965. By the nineteenth century, efforts of a number of mathematicians, most notedly Riemann, Grünwald, Letnikov, Liouville, lead to a consistent theory of fractional calculus for real variable functions. Although there are many definitions of fractional derivatives, the most known definitions are Riemann-Liouville and Caputo derivatives.

When it cames to the theory of discrete fractional calculus, we mention the paper presented by Diaz and Osler in 1974. In this paper, the authors introduced a fractional difference operator using an infinite series. In 1988, Gray and Zhang introduced a new definition of a fractional difference operator and they proved a Leibniz formula, composition rule and power rule. Whereas Diaz et al. gave a definition for the delta (forward) difference operator, Gray et al. gave their definition for the nabla (backward) difference operator.

Mathematicians have began to pay attention to this theory for last three decades. As a pioneering work, Atici and Eloe presented properties of a generalized falling function that plays a major role in difference calculus as exponential function, power rule and commutativity of fractional sums.

Since inequalities are useful tools in mathematics, authors began to establish fractional analogues of those being published in difference calculus.

In this presentation, we will fractional analogues of some well-known inequalities using discrete operators.

Key Words: Fractional difference operator, inequalities.

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Interior Structural Bifurcation near Simple-Degenerate Singular Point with Symmetry

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ABSTRACT

Structural bifurcation of divergence free vector fields near an interior point have been studied by Ma and Wang [1]. They give the necessary and sufficient conditions for structural stability of a divergence free vector field. In this study, their method is extended to a simple degenerate critical points under symmetry conditions. Divergence free vector fields u(.,t) near degenerate singular points are simplified by the homotopy invariance of the index. Their structural bifurcation with index 1 and -1 are analyzed by deriving kinematic conditions. From this, flow patterns such as a pair of co-rotating vortices and a double saddle connection are obtained.

Also, we obtain the necessary conditions for the appearance of the degenerate critical points in the flow field for one parameter divergence free vector fields. We consider the connection between the index of a divergence free vector field $\vec{u}$ at x_0 and different orders of $\vec{u}$ in its Taylor expansion near x_0 with symmetry about y-axis and we obtain flow structure corresponding to these indices. One can think x_0 has another structure of the flow when the orbits of x_0 anti-symmetric about the origin $O(0,0)$. For this purpose we consider the degenerate structures of divergence free vector field $\vec{u} \in D^r(TM)$ with anti-symmetry. Our results consistent with previous experimental and computational results.

Key Words: Flow structures, structural stability, divergence-free vector field and bifurcation.

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Interval Estimation for Population Variance with Kurtosis Coefficient Based on Trimmed Mean

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ABSTRACT

Let $x_1, x_2, ..., x_n$ be a random sample of size n from normally distribution. It is known that the sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator for σ^2. An improved estimator of the variance that utilizes the kurtosis was derived by Searls and Intarapanich (1990). This estimator is biased and has the minimum possible mean-squared error and it is defined as the "minimum mean-squared error biased estimator" (MBBE). The estimator has the form,

$$S_w^2 = w\sum_{i=1}^{n}(X_i - \bar{X})^2, \ w \in (0,1) \tag{1}$$

where the weight is given as;

$$w = [(n+1) + (\gamma_4 - 3)n^{-1}(n-1)]^{-1} \tag{2}$$

It's a function of the sample size *n* and the kurtosis coefficient γ_4. This estimator is always a more efficient estimator compared to unbiased S^2 estimator of the variance even in nonnormal distribution assumption (Wencheko and Chipoyera, 2009). For large n_i, when random sampling from any distribution with a finite fourth moment, and by the central limit theorem, the mbbe of variance is approximately standard normal with $E(S_w^2)$ and $\mathrm{MSE}(S_w^2)$.

Expected value and mean squared error of a biased estimator S_w^2 are given as;

$$E(S_w^2) = \mathrm{w}(\mathrm{n}-1)\sigma^2 \tag{3}$$

and

$$\mathrm{MSE}(S_w^2) = \mathrm{w}^2(n-1)^2\,\mathrm{var}(S^2) + [(\mathrm{n}-1)\,\mathrm{w}-1]^2\sigma^4 \tag{4}$$

It is known that S^2 sample variance estimator does not display robust statistics features in the estimation of nonnormal population variance and the coverage probabilities of the confidence intervals obtained with this estimator have much lower values compared to the nominal confidence interval [Scheffé , 1959; Casella and Berger, 2001]. In such cases, it is necessary to use robust scale estimators for estimation of population variance.

For nonnormal populations, sample kurtosis coefficient is a quite biased estimator (Royston, 1992). In this study, it is aimed that the kurtosis coefficient obtained with $0.5/\sqrt{n-4}$ trimming proportion which was suggested by Bonett (2006a, 2006b) is used instead of sample kurtosis coefficient in obtaining MBBE estimator of variance. With this information, confidence intervals based on this estimator were obtained for nonnormal population variance. Coverage probabilities and average length widths of these confidence intervals were compared with the confidence interval coverage probabilities and average length widths obtained when sample kurtosis coefficient is used.

When confidence intervals based on S_w^2 robust estimator which was obtained using both the kurtosis coefficient based on trimmed mean and the sample kurtosis coefficient for nonnormal population variance are compared in terms of coverage probabilities, it was determined that coverage probabilities of confidence interval obtained with the S_w^2 estimator which is obtained by using the kurtosis coefficient based on trimmed mean when type I error is both 0.05 and 0.10 are quite approximate to the nominal confidence level even in small sample sizes where confidence interval coverage probabilities are higher. When these confidence intervals are compared in terms of average length widths, it was determined that the average length widths of the S_w^2 estimator which was obtained by using the kurtosis coefficient based on trimmed mean are narrower. According to this result, if it is

desired to create a narrower confidence interval for nonnormal population variance, kurtosis coefficient which is obtained by using the trimmed mean should be preferred in obtaining the S_w^2 estimator which has high coverage probability.

Keywords: Coverage probability, Kurtosis coefficient, Minimum mean-squared error.

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Inverse Eigenvalue Problems For Dirac Operator With Finite Number Of Transmission Conditions

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ABSTRACT

In this presentation, we consider the problem $L\left(\Omega(x), a_i(\lambda), \mathrm{b_i}(\lambda), \theta_i, \mathrm{\rho_i}, \gamma_i(\lambda)\right)$ of the form

$$l(Y) := BY'(x) + \Omega(x)Y(x) = \lambda\rho(x)Y(x) \quad x \in I := \bigcup_{i=0}^{n}(\xi_i, \xi_{i+1}), \xi_0 = a, \xi_{n+1} = b \tag{1}$$

where $y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}, B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Omega(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & r(x) \end{pmatrix} p(x), q(x), r(x) \in L_2(I, \Re)$

$,\rho(x) = \begin{cases} \rho_0, & \xi_0 \leq \mathrm{x} < \xi_1 \\ \rho_i, & \xi_i < x < \xi_{i+1} \\ \rho_n, & \xi_n < x \leq \xi_{n+1} \end{cases}, \rho = \rho_i \in \Re^+, \left(i = \overline{0,n}\right)$ and λ is the spectral parameter and with boundary conditions

$$l_1(y) := a_2(\lambda)y_2(a) - a_1(\lambda)\mathrm{y}_1(\mathrm{a}) = 0 \tag{2}$$

$$l_2(y) := b_2(\lambda)y_2(b) - b_1(\lambda)\mathrm{y}_1(\mathrm{b}) = 0 \tag{3}$$

where$a_i(\lambda)$and $\mathrm{b_i}(\lambda)$, $\mathrm{i} = 1{,}2$are polynomials with respect to λ and with transmission conditions

$$\begin{aligned} &l_3(y) := y_1(\xi_i + 0) - \theta_i y_1(\xi_i - 0) = 0 \\ &l_4(y) := y_2(\xi_i + 0) - \theta_i^{-1} y_2(\xi_i - 0) - \gamma_i(\lambda) y_1(\xi_i - 0) = 0, i = \overline{1,n}, \end{aligned} \tag{4}$$

where $\xi_i \in [\mathrm{a,b}]$, $i = \overline{1,n}$, $\theta_i \in \Re^+$, $i = \overline{1,n}$ and $\gamma_i(\lambda)$ $\left(i = \overline{1,n}\right)$ are also polynomials with respect to λ.

Some operators with eigenparameter linearly or non-linearly dependent boundary conditions and transmission conditions come up in various problems of mathematics as well as in applications. This kind of studies for ordinary differential

operators depending on the parameter exist in various papers[1-8]. In 1977, Fulton studied the Sturm-Liouville problem with the boundary conditions dependent on the spectral parameter linearly and obtained the spectral theory of these kinds of problems. After that, Such spectral problems have continued to increase. These kinds of works are more difficult to investigate and there exist some papers in this sense.

We study the inverse spectral problem for (1)-(4) on a finite interval. Inverse problems of spectral analysis consist in recovering operators from their spectral characteristics. Such problems often appear in mathematical physics, electronics, mechanics, geophysics and other branches of natural sciences. In accordance with this purpose, first, we get some properties of eigenvalues and eigenfunctions. Then, we introduce the Weyl function for considered Dirac operator and investigate some uniqueness theorems by using Weyl function and some spectral data.

Key Words: Dirac Operator, Inverse Problem, Eigenvalue, Eigenfunction, Characteristic Function, Weyl Function, Weyl Solution

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Inverse Nodal Problem For *P*-Laplacian Bessel Equation With Polynomially Dependent Spectral Parameter

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ABSTRACT

In this study, solution of inverse nodal problem for p-Laplacian Bessel equation is extended to the case that boundary condition depends on polynomial eigenparameter. To find the spectral datas as eigenvalues and nodal parameters of this problem, we used a modified Prüfer substitution. Then, reconstruction formula of the potential function is also given by using nodal lenghts. However, this method is similar to used in [1], our results are more general. Inverse spectral problem consists in recovering differential equation from its spectral parameters like eigenvalues, norming constants and nodal points (zeros of eigenfunctions). These type problems divide into two parts as inverse eigenvalue problem and inverse nodal problem. They play important role and also have many applications in applied mathematics. Inverse nodal problem has been firstly studied by McLaughlin in 1988 [2]. She showed that the knowledge of a dense subset of nodal points is sufficient to determine the potential function of Sturm-Liouville problem up to a constant. Nowadays, many authors have given some interesting results about inverse nodal problems for different type operators. In this study, we concern ourselves with the inverse nodal problem for *p*-Laplacian Bessel equation [3-4] with boundary condition polynomially dependent on spectral parameter. As far as we know, this problem has not been considered before. Furthermore, we give asymptotics of eigenparameters and reconstructing formula for potential function.

Key Words: Inverse Nodal Problem, Prüfer Substitution, p-Laplacian Bessel equation.

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Inverse Problem on Sturm-Liouville Operator with Parameter Dependent Boundary and Finite number of Transmission Conditions

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ABSTRACT

Sturm-Liouville equations with boundary conditions linearly dependent on the spectral parameter were addressed by many authors. Such problems often arise from physical problems, for example, vibration of a string, quantum mechanics and geophysics. In [1]-[6], an operator-theoretic formulation of the problems with the spectral parameter contained in only one of the boundary conditions has been given and inverse problems according to various spectral data for eigenparameter linearly or nonlinearly dependent Sturm-Liouville operator were investigated

In this study, we consider a boundary value problem $L = L(q, h, H, s_1, s_2, \dots, s_N)$ of the form

$$\ell y := -y'' + q(x)y = \lambda w(x)y, \;\; x \in \cup_{i=1}^{N}(d_i, d_{i+1})$$

with the boundary conditions

$$U(y) := \lambda(y'(0) + h_0 y(0)) - h_1 y'(0) - h_2 y(0) = 0$$

$$V(y) := \lambda(y'(1) + H_0 y(1)) - H_1 y'(1) - H_2 y(1) = 0$$

and a finite number of discontinuity conditions

$$\begin{cases} y(d_i + 0) = \alpha_i y(d_i - 0) \\ y'(d_i + 0) = \alpha_i^{-1} y'(d_i - 0) - (w_i \lambda + \beta_i) y(d_i - 0) \end{cases}$$

where $q(x)$ is real valued function in $L^2(0,1)$; β_i , h_j and H_j, $j = 0, 1, 2$, are real numbers; $\alpha_i \in \mathbb{R}^+$, $d_0 = 0$, $d_i \in (0,1)$, $d_{N+1} = 1$, $\rho_1 := h_2 - h_0 h_1 > 0$, $\rho_2 := H_0 H_1 - H_2 > 0$, $w(x) = 1/\sigma_k^2$, $d_k < x < d_{k+1}$, $\sigma_k \in \mathbb{R}$ for $k = 0, , \cdots, N$, $\sigma_0 = 1$ and λ is a spectral parameter, $h = (h_0, h_1, h_2)$, $H = (H_0, H_1, H_2)$, $s_i = (d_i, \alpha_i, \omega_i, \beta_i)$, $i = 1, \cdots, N$.

For the problem L, it is obtained a represantation for the solution and some important properties of eigenvalues are studied. Moreover, It is proven that the coefficients of the problem can be uniquelly determined by Weyl function. The obtained results are generalizations of the similar results for the classical Sturm-Liouville operator on a finite interval.

Key Words: Inverse problem, Sturm-Liouville operator, Weyl function, Transmission conditions, Eigenparameter dependent boundary conditions.

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Inverse Problems For Sturm-Liouville Operators With Discontinous Coefficients And Spectral Parameter Dependent To Boundary Conditions

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ABSTRACT

In this study, Sturm-Liouville operators which have discontinuous coefficients with spectral parameter dependent to boundary conditions has been investigated. Also Weyl function for this problem under consideration has been defined and uniqueness theorems for solution of inverse problem according to this function have been proved.

We consider the problem

$$l(y) = -y'' + \left(\frac{A}{x^{\alpha}} + q_0(x)\right)y = \lambda^2 \rho(x) y \quad ,0 < x \leq \pi$$

$$(\Gamma_{\alpha} y)(0) - hy(0) = 0$$

$$\lambda\big(y'(\pi) + H_1 y(\pi)\big) = y'(\pi) + H_2 y(\pi)$$

where

$$\rho(x) = \begin{cases} 1 \,, 0 < x \leq a \\ \beta^2, a < x \leq \pi \end{cases}$$

$A \in R^{+}, 1 \leq \alpha < \frac{3}{2}, \beta > 0, \beta \neq 1$ are real numbers, $q_0(x)$ is real valued bounded function in $L_2(0,\pi)$, $\boldsymbol{\lambda}$ is a spectral parameter.

We denote that in spectral theory, the inverse problem is the usual name for any problem in which it is required to ascertain the spectral data that will determine a differential operator uniquely and a method of construction of this operator from the

data. This kind of problem was first formulated and investigated by Ambartsumyan in 1929 [1]. Since 1946, various forms of the inverse problem have been considered by numerous authors G. Borg[2], N. Levinson [3], B.M. Levitan [4], etc. The inverse problems having specified singularities were considered by a number of authors [5], [6], [7]. The method of spectral mappings is an impressive device for investigating a profound class of inverse problems not only for Sturm-Liouville operators, but also for other more complicated classes of operators such as differential operators of arbitrary orders, differential operators with singularities and others. We apply the method of spectral mappings to the solution of the inverse problem for the Sturm-Liouville operator on a finite interval. In the method of spectral mappings we begin from Cauchy's integral formula for analytic functions. We apply this theorem in the complex plane of the spectral parameter for specially constructed analytic functions having singularity connected with the given spectral characteristics. This permits us to reduce the inverse problem to the so-called main equation which is a linear equation in a corresponding Banach space of sequences.

In this study, first it is mentioned about integral representation for solution which satisfies certain initial conditions of differential equation generated by singular Sturm-Liouville operator, properties of spectral characteristics and uniqueness theorems for solution of inverse problem are discussed. After that we give a derivation of the main equation and prove its unique solvability.

Key Words: Integral representation, Inverse problem, Sturm-Liouville, Singular potential.

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Inverse Sturm-Liouville Problem With Energy Dependent Potential

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ABSTRACT

Consider the boundary value problem

$$-y'' + \left[q(x) + 2\lambda p(x)\right] y = \lambda^2 y,\ x \in (0,\pi) \tag{1.1}$$

$$y'(0) = 0,$$

$$(a_0 + a_1\lambda) y(\pi) + y'(\pi) = 0,$$

where a_0 and a_1 are real numbers and $q(x) \in W_2^1[0,\pi],\ p(x) \in W_2^2[0,\pi]$.

The determination of differential operators using spectral data has become an attractive problem recently. One of these operators is Sturm-Liouville (SL) operator. In inverse SL problem, one tries to recover both potential function and constants by using the eigenvalues with another piece of spectral data as norming constants and spectral function. Also some authors proposed a new way which is related to nodal points to recover this operator. This effective procedure is called inverse nodal problem.

In the literature, equation (1.1) is called as quadratic of differential pencil and it is very important in quantum theory. For instance, these type equations come to light in Klein-Gordon equations by seperation of variables, which define the motion of particles. By the way, Sturm--Liouville energy-dependent potential is also used in viscous vibration of rope. We also emphasize that problems including the spectral parameter λ in boundary condition is related to the energy of the system. Inverse problems of quadratic pencil have been solved by many authors [1-4].

In this study, we give asymptotics of eigenvalues and some uniqueness theorem which is known Ambarzumyan's theorem in the literature for energy dependent potential problem with boundary condition including spectral parameter.

Key Words: Energy Dependent potential, Spectrum, Ambarzumyan Theorem

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Investigation of various travelling wave solutions to the Extended (2+1)-dimensional Quantum ZK equation

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ABSTRACT

In this paper, we devoted our effort in constructing new solutions to the extended (2+1)-dimensional quantum Zakharov-Kuznetsov equation by using the powerful sine-Gordon expansion method with help of Wolfram Mathematica 9. Sine-Gordon expansion method is based on the travelling wave transformation and the sine-Gordon equation. We first reduce the extended (2+1)-dimensional quantum Zakharov-Kuznetsov equation to nonlinear ordinary differential equation, we then balanced the obtained nonlinear ordinary differential equation by applying the balancing principle.We obtain new results with the good solution structures such as complex, trigonometric and hyperbolic structures. All the obtained results in this paper have some physical meaning, for instance hyperbolic tangent arises in the calculation of magnetic moment and rapidity of special relativity, hyperbolic secant arises in the profile of a laminar jet, hyperbolic sine arises in gravitational potential of a cylinder and the calculation of a Roche limit and the hyperbolic cosine function is the shape of a hanging cable. All the obtained solutions in this paper verified the extended (2+1)-dimensional quantum Zakharov-Kuznetsov equation. We plot the two- and three-dimensional graphics of all the obtained solutions in this paper, all with help of Wolfram Mathematica 9. We finally, submit a comprehensive conclusion.

Keywords: The SGEM,the (2+1)-dimensional QZK equation, the complex function solution,the hyperbolic function solution, the trigonometric function solution.

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$L^{p(x)}$-Approximation by Bernstein- Kantorovich Operators

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ABSTRACT

For a real-valued bounded function F defined on $I=[0,1]$ the Bernstein polynomials are given by

$$B_n(F,x) = \sum_{k=0}^{n} F\left(\frac{k}{n}\right) q_{n,k}(x), \quad x \in I, \quad n \in N,$$

where

$$q_{n,k}(x) = \binom{n}{k} x^k (1-x)^{n-k}.$$

The following modification of these Kantorovich polynomials for $f \in L^1(I)$ is due to $[2]$:

$$(K_n f)(x) = (n+1) \sum_{k=0}^{n} q_{n,k}(x) \int_{I_{k,n+1}} f(t)dt,$$

where

$$I_{k,n+1} = [k/(n+1), (k+1)/(n+1)].$$

If $F(x) = \int_0^x f(t)dt$, then

$$\frac{d}{dx} B_{n+1}(F,x) = (K_n f)(x).$$

In 1953 Lorentz $[2, \text{Theorem } 2.1.2]$ and Maier $[3]$ proved that for $f \in L^p(I), p \geq 1$, we have

$$\lim_{n\to\infty} \|K_n f - f\|_p = 0. \tag{1}$$

Let $p: I \to [1,\infty)$ be a measurable bounded function called the variable exponent on I. We write

$$p^- = \inf_{x\in I} p(x), \qquad p^+ = \sup_{x\in I} p(x).$$

We define the variable exponent Lebesgue space $L^{p(x)}(I)$ to consist of all measurable functions $f: I \to \mathbb{R}$ such that the modular

$$\rho_{p(x)}(f) = \int_I |f(x)|^{p(x)} dx < \infty.$$

If $p^+ < +\infty$ then

$$\|f\|_{L^{p(x)}(I)} := \|f\|_{p(x)} = \inf \left\{ \lambda > 0 : \rho_{p(x)}\left(\frac{f}{\lambda}\right) \leq 1 \right\}$$

defines a norm on $L^{p(x)}(I)$. Let $L^{2,p(x)}(I) = \{f : f' \text{absolutely continuous on } I,\ f'' \in L^{p(x)}(I)\ \}$. We say that the exponent p is (locally) log-Hölder if there exists a constant $C > 0$ such that

$$-|p(x) - p(y)| log|x - y| \leq C, \quad (2)$$

for every $x, y \in I$ with $|x - y| \leq \frac{1}{2}$. We denote by $\Re(I)$ the class of all exponents satisfying the condition (2).

Our purpose in this presentation is to generalize the results in [2] and [3] in variable exponent Lebesgue spaces (see $[1], [4]$). We consider the following result: If $p \in \Re(I)$, $p^- > 1$ and $f \in L^{2,p(x)}(I)$, then there exists positive constants $C_1(n,p)$ and $C_2(n,p)$ such that

$$\|_{K_n f} - f\|_{p(x)} \leq C_1(n,p)\|f'\|_{p(x)} + C_2(n,p)\|f''\|_{p(x)}.$$

Keywords: Variable exponent Lebesgue spaces; Bernstein-Kantorovich operators.

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Lucas Matrix Polynomial Method for Linear Complex Differential Equations

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ABSTRACT

Complex differential equations have been studied since 1960s by many scientists and applied to different areas. Some of these studies are a geometric approach based on meromorphic function in arbitrary domains, a topological description of solutions of some complex differential equations with multivalued coefficients, the zero distribution, growth estimates of linear complex differential equations, and also the rational together with the polynomial approximations of analytic functions in the complex plane. In this paper, the matrix relations between the Lucas polynomials and their derivatives, we develop a method for solving linear complex differential equation.

$$\sum_{n=0}^{m} P_n(z) f^{(n)}(z) = g(z) \tag{1}$$

with the initial conditions

$$f^{(t)}(\alpha) = \vartheta_t \quad t = 0,1, \ldots., m-1 \tag{2}$$

We will let $f(z)$ is unknown function, $P_n(z)$ and $g(z)$ are analytical functions in the circular domain which $D = \{z = x + iy, z \in C, |z - z_0| \leq r, r \in R^+\}$, $\alpha, z_0 \in D$, is appropriate complex or real constant. We assume that the solution of (1) under the conditions (2) is approximated in the form

$$f(z) = \sum_{n=0}^{N} a_n L_n(z),\ z \in D \tag{3}$$

which is the truncated Lucas series of the unknown function, where all of are the Lucas coefficients to be determined. We also use the collocation points

$$z_{pp} = z_0 + \frac{r}{N} p e^{\frac{i\theta}{N} p}\ , 0 < \theta \leq 2\pi, r \in R^+, p \in 0,1, \ldots, N \tag{4}$$

Consequently, we have obtained the numerical solutions of linear complex differential equations by using the Lucas Polynomials and performed it two test problems. When we have compared exact solutions and numerical solutions of tables and graphs, we realized that our method is reliable, practical and functional.

Keywords: Linear complex differential equations, Lucas Polynomials, Collocation Method, Numerical solution.

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MHD Flow For Different Flow Geometries

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ABSTRACT

This study investigates the unsteady magneto hydrodynamic (MHD) flow of a viscous incompressible fluid in different flow geometries. We consider here two different cases, where the first one involves the unsteady MHD flow between parallel plates in the absence of heat transfer and the second case considers the same flow between circular disks with the presence of the heat transfer. In both cases the flow is considered as 2-D and Navier Stokes equations with continuity equation govern the fluid flow. Additionaly, heat transfer equation is taken into account for the disk problem. Aforementioned types of problems have many applications in technology and have attracted the concern of many researchers so far. Therefore, the solution of the model problem is a vital issue due to non-linear nature of the equations.

In the case of flow through two parallel plates, 2-D Navier Stokes equations, involving magnetic field of the strength term, together with the continuity are reduced to the fourth order non-linear ordinary differential equation by using suitable transformations. The solution of the problem is obtained by the help of the differential transform method (DTM), which is the generalized iterative way for obtaining Taylor series coefficients of a smooth function. The solution of the problem is considered as a truncated Taylor series and the coefficients are evaluated easily by the help of DTM using any mathematical software packages such as Mathlab or Maple. The obtained results are presented for different flow parameters such as Reynolds and Hartman numbers.

The second flow geometry is related to the flow past through two disks. The governing Equations are again given in cylindrical coordinates and by using the known transformations, equations are reduced to a coupled ordinary differential

equations in terms of velocity and temperature distribution. The solution technique is again the DTM and the obtained results are presented for different flow parameters.

Key Words: MHD flow, Navier Stokes Equation, Heat Transfer Equation, Differential Transform Method, Reynolds Number, Hartman Number

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Multi Time Step Method for Numerical Solution of the Equal Width Wave Equation

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ABSTRACT

Nonlinear partial differential equations are useful in describing the various phenomena in disciplines. Analytical solutions of these equations are usually not available, especially when the nonlinear terms are involved. The equal width wave (EW) equation is a model nonlinear partial differential equation used for the simulation of one-dimensional nonlinear waves propagating in dispersive media. This equation is also an alternative form of nonlinear dispersive waves to the well-known regularized long wave equation and Korteweg–de Vries equation.

The EW equation is a partial differential equation (PDE) given by

$$u_t + \varepsilon u u_x - \mu u_{xxt} = 0, \tag{4}$$

in which $u = u(x,t)$ is a function of the two independent variables x and t that normally denote space and time, respectively [1]. The boundary and initial conditions for our method are approximated on the finite computational domain as

$$\begin{aligned} u(a,t) = u(b,t) = 0, \\ u_x(a,t) = u_x(b,t) = 0, \end{aligned} \tag{5}$$

and

$$u(x,0) = f(x), \tag{6}$$

which have been used in previous studies.

Analytical solutions of the EW equation are known with only a restricted set of boundary and initial conditions. Therefore, the MEW equation has been studied by some authors used various kinds of numerical methods based on method of lines, finite-difference and finite-element methods [2-6]. In this paper, a numerical solution of the EW equation based on the multistep Adams−Moulton method for the time discretization and the quasi cubic B-spline method for the space discretization is presented. The main advantage of the resulting scheme is that the proposed algorithm is very simple, so it is very easy to implement. The proposed method is tested on the problems of propagation of a solitary wave and interaction of two solitary waves. The three conservation quantities of the motion are also calculated to determine the conservation properties of the proposed algorithm. From the numerical experiments, the obtained results indicate that the present method is remarkably successful numerical technique for solving the EW equation and the method can be also used efficiently for solving a large number of physically important nonlinear problems.

Key Words: Adams Moulton, Quasi Spline, Solitary wave.

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Multiple Small Solutions for $p(x)$-Schrödinger Equations With Local Sublinear Nonlinearities Via Genus Theory

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ABSTRACT

In the present, we are concerned with the following $p(x)$-Schrödinger problem:

$$-\Delta_{p(x)}u + V(x)u^{p(x)-2}u = f(x,u) \text{ in } \mathbb{R}^N, u \in W^{1,p(x)}(\mathbb{R}^N),$$

where $N \geq 2$, $\Delta_{p(x)}u := \operatorname{div}\left(|\nabla u|^{p(x)-2}\nabla u\right)$ is the $p(x)$-Laplacian operator, $p\colon \mathbb{R}^N \to \mathbb{R}$ is Lipschitz continuous, $1 < p^- := \inf_{x\in\mathbb{R}^N} p(x) \leq p^+ := \sup_{x\in\mathbb{R}^N} p(x) < N$.

(V_1): $V \in C(\mathbb{R}^N, \mathbb{R})$, $\inf_{x\in\mathbb{R}^N} V(x) > 0$, there exist constants $r > 0$, $b > 0$ such that $\lim_{|y|\to+\infty} \mu\left(x \in B_r(y) \colon \frac{V(x)}{|x|^{\sigma(x)}} \leq b\right) = 0$, where $B_r(y) = \{x \in \mathbb{R}^N \colon |x - y| < r\}$, $\mu(.)$ denotes the Lebesgue measure and σ satisfies the log-Hölder continuous ([1],[2]).

(f_1): There exist an $x_0 \in \mathbb{R}^N$ and a constant $r > 0$ such that

$$\liminf_{t\to 0}\left(\inf_{x\in B_r(x_0)} \frac{\int_0^t f(x,s)ds}{|t|^{p^-(B_r(x_0))}}\right) > -\infty, \limsup_{t\to 0}\left(\inf_{x\in B_r(x_0)} \frac{\int_0^t f(x,s)ds}{|t|^{p^+(B_r(x_0))}}\right) > \infty,$$

with $p^+(B_r(x_0)) < ap^-(B_r(x_0))$, $1 < a < p^-$;

(f_2): $f \in C(\mathbb{R}^N \times [-\delta, \delta], \mathbb{R})$ with $\delta > 0$ and a nonnegative functions $g \in L^{\frac{a^2}{a^2-1}}(\mathbb{R}^N) \cap L^{\frac{ap(x)}{(a-1)p(x)}} \cap L^\infty(\mathbb{R}^N)$ and $p \in C(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$, $p^+ < ap^-$ such that $|f(x,t)| \leq g(x)|t|^{p(x)/a-1}$ for all $|t| \leq \delta$ and $x \in \mathbb{R}^N$;

(f_3): f is an odd function according to t, that is $f(x,t) = -f(x,-t)$ for all $t \in \mathbb{R}$ and $x \in \mathbb{R}^N$.

V potential function satisfies the (V_1), f function satisfies the $(f_1) - (f_3)$ conditions, without any growth condition for the nonlinear term $f(x,u)$ on u at infinity, by using variational approach and Krasnoselskii's genus theory, we obtain a sequence of solutions converging to zero in some variable exponent Sobolev spaces.

Keywords: Variable exponent Sobolev spaces; $p(x)$-Laplace operator; Schrödinger equation; Krasnoselskii's genus

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Multivariate Semiparametric Errors in Variables Model

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ABSTRACT

Bias resulting from the presence of measurement error in a regression analysis, which mostly appears in Econometrics and several other fields when the samples have errors stem from a deficiency of the measuring techniques, has been derived for the solutions in parametric and nonparametric regression and has remained undetermined in semiparametric regression [1,2,3,4,5]. In literature, all semiparametric errors in variables regression models are univariate models. However, in some applications, such as financial applications, working with a multidimensional response variable may be interested in. For these cases the expanded model to a multidimensional case is needed.

This study investigates the multivariate partially linear semiparametric model when the independent variable of nonparametric function of this model has measurement error. In this model it is assumed that the measurement error stems from wrong measurements has a known distribution. We derived an estimator of parametric part. It is important in statistics to observe whether the distribution around the parameter which they converge while the sample size of the estimators obtained by this method converges to infinity fits to the normal distribution or not. Hence at the last section of the study, we showed the asymptotic normality properties of presenting estimator. The resulting estimator is shown to be asymptotically normal.

Key Words: Multivariate semiparametric regression, errors in variables model, asymptotic normality.

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New exact solutions of some fractional order differential equations via improved fractional sub-equation method

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ABSTRACT

Fractional calculus is an important mathematical field that has been growing rapidly from the past to the future. And in the nature, many complex mechanical and engineering systems can be characterized by it. During its development stages, many different fractional operators are introduced such as Grünwald Letnikov, Caputo and Riemann Liouville fractional derivatives. Recently, Jumarie has developed a new alternative definition for the Riemann Liouville derivation. This newly introduced derivative definition has many advantages over both of the modified Riemann Liouville derivative and the Caputo derivative. While in one of them the derivative of a constant is equal to zero, in the other one; we can apply for any continuous (non-differentiable) functions. Therefore, by the help of this new concept, we can overcome some of the shortcomings encountered in the previous ones.

In the present paper, an improved fractional sub-equation method has been proposed in order to obtain a new analytical solution for some nonlinear fractional differential equations by means of Jumarie's modified Riemann Liouville derivative. The method is applied to a time-fractional biological population model and a space-time fractional Fisher equation successfully. Finally, the simulations of new analytical solutions have been presented graphically. In addition, it can be seen from the newly obtained results that the method is more efficient, effective and applicable one for solving a wide range of nonlinear fractional differential equations.

Key Words: Improved fractional sub-equation method, Riemann Liouville, analytical solution, time-fractional biological population model, space-time fractional Fisher equation.

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Nontrivial Solutions for a Dirichlet Problem in Orlicz-Sobolev Spaces

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ABSTRACT

In this study, we are interested in the existence and uniqueness of solutions to the quasilinear elliptic equation with Dirichlet boundary condition of the following form

$$\begin{cases} div\left(a\left(|\nabla u|\right)\nabla u\right)+a\left(|u|\right)u=f\left(x,u\right) & in\ \Omega, \\ u=0 & on\ \partial\Omega, \end{cases} \quad \text{(P)}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, f is a Carathéodory function, and $\varphi(t):=a\left(|t|\right)t$ is an increasing homeomorphisms from $\mathbb{R}$ onto $\mathbb{R}$. We want to remark that in problem (P) if we let $a(t):=|t|^{p-2}$ problem (P) turns into the well-known p-Laplace equation and in case of $p=p(x)$, i.e., $a(t):=|t|^{p(x)-2}$, problem (P) becomes the $p(x)$-Laplace equation, the generalization of p-Laplace equation. Since problem (P) contains a nonhomogeneous function φ in the differential operator, problem (P) is settled in the Orlicz-Sobolev spaces. The main tools used here are Variational Method and Critical Point Theory. We show that the energy functional corresponding to problem (P) is coercive and weakly lower semicontinuous. Therefore, it has a global minimum in the Orlicz-Sobolev space, which is in turn a weak solution to problem (P). Then, by using two assumptions regarding the growth condition of function f, we show that the solution is not identically zero, that is, problem (P) has a nontrivial solution.

We want to remark that the importance of problem (P) is that it generalizes the well-known p-Laplace equation and $p(x)$-Laplace equation. The study of variational problems in the classical Sobolev and Orlicz-Sobolev spaces is an interesting topic of research due to its significant role in many fields of mathematics, such as approximation theory, partial differential equations, calculus of variations, non-linear potential theory, the theory of quasiconformal mappings, non-Newtonian fluids, image processing, differential geometry, geometric function theory, and probability theory. Moreover, problem dealt this presentation possess more complicated nonlinearities, for example, it is inhomogeneous, so in the discussions, some special techniques will be needed. However, the inhomogeneous nonlinearities have important physical background. Therefore, problem (P) may represent a variety of mathematical models corresponding to certain phenomenon.

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Numerical Solution with a matrix stability via Difference Scheme Method for Telegraph Partial Differential Equation

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ABSTRACT

In this study a second order of a telegraph partial differential equation was presented, we studied the following local boundary initial value problem for a two dimensional telegraph equation

$$\begin{cases} u_{tt}(t,x)+u(t,x)=u_{xx}(t,x)+u_x(t,x)+f(t,x) \quad 0\le t\le \pi \\ u(0,x)=u_t(0,x)=0\,,u(t,0)=\varphi_1(t)\,,u(t,1)=\varphi_2(t)\,,0<x<1 \end{cases}$$

The general work in this research is that solving telegraph differential equation by difference scheme method to get a good accuracy, this method based on reduce two dimensional telegraph to one dimensional telegraph equation, for the one and two dimensional telegraph differential equation the first and second order of accuracy difference schemes for the approximate solution of this problem are presented, those difference scheme was founded by expanding derivatives with respect to $(x\ and\ y)$ by Taylor series, in order $(t_{k-1}, t_k\,,$ $t_{k+1}\,, and\ x_{n-1}\,, x_n\,, x_{n+1})$ where $(0\le t\le T\ and\ 0<x<l)$from the difference schemes the matrix stability was established by putting the initial values and it was computed, the difference scheme was reduced to matrices, it was wrote and programmed on matlab the program can be used to get numerical solution, it has a more accuracy, some results are presented in order to support theoretical ,illustrative examples are included to demonstrate the validity and applicability of the presented technique, for those examples the exact solution was founded by using Laplace transform, the least square approximation was estimated for the analytic solutions and the numerical solution which founded by matlab program, the result has a good accuracy.

Key Words: Telegraph equations, Difference Scheme, Partial differential equation.

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Numerical solution of inverse problem for multidimensional elliptic equation with multipoint nonlocal and Neumann boundary conditions

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ABSTRACT

The papers [1] - [5] are devoted to theory and methods of solving various inverse problems for elliptic differential and difference equations.

In this work, we study approximation of inverse problem of finding $v(t,x)$ and $p(x)$ $(x=(x_1,\dots,x_n))$ for the multidimensional elliptic equation with the following multipoint nonlocal and Neumann boundary conditions

$$\begin{cases} -v_{tt}-\sum_{r=1}^{n}\left(a_r(x)v_{x_r}\right)_{x_r}+\sigma v(t,x)=g(t,x)+p(x), \in \Omega,\ 0<t<T, \\ v(0,x)=\phi(x), v(T,x)-\sum_{i=1}^{q}k_i v(\lambda_i,x)=\eta(x), v(\lambda_0,x)=\zeta(x), x\in\overline{\Omega}, \\ \frac{\partial v(t,x)}{\partial \vec{n}}=0,\ 0\le t\le T, x\in S. \end{cases} \quad (1)$$

Here $a_r(x)$ $(\boldsymbol{x}\in\Omega)$, $\phi(x)$, $\eta(x)$, $\zeta(x)$ $(x\in\overline{\Omega})$, $g(t,x)$ $(t\in(0,T), x\in\Omega)$ are given smooth functions, $a_r(x)\ge 0, \Omega=(0,l)^n$, $S=\partial\Omega, \overline{\Omega}=\Omega\cup S$, the numbers $\sigma>0$, $T>0, l>0, k_i, \lambda_i, i=1,\dots,q$, and λ_0 are known. Suppose that $\sum_{i=1}^{q}k_i=1, k_i\ge 0, 0<\lambda_1<\lambda_2<\cdots<\lambda_q<1, 0<\lambda_0<1$.

Well posedness of problem (1) has been established in the paper [4].

In this presentation, we construct the first and second order of accuracy with respect t and second order accuracy with respect x difference schemes for approximate solution of this problem. Difference schemes for inverse problem are reduced to corresponding auxiliary difference schemes for multipoint nonlocal boundary value problem. Operator method is used to study these difference problems. To find solution of difference problems, we use algorithm which includes three stages. Stability and coercive stability estimates for solutions of both difference schemes are established. Numerical results with explanation on the realization for two dimensional and three dimensional inverse elliptic problem with multipoint Bitsadze-Samarskii type nonlocal and Neumann boundary conditions are presented by using MATLAB program.

Key Words: Inverse elliptic problem, difference scheme, stability.

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Numerical Solution of the Advection Diffusion Equation Using Trigonometric Cubic B-spline Galerkin Method

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ABSTRACT

We consider the following one dimensional advection-diffusion equation

$$u_t + \alpha u_X - \mu u_{XX} = 0. \tag{7}$$

Boundary conditions

$$\begin{aligned} u(a,t) &= u(b,t) = 0, \\ u_x(a,t) &= u_x(b,t) = 0, t \in [0,T] \end{aligned} \tag{8}$$

and initial condition

$$u(x,0) = f(x),\ a \le x \le b. \tag{9}$$

are chosen over space/time interval $[a,b] \times [0,T]$. In the one dimensional linear advection diffusion equation, α is advection (velocity) coefficient, μ is the diffusion coefficient and $u = u(x,t)$ is a function of two independent variables t and x, which generally denote time and space, respectively. The advection-diffusion equation is the basis of many physical and chemical phenomena, and its use has also spread into economics, financial forecasting and other fields [1]. Various numerical techniques including spline approximation methods have been proposed for solving the one dimensional advection-diffusion equation with constant coefficient so far [2,3].

In this study, the advection diffusion equation will be solved numerically using the trigonometric cubic B-spline Galerkin finite-element method, based on second, third and fourth order single step methods for time integration. The test problem modelling fade out of an initial bell-shaped concentration is studied and accuracy of the numerical results are measured by the computing the order of convergence and error norm L_∞ for the proposed three methods. The numerical results of this study demonstrate that the proposed algorithms especially the fourth order single step method are a remarkably successful numerical technique for solving the advection-diffusion equation. It can also be efficiently applied to similar physically important equations.

Key Words: Trigonometric Cubic B-spline, Advection diffusion equation, Galerkin method

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Obtaining Three-Periodic Wave Solutions of (3+1) Dimensional BKP Equation Using Hirota-Riemann Method

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ABSTRACT

Knowing the exact solution of PDE's can be helpful in understanding complicated physical models and reveals the general structure of complex nonlinear phenomena. In the 1980's, Nakamura proposed a method to construct periodic wave solutions of nonlinear equations by combining Riemann theta function and Hirota's bilinear method in Refs. [3, 4]. Owing to this method it is possible to obtain periodic wave solutions and soliton wave solutions directly.

Recently, Hirota-Riemann method has been further expanded to investigate periodic wave solutions of certain nonlinear differential, discrete and supersymmetric equations [2, 5, 6, 7]. It is known that if an equation has a soliton solution including at least three solitons, then this will coincide with integrability. Because of certain difficulties in the calculations, it is not possible to find three-periodic solutions everytime. But in Demiray's Phd Thesis [1], she showed a way how to find N = 3 periodic solutions.

In this study, three-periodic solution of (3+1) dimensional BKP equation was obtained. The asymptotic behavior of the periodic wave solution was analysed and periodic solution was compared to the known soliton solutions under a small amplitude limit. Finally, to show the propogation of the wave, graphics of three-periodic wave solution were plotted.

Key Words: Riemann Theta functions, Hirota-Riemann Method, Quasi periodic solutions, Three-periodic solutions, (3+1) dimensional BKP equations.

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On A Numerical Solution of Cauchy Type Singular Integral Equation

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ABSTRACT

In this study, numerical solution methods with the help of Gauss-Jacobi quadrature formulas for singular integral equations of the second kind are considered. The singular integral equation is handled as follows:

$$a\varphi(x)+\frac{b}{\pi}\int_{-1}^{1}\frac{\varphi(t)}{t-x}dt+\int_{-1}^{1}K(x,t)\varphi(t)dt=f(x), \qquad (-1<x<1),$$

where, *a* and *b* are arbitrary real constants. It is assumed that the functions *f(x)* satisfy Holder condition on the interval *1≤x≤1* and the kernel *K(x,t)* satisfy Holder condition on the square *-1≤x,t≤1*. Also, while the first integral has Cauchy type singularity; the second integral is regular term. Cauchy type singular integral equations whose indexes are equal to 1 are solved by reducing the singular integral equations to a system of linear equations through Gauss-Jacobi quadrature formulas by ignoring remaining terms. To encourage of the proposed method, a few examples of Cauchy type singular integral equations are presented. The numerical solutions are given by using Gauss-Jacobi quadrature formulas. Here the collocation points should not coincide with any of the integration points. For example, the collocations points may be chosen as the midpoint of two successive integration points. To show the effectiveness of the method, numerical results and exact solutions are given in the tabular form and compared. Also, the results are presented for the different values of total number of integration point *n* and the accuracy of the numerical results for quadrature formulas are interpreted.

Key Words: Singular integral equation, quadrature formulas, Jacobi polynomials.

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On Applications of the Fractional Calculus for Some Singular Differential Equations

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ABSTRACT

Claim of the derivatives and integrals with any arbitrary order (*that is, fractional calculus*) was born in 1695 and, this new and remarkable subject has intensive work fields such as mathematics, physics, chemistry, biology, medicine, engineering and so on since that day.

In present study, generalized Leibniz rule and some theorems in the calculus of the fractional derivatives and integrals are used to obtain the fractional solutions of the radial Schrödinger equation transformed into a singular differential equation and, these solutions are also exhibited as hypergeometric notations.

We study on the Schrödinger equation's radial components in the β-dimensional space and under the Coulomb potential respectively, and we first transform these equations to the singular differential equations by means of some assumptions. After, the generalized Leibniz rule and some fractional calculus theorems are applied to these singular equations due to find the fractional solutions, and hypergeometric solutions are also obtained. Thus, we exhibit two different solution methods for two different equations.

In the β-dimensional space, reduced radial Schrödinger equation is

$$z\psi_2 + (\lambda - z)\psi_1 + \left(\frac{\omega}{z} - \frac{\lambda}{2}\right)\psi = 0. \tag{1}$$

Equ. (1) has the following fractional solutions:

$$\psi^I(z) = Kz^{\frac{1-\lambda+\xi}{2}}\left[z^{-\left(\frac{1+\xi}{2}\right)}\mathrm{e}^z\right]_{-\left(\frac{1-\xi}{2}\right)},$$

and,

$$\psi^{II}(z) = Lz^{\frac{1-\lambda-\xi}{2}}\left[z^{-\left(\frac{1-\xi}{2}\right)}\mathrm{e}^{z}\right]_{-\left(\frac{1+\xi}{2}\right)}.$$

Under the Coulomb potential, reduced radial Schrödinger equation is

$$z^2\psi_2 + z\psi_1 - \left(\frac{z^2}{4} - bz + \frac{k^2}{4}\right)\psi = 0. \qquad (2)$$

And, the fractional solution of Equ. (2) is

$$\psi(z) = Kz^{\nu}\mathrm{e}^{\varepsilon z}\left[z^{-(2\nu+1-\kappa)}\mathrm{e}^{-2\varepsilon z}\right]_{\kappa-1}.$$

Key Words: Fractional calculus, generalized Leibniz rule, fractional calculus theorems, radial Schrödinger equation.

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On Basis Property of Eigenfunctions of a Discontinuous Sturm-Liouville Operator

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ABSTRACT

Let us consider the discontinuous Sturm Liouville equation

$$\ell(y) \equiv -p(x)y'' + q(x)y = \lambda y, \ x \in [a,c) \cup (c,b], \tag{1}$$

with boundary conditions:

$$\alpha_{11} y(a) - \alpha_{12} y'(a) = \lambda\left(\alpha_{21} y(a) - \alpha_{22} y'(a)\right), \tag{2}$$

$$\beta_{11} y(b) - \beta_{12} y'(b) = \lambda\left(\beta_{21} y(b) - \beta_{22} y'(b)\right), \tag{3}$$

and transmission conditions:

$$y(c+0) = y(c-0), \tag{4}$$

$$y'(c+0) - \gamma y'(c-0) = (\delta_0 + \delta_1 \lambda) y(c), \tag{5}$$

where λ is complex parameter; q(x) is real-value continuous function on the intervals $[a,c)$ and $(c,b]$ and has a finite limits $\lim\limits_{x \to c \pm 0} q(x)$;

$$p(x) = \begin{cases} \dfrac{1}{p_1^2}, & a \le x < c \ , \\ \dfrac{1}{p_2^2}, & c < x \le b \ ; \end{cases}$$

$p_i, \alpha_{ij}, \beta_{ij}, \gamma, \delta_i \ (i, j = 1,2)$ are real constants.

In this paper, we investigate the completeness, the minimality and the basis properties of the system eigenfunctions of the discontinuous boundary-value problem (1)-(5) .

When the boundary conditions contained spectral parameter in continuous case, the eigenvalue problem for the Sturm-Liouville operator with physical

applications was examined in [1,2,3] and in other studies. The boundary value problem when the boundary condition contained spectral parameter for the n-th order ordinary differential equations was given systematically in [4]. The boundary value problem which is reduced to equivalent operator equation in a suitable Hilbert space was examined in these works. It is shown that the eigenfunctions for this class problems form a Riesz basis of the corresponding Hilbert space, a corresponding Rayleigh-Ritz formula is developed, and a lower bound estimation for eigenvalues is found in [5]. The eigenfunctions form a Riesz basis in suitable Hilbert space was shown in [1].

The completeness, the minimality and the basis properties of the system eigenfunctions of the discontinuous Sturm Liouville operator was learned in [6].It has been shown that the eigenvalues of the boundary-value problem (1)-(5) coincide with the zeros of an entire function and form at most countable and bounded below set which is convergent to the infinity at the infinity in [7].

To investigate the basic property of the problem (1)-(5) we define the special Hilbert space $H = L_2 \times \mathbb{C}^3$ of all elements $\tilde{y} = \left(y(x), y_1, y_2, y_3\right)$ with a scalar product defined by

$$\left\langle \tilde{y}, \tilde{z} \right\rangle = p_1^2 \int_a^c y(x) z(x) dx + p_2^2 \int_c^b y(x) z(x) dx + \frac{1}{p_1} y_1 \bar{z}_1 + + \frac{1}{p_2} y_2 \bar{z}_2 + \frac{1}{\delta_1} y_3 \bar{z}_3$$

where $y(x) \in L_2[a,b]$ and $y_i, z_i \in \mathbb{C}$, $i = \overline{1,3}$. Notice that the scalar product $\left\langle \tilde{y}, \tilde{z} \right\rangle$ will be defined in different form depending on the sign of numbers p_1, p_2, δ_2.

Regarding the problem (1)-(5) ,let A be a operator defined by the formula

$$A\tilde{y} = \left\{-p(x)y'' + q(x)y, \alpha_{11}y(a) - \alpha_{12}y'(a), \beta_{11}y(a) - \beta_{12}y'(a), y'(c+0) - \gamma y'(c-0) - \delta_0 y(c)\right\}$$

in the domain

$$D(A) = \left\{\tilde{y} \in H, \tilde{y} = \left\{y(x), y_1, y_2, y_3\right\}, y(x) \in AC[a,b], y'(x) \in AC[a,c), y'(x) \in AC(c,b], \right.$$
$$y'(c \pm 0) \equiv \lim_{x \to c \pm 0} y(x),\ \ell(y) \in L_2[a,b], y_1 = \alpha_{11}y(a) - \alpha_{12}y'(a),$$
$$\left. y_2 = \beta_{11}y(a) - \beta_{12}y'(a), y_3 = \delta_1 y(c)\right\}$$

where $AC[a,b]$ denotes the space of all absolutely continuous functions on the interval $[a,b]$.

From the properties of the operator A and a J-indefinite metric, the operator A has infinitely many non-real eigenvalues.

It is clear that the spectral problem (1)-(5) is equivalent to the eigenvalues

$$A\tilde{y} = \lambda \tilde{y}$$

for the operator A and it is obtained from Lemma1.4 in [4].

In this paper the completeness, the minimality and the basis of eigenfunctions of problem (1)-(5) for depending numbers $p_1, p_2, \delta_0, \delta_1$ in $L_2[a,b]\times \mathbb{C}^k$, $(k = 0,1,2,3)$ spaces .

Key Words: Discontinuous Sturm-Liouville operator, basis property of eigenfunctions

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On Boundary Discontinuous Fourier Analysis of Laminated Composite Structures

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ABSTRACT

Composite structures in the form of laminated plates or shells are extensively used in many areas such as aero, marine and land vehicles and micro/nano-electromechanical structures. There is a high demand on analysis and design of laminated composite structures. Mechanical modelling of these structures requires novel methods. Existing methods can be classified as experimental, numerical and analytical. Numerical methods consist of finite difference, finite element, boundary element, variational methods (Rayleigh-Ritz) and meshless methods, etc. Analytical methods include closed form solutions, i.e. elasticity solutions for limited geometries or loading/boundary conditions, and Navier-Levy type Fourier series solutions.

The present solution technique utilizes the double Fourier series approach, first discussed by Hobson [1], and used by Goldstein [2,3] to solve the stability problems of fluid flow. For the solution to the problem of a clamped isotropic plate, Green [4] used the double Fourier series approach. After this pioneering works, there were many works related to the static and dynamic analysis of composite laminated plates and shells. Recently, a novel discontinuous double Fourier series approach developed by Chaudhuri [5,6] in solving highly coupled partial differential equations.

In this paper, the application of this method will be explained through the class of problems of symmetric and antisymmetric cross-ply plates and doubly-curved shells, subjected to specific combinations of admissible boundary conditions (for example see [7,8]). In addition to that, the method will be discussed in detail with mathematicians, hoping a further improvement in the method.

Key Words: Boundary-discontinuous Fourier analysis, laminated composite structures.

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On solutions of Epidemic System

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ABSTRACT

In this talk, we interest in analytical and numerical solutions of an epidemic system. The following model is given (Abramson and Kenkre,2002) has been used in the study of Hantavirus epidemics:

$$\frac{dM_S}{dt} = b(M_s + M_I) - cM_s - \frac{M_S(M_S+M_I)}{K} - aM_S M_I$$

$$\frac{dM_I}{dt} = -cM_I - \frac{M_I(M_S+M_I)}{K} + aM_S M_I \qquad (1)$$

In the system (1), Logistic equation is obtained if we side by side addition the equations. The total population becomes $M = M_s + M_I$. In the analytical solution of the Logistic differential equation, the Lie Symmetry Method obtained with the help of a parameterized Lie Group will be used. Explicit numerical methods using Theta Method (θ=0) and Nonstandart Finite Difference Schemes (NSFD) will be used for the numerical solution of the Logistic differential equation. Then these solutions will be compared with some graphics.

Key Words: Logistic differential equation, Lie symmetries, Theta method, Nonstandard finite difference scheme, Hanta epidemics.

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On Spectral Problem for Discrete Discontinuous Sturm-Liouville Problem

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ABSTRACT

One of the studies related to Sturm-Liouville problems is the investigation of spectrum, by finding eigenvalues and eigen functions which help us to understand the characteristic of differential equations. In general, these studies can be made through two methods. One of them is the continuous form and the other is discrete form. In discrete form, first, the considered equation is transformed to finite difference equation and then the eigenvalues and eigen functions could be found from the coefficients matrix of the difference equation. In this presentation, we study the direct and inverse problem for difference equations which are constructed by the Sturm-Liouville equations with generalized function potential from the generalized spectral function. We will begin by defining the generalized spectral function, and determine the conditions it must satisfy. Then, some formulas are given in order to obtain the matrix J, which need not be symmetric, by using the initially given generalized spectral function and must satisfies some conditions. In addition, we will show that the generalized spectral function of J has a special form and we will give a structure of the generalized spectral function. At the end of this presentation, some examples are given in order to illustrate the method we used.

Key Words: Difference equation, Inverse problems, Generalized spectral function.

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On The Behavior of Solutions of Difference Equations

$$x_{n+1}=\frac{x_{n-5}x_{n-7}x_{n-9}}{x_{n-1}x_{n-3}\left(\mp 1\mp x_{n-5}x_{n-7}x_{n-9}\right)}$$

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ABSTRACT

Difference equations are widely used to investigate equations arising in mathematical models describing real-life situations such as population biology, probability theory, and genetics. Also, they are used to numerical solutions of differantial equations using in physics and engineering.

In this paper, we investigate well-defined solutions of the difference equations

$$x_{n+1}=\frac{x_{n-5}x_{n-7}x_{n-9}}{x_{n-1}x_{n-3}\left(\mp 1\mp x_{n-5}x_{n-7}x_{n-9}\right)} \tag{1}$$

where the initial conditions $x_0, x_{-1}, x_{-2}, x_{-3}, x_{-4}, x_{-5}, x_{-6}, x_{-7}, x_{-8}, x_{-9}$ are arbitrary nonzero real numbers. Really, (1) consist from 4 rational difference equations. Firstly, we obtain well-defined solutions of the difference equation

$$x_{n+1}=\frac{x_{n-5}x_{n-7}x_{n-9}}{x_{n-1}x_{n-3}\left(1+x_{n-5}x_{n-7}x_{n-9}\right)}. \tag{2}$$

The equation (2) has only an equilibrium point which is the number zero and this equilibrium point is not locally asymptotically stable. Second equation studied is the rational difference equation

$$x_{n+1}=\frac{x_{n-5}x_{n-7}x_{n-9}}{x_{n-1}x_{n-3}\left(-1+x_{n-5}x_{n-7}x_{n-9}\right)}. \tag{3}$$

We give the solutions of the equation (3) exactly. Also, the equation (3) has two equilibrium points which are $0,\sqrt[3]{2}$ and these equilibrium points are not locally asymptotically stable. Thirdly, we study the rational difference equation

$$x_{n+1}=\frac{x_{n-5}x_{n-7}x_{n-9}}{x_{n-1}x_{n-3}\left(1-x_{n-5}x_{n-7}x_{n-9}\right)}. \tag{4}$$

The equation (4) has an unique equilibrium point which is the number zero and this equilibrium point is not locally asymptotically stable. Also, we obtain well- defined solutions of equation (4) exactly. Finally, we obtain the solutions of the rational difference equation

$$x_{n+1}=\frac{x_{n-5}x_{n-7}x_{n-9}}{x_{n-1}x_{n-3}\left(-1-x_{n-5}x_{n-7}x_{n-9}\right)}. \tag{5}$$

Furthermore, we show that the equation has two equilibrum points which are $0,-\sqrt[3]{2}$ and these equilibrium points are not locally asymptotically stable.

Key Words: Difference equation, well-defined solution, stability.

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On the Existence and Uniqueness of Solutions to Nonlinear Wave Equations

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ABSTRACT

This work is concerned with the existence and uniqueness of a generalized solution for one dimensional nonlinear wave equations. The unique solvability of boundary value problems for the nonlinear wave equation in a Hilbert space is investigated.

First, the abstract Cauchy problem for the nonlinear wave equation

$$\begin{cases} \frac{d^2u(t)}{dt^2} + Au(t) = f(t,u(t)) \ \ 0 < t < T, \\ u(0) = \phi, \\ u_t(0) = \psi \end{cases}$$

in a Hilbert space H with the self-adjoint positive definite operator A is considered. A necessary condition for finding a unique solution is obtained by using theoretical statements about k-contractivity, Lipschitz functions and several well-known fixed point theorems, mainly Banach fixed point theorem, which gives a generalization of the successive approximation method.

Second, the Cauchy problem for the nonlinear hyperbolic equation

$$\begin{cases} u_{tt}(t,x) + u_{xx}(t,x) = f(t,x,u,u_t,u_x) \ \ 0 < t < a, x \in (-\infty,\infty), \\ u(0,x) = \varphi(x), x \in (-\infty,\infty), \\ u_t(0,x) = \psi(x), x \in (-\infty,\infty), \end{cases}$$

is considered.

First and second order of accuracy difference schemes for the solution of one dimensional linear hyperbolic equation constructed in [4] are used. Both fixed point iteration and these difference schemes are utilised to find an approximate solution to boundary value problem for the nonlinear wave equation.

Numerical implementation on unique solvability is considered to verify theoretical results.

Key Words: Abstract hyperbolic equations, fixed point theorems, difference equations.

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On The Fitted Difference Schemes for Ordinary Linear Differential Equations

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ABSTRACT

Consider the initial/boundary value problem that depends(smoothly) on a small parameter $\varepsilon > 0$. The problem obtained by setting $\varepsilon = 0$ in the equation and data is called the reduced(unperturbed) problem. If the reduced problem is of the same type and order as the given one and both have unique solutions, then the given problem is called a regular perturbation problem; otherwise, it is called a singular perturbation problem. The parameter ε termed the perturbation parameter. In mathematical terminology these are the DEs with a small parameter ε multiplying highest order derivatives. The parameter ε termed the perturbation parameter. If we let ε approach 0, the order of the DEs reduced, and some of the initial/boundary conditions will become superfluous (unnecessary). The solution exibits narrow regions of vary fast variation, so-called initial/boundary layers, while away from the layers the solution behaves regularly and varies slowly. More exactly, outside the layers the solution is mainly dominated by that of reduced problem. But inside the layers the smoothness of the solution of SPP aggravates (deteriorates, spoils, gets worse, the solution badly behaved, reducing the smoothness takes place) and the solution has derivatives which blow up (are unbounded) as $\varepsilon \to 0$.

That is why, the use of standard numerical methods for solving such problems may give rise to difficulties (presents severe difficulties) when the perturbation parameter ε is small. Often the root of these difficulties lies in the instability, or in the convergence absence of the numerical process. For instance, classical difference methods for solving such problems, in general, do not converge uniformly with respect to the small parameter. Therefore these is a need to develop, for such type of problems, special numerical methods, whose accuracy does not depend on the parameter value ε i. e. , methods that are convergence ε uniformly.

In this work we investigate the numerical solutions of first and second-order differential equations and singularly perturbed linear initial and boundary-value problems. We analyse a fitted difference scheme on a uniform mesh for the numerical solution of this problems. We state some important properties of the exact solution and present the difference scheme and obtain uniform error estimates for the truncation term and appropriate solution on a uniform mesh. The uniform convergence analysis in small parameter is given. We prove the first order uniform convergence of the scheme in the sense of discrete maximum norm. By the method of integral identities with the using exponential basis functions and interpolating quadrature rules with the weight and remainder term in integral form an exponentially fitted difference scheme on an uniform mesh is developed which is shown to be original ε-uniformly first order accurate in the discrete maximum norm for original problem. Numerical results are also presented. It is shown that numerical results are in agreement with the theoretical results.

Keywords: Ordinary linear differential equations, Fitted difference scheme, Singularly perturbation problem, Initial and Boundary-value problems.

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On the Limit Distribution of a Semi – Markovian Random Walk with a Generalized Reflecting Barrier

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ABSTRACT

Random walk processes with one or two barriers find a massive place to express many interesting problems in the fields of inventory, mathematical biology, queues and reliability theories, etc. These barriers can be reflecting, delaying, absorbing, elastic, etc., depending on concrete problems at hand. Numerous studies have been done about random walks with one or two barriers because of their practical and theoretical importance (Feller (1971), Borovkov (1984), Gihman and Skorohod (1975)). In this study, a semi – Markovian random walk process $(X(t))$ with a generalized reflecting barrier is investigated. First, the considered process is constructed mathematically and given a definition of the process. Next, under some weak conditions, the ergodicity of the process is proved. Then, the explicit form of the ergodic distribution is found and after standardization, it is shown that the ergodic distribution weak converges to limit distribution $R(x)$:

$$Q_X(\lambda x) \equiv \lim_{t\to\infty} P\{X(t) \le \lambda x\} \underset{\lambda\to\infty}{\longrightarrow} R(x) = \frac{2}{\mu_2}\int_0^x \left\{\int_z^\infty \big(1 - F_+(v)\big)dv\right\} dz$$

Here, $F_+(x)$ is the distribution function of the first ladder height (χ_1^+) generated by $S_n = \sum_{i=1}^n \eta_i$ and $\mu_2 = E(\chi_1^{+2})$. Moreover, $\{\eta_i, i = 1,2, ...\}$ random sequence represents the jumps of the process $X(t)$.

Finally, in order to evaluate asymptotic rate of the weak convergence, the following inequality is obtained, when λ is sufficiently large:

$$|Q_Y(x) - R(x)| \le \frac{2\mu_1 m_1\big(1 - \pi_+(x)\big) + m_2\big(1 - F_+(x)\big)}{\lambda m_1 \mu_2}$$

Here, $\pi_+(x) = \frac{1}{\mu_1}\int_0^x\big(1 - F_+(t)\big)\,dt; F_+(x) \equiv P\{\chi_1^+ \leq x\};\ \mu_k = E\big(\chi_1^{+k}\big), k = 1,2;$
$m_1 = E(\eta_1)$.

Key Words: Asymptotic rate, Random walk, Reflecting barrier, Weak convergence, Limit distribution.

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On the Numerical Solution of a Class of Interval Differential Equations

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ABSTRACT

The behaviour of a dynamical system is described mainly by a differential equation. In this, the exact values of some parameters in the equation are unknown, but often the intervals in which these values lie can be determined. This gives rise to interval differential equations. In this study, we consider Initial Value Problem (IVP) for a linear differential equation with interval coefficients. In the considered problem, the initial values are also intervals. For such an interval IVP we propose a new approach, which is different from the commonly used approaches. We interpret the interval IVP as a set of classical IVPs, and we investigate the bunch (set) of their solutions. We define this bunch to be the solution of the interval IVP. We develop a numerical method to find the upper and lower bounds of the solution bunch. For second order homogeneous linear differential equations, the complexity of the method is $O(n^2)$, in terms of classical IVPs to be solved. Therefore, it is essentially better than a straightforward method, the complexity of which is $O(n^4)$.

We apply the developed method to some examples and demonstrate the results. We provide a sufficient condition under which the solution is obtained by an analytical formula.

Key Words: Interval differential equation; Linear differential equation; Bunch of functions.

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On the Numerical Solution of a Fractional Telegraph Partial Differential Equation by Difference Scheme

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ABSTRACT

In this study the following initial value problem for fractional partial differential equation

$$\frac{\partial^2 u(t,x)}{\partial t^2} + \lambda \frac{\partial^\alpha u(t,x)}{\partial t^\alpha} - \frac{\partial^2 u(t,x)}{\partial x^2} + \beta u(t,x) = f(t,x),$$

$$0 \le t \le T,\ 0 \le x \le \pi,\ \lambda > 0,\ 0 < \alpha \le 1,$$

$$u(0,x) = \varphi(x), u_t(0,x) = \psi(x), u(t,0) = u(t,\pi) = 0,$$

is investigated. The numerical and analytic solutions of above fractional telegraph partial differential equations with the initial condition are presented. Difference scheme is constructed for solving fractional partial differential equation with Caputo fractional derivative. Theorem on stability estimates for the solution of the fractional telegraph differential equation is established. First order of accuracy difference scheme methods are presented. The stability inequalities for the solutions of this difference schemes are established. For some positive M and N, the grid sizes in space and time for the finite difference algorithm are defined by $h = \frac{\pi}{M}$ and $\tau = \frac{T}{N}$ respectively. The grid points in the space interval $[0,\pi]$ are the numbers $x_n = nh, n = 0,1,2,\dots,M$ and the grid points in the time interval $[0,T]$ are labeled $t_k = k\tau, k = 0,1,2,\dots,N$. A procedure of modified Gauss elimination method is used for solving this difference scheme in the case of one dimensional fractional hyperbolic telegraph partial differential equations. Some results of numerical example are presented in order to support theoretical statements. Different several methods are used to solve these type of fractional differential equations, fractional partial differential equations, fractional integro-differential equations and dynamic system containing fractional derivatives, such as Adomian's decomposition method, variational iteration method, homotopy perturbation method, spectral method. The analytical solution of above equation is derived by using Laplace transform method. The obtained results are analyzied by comparing with exact solutions. Using Matlab programming, numerical results of this equation are obtained. The results obtained show the efficiency and accuracy of this work.

Key Words: Fractional telegraph differential equation, initial value problem, stability, difference scheme, Laplace transform method, numerical solution.

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On The Periodic Behaviour of a Nonlinear Difference Equation

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ABSTRACT

Within the past years, difference equations, also referred to iterative sequence or recursive sequence, are a favourite topic between researchers. Therefore, there has been a rising interest in the research of qualitative analysis of difference equations. Also, difference equations come in sight naturally as discrete analogues and as numerical solutions of differential and delay differential equations having applications in population's dynamics, health sciences, biology, ecology, economics, physics, computer sciences, engineering and so on. Although the linear difference equations a lot studied in the beginning, the non-linear difference equations have been much more studied for the last years. In particular, some authors have published a number of quality articles on the nonlinear difference equations. More and more, some of the authors were interested in the class of difference equations of the form $x_{n+1} = x_{n-k}x_{n-l} - 1$. Further, many authors investigated to dynamics of various forms of the related difference equation such as $k = 0, l = 1$ in [1]; $k = 0, l = 2$ in [3]; $k = 1, l = 2$ in [2]; $k = 0, l = 3$ in [4]. Besides, Stević and Iricanin in [5] have obtained some results regarding the general form of the related difference equation.

In this work we will study dynamic behaviours of the nonlinear difference equation $x_{n+1} = x_{n-2}x_{n-3} - 1$. Especially, we will investigate in existence of the solutions of the nonlinear difference equation which are periodic or non-periodic.

Key Words: Difference equations, Periodic solutions.

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On The Periodic Solutions of a Difference Equation

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ABSTRACT

During the last years, the worldwide growing interest to examination of long term behaviours and dynamics of the non-linear difference equations as good as periodicity, boundedness property and stability, is in coincident. The reason for this, these form of non-linear difference equations have a lot of applications not only in the branches of mathematics but also in interrelating sciences, especially in discrete system biology, genetic, ecology, bioengineering, probability theory, physics, engineering and so on. Some mathematicians have preferred to investigate the solutions of the difference equations. However, the periodic character of the difference equations have become a special chapter of the articles of these mathematicians. As well, Amleh et al in [1] and [2] study the periodic nature and the boundedness of some class of the difference equations. What is more, in [3] Kent et al investigate that the periodicity and the boundedness of solutions of a non-linear difference equation of $x_{n+1} = x_n x_{n-2} - 1$. Besides, in [4], Taşdemir and Soykan studied the periodicity of difference equation $x_{n+1} = x_n x_{n-1} + \alpha$. In this work, we will widen one's viewpoint related a non-linear difference equation.

In this paper we will study the periodic solution of the following non-linear difference equation $x_{n+1} = x_n x_{n-2} + \alpha, n \in \mathbb{N}_0$ and the initial conditions are real numbers. In other words, we will investigate that there are periodic solutions of related non-linear third order difference equation under which conditions.

Key Words: Difference equations, Periodic solutions.

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On the periodic solutions of some systems of higher order difference equations

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ABSTRACT

Difference equations and systems have been investigated by many researchers in the last few decades. There is no doubt that the theory of difference equations will continue to play an important role in mathematics as a whole. The reason is that difference equations have a lot of applications in several mathematical models in biology, economics, genetics, population dynamics, medicine, information system, ecological balance, engineering control, physiology and so forth. Nonlinear difference equations of order greater than one are of paramount importance in applications. As typical nonlinear difference equations, rational difference equations have become a research hot spot in mathematical modelling. Moreover, studying the qualitative analysis of difference equations and systems is a very rich research field. It is very interesting to investigate the solutions of a system of higher order rational difference equations and to discuss the periodicity nature of solutions. There are many papers related to the periodicity of the positive solutions of the rational difference equation systems.

In this work we obtain the general form of the periodic solutions of some higher order rational difference equations system

$$x_{n+1} = \frac{y_{n-k}}{x_{n-(2k+1)}(\pm 1 + y_{n-k})}, \qquad y_{n+1} = \frac{x_{n-k}}{y_{n-(2k+1)}(\pm 1 + x_{n-k})}, \qquad n, k \in \mathbb{N}_0$$

where the initial values are arbitrary real numbers such that the denominator is always nonzero. Moreover, some numerical examples are presented to verify our theoretical results.

Key Words: periodicity, systems of higher order difference equations, form of the solutions.

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On The Solution Of Eigenvalue Problem Of Complete Conical Shell With Functionally Graded Coatings And Metal-Rich Core

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ABSTRACT

The sandwich conical shells comprise a class of structural components, which is applied on a daily basis in engineering. Typical sandwich structure consists of one or more layers of high strength and high rigidity linings associated with a flexible core [1]. Functionally graded materials (FGMs) are often used at high temperatures, such as nuclear, aerospace and many other applications [2,3]. The number of studies to address the eigenvalue problem of FGM sandwich structures in the framework of shear deformation shell theory (SDSTs) has increased in recent years [4-6]. The review clearly points out that there is no research in literature on the solution of the eigenvalue problem of FGM sandwich complete conical shells under lateral pressure in the framework of the first order shear deformation theory (FOSDST). In this work, the solution of the eigenvalue problem of complete conical shells with FGM coatings and metal-rich core under lateral pressure in the framework of the FOSDST is investigated It is assumed that the volume fractions of FGM coatings vary according to a simple power law function of thickness coordinate while that of the core equals unity. The basic partial differential equations for simply supported sandwich truncated conical shells with FGM coatings are displayed based on the Donnell shell theory including the transverse shear deformations and solved by use the Galerkin's method. The formula for the critical lateral pressure of metal-rich complete conical shells with FGM coatings in the framework of the FOSDST is obtained. The

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influences of functionally graded coatings and transverse shear deformations on the critical lateral pressure for sandwich complete conical shells with FGM coatings are investigated numerically by using Maple 14 software.

Key Words: Complete Conical Shells, Functionally graded materials (FGMs), Eigenvalue problem

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On The Solutions of Difference Equations

$$x_n=\left(\prod_{k=4}^{7}x_{n-k}\right)/\prod_{k=1}^{3}x_{n-k}\left(\mp1\mp\prod_{k=4}^{7}x_{n-k}\right)$$

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ABSTRACT

Difference equations are mostly used in numerical solutions of differential equations arising in physics, engineering etc.

In this paper, we investigate behaviour of well-defined solutions of the difference equations for $n=1,2,3,\ldots$,

$$x_n=\left(\prod_{k=4}^{7}x_{n-k}\right)/\prod_{k=1}^{3}x_{n-k}\left(\mp1\mp\prod_{k=4}^{7}x_{n-k}\right), \qquad \textbf{(1)}$$

where $k\in\mathbb{N}$ and the initial conditions are arbitrary nonzero real numbers. Really, we investigate four different difference equations such that

$$x_n=\frac{x_{n-4}x_{n-5}x_{n-6}x_{n-7}}{x_{n-1}x_{n-2}x_{n-3}\left(1+x_{n-4}x_{n-5}x_{n-6}x_{n-7}\right)}, n=1,2,3,\ldots \qquad \textbf{(2)}$$

$$x_n=\frac{x_{n-4}x_{n-5}x_{n-6}x_{n-7}}{x_{n-1}x_{n-2}x_{n-3}\left(-1+x_{n-4}x_{n-5}x_{n-6}x_{n-7}\right)}, n=1,2,3,\ldots \qquad \textbf{(3)}$$

$$x_n=\frac{x_{n-4}x_{n-5}x_{n-6}x_{n-7}}{x_{n-1}x_{n-2}x_{n-3}\left(1-x_{n-4}x_{n-5}x_{n-6}x_{n-7}\right)}, \ n=1,2,3,\ldots \qquad \textbf{(4)}$$

$$x_n=\frac{x_{n-4}x_{n-5}x_{n-6}x_{n-7}}{x_{n-1}x_{n-2}x_{n-3}\left(-1-x_{n-4}x_{n-5}x_{n-6}x_{n-7}\right)}, \ n=1,2,3,\ldots \qquad \textbf{(5)}$$

Firstly, we obtain the solutions of the difference equation (1). We find equilibrium points of difference equation (1) which has only one equilibrium point $x^*=0$. Because of to determine stability of equilibrium point, we determine the

linearized equation of the equation (1) about $x^{*}=0$. Secondly, we give the well-defined solutions of rational difference equation (2) exactly. Then, we investigate stability of equilibrium points of the equation (2) which has two equilibrium points $x^{*}=0$ and $x^{*}=\sqrt[4]{2}$. Thirdly, we investigate the well-defined solutions of rational difference equation (4). We obtain formulas of the well-defined solutions of rational difference equation (4). Also, we investigate stability of the equilibrium points of difference equations (4) which has an unique equilibrium point $x^{*}=0$. Fourthly, we give formulas of solutions of rational difference equation (5). We show that the rational difference equation (5) has only one equilibrium point $x^{*}=0$ and then we investigate stability of $x^{*}=0$. Finally, we give some numerical results graphically about some special cases of rational difference equations (2), (3), (4) and (5).

Key Words: Difference equation, solution, stability.

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On the Spectral Expansions of a Boundary Value Problem with eigenparameter in both Boundary Conditions

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ABSTRACT

In this talk, we consider the boundary value problem

$$-y'' + q(x)\,y = \lambda y,\ 0 < x < 1,$$

$$(a_0\lambda + b_0)\,y(0) = y'(0),\ (a_1\lambda + b_1)\,y(1) = y'(1),$$

where λ is a spectral parameter, $q(x)$ is a real-valued continuous function on the interval $[0,1]$, $a_k, b_k\ (k = 0,1)$ are arbitrary real constants and $a_0 < 0$, $a_1 > 0$.

This paper is devoted to studying the uniform convergence of Fourier series expansions for continuous functions in the system of eigenfunctions, after the elimination of two arbitrary functions with indices of opposite parity, of the above boundary value problem. The convergence of the spectral expansions in terms of root functions of some differential operators with a spectral parameter in the boundary conditions is handled as a current topic in many articles.

The above boundary value problem with the condition $a_0 < 0$ and $a_1 > 0$ was considered; the existence and simplicity of eigenvalues, the oscillation properties of eigenfunctions, the asymptotic formulae of eigenvalues and eigenfunctions, the basis properties of the system of eigenfunctions in $L_p(0,1)\ (1 < p < \infty)$ were studied. Furthermore, there were some studies above about the spectral properties of this boundary value problem with the conditions $a_0 > 0$ and $a_1 < 0$, $a_0 < 0$ and $a_1 < 0$.

Note that the condition $a_0 < 0$ and $a_1 > 0$ plays an important role for investigating spectral properties of above boundary value problem. If $a_0 < 0$ and $a_1 > 0$, then this problem can be treated as a spectral problem for a self-adjoint operator in the Hilbert space $L_2(0,1) \oplus \mathbb{C}^2$. In this case, all the eigenvalues of this problem are real and simple; in addition, the system of root functions of this problem consist only from eigenfunctions.

This study is related to the articles [1]-[3].

Key Words: Sturm-Liouville problems, Fourier series, Uniform convergence.

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On the Uniform Convergence Fourier Series Expansions of a Spectral Problem with Eigenparameter in a Boundary Condition

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ABSTRACT

In recent years, there has been a growing interest in spectral theory of differential operators. Proving existence of eigenvalues, obtaining oscillation of eigenfunctions, giving asymptotic formulae of eigenvalues and eigenfunctions, investigating the basis properties of the system of root functions in different functional spaces and the convergence of the spectral expansions in terms of root functions in some linear spaces are the goals of some of important problems of linear differential operator theory.

In this talk, we research the uniform convergence of the Fourier Series expansions in terms of eigenfunctions for the boundary value problem

$$-y'' + q(x)y = \lambda y,\ \ 0 < x < 1,$$

$$y(0) = 0,\ \ y'(0) = \lambda\left(ay(1) + by'(1)\right),$$

where λ is a spectral parameter, $q(x) \in L_1(0,1)$ is a complex-valued function, a and b are arbitrary complex numbers which satisfy the condition $|a| + |b| \neq 0$. Firstly, to prove this problem, we investigated that the system of eigenfunctions of the corresponding operator, with an arbitrary eigenfunction removed, form a basis in the space $L_2(0,1)$. For the basicity in $L_2(0,1)$, we proved that the system of eigenfunctions is quadratically close to corresponding sine or cosine systems which are orthonormal bases in $L_2(0,1)$.

Note that, the sharpened asymptotic formulae for eigenvalues and eigenfunctions of above-mentioned spectral problem are need to obtain and biorthogonally conjugate systems to eigenfunctions system are need to know for investigate the uniform convergence of the spectral expansions in terms of eigenfunctions for this spectral problems. Therefore, minimality of eigenfunctions system is need to prove in the space $L_2(0,1)$. Moreover, we shall use linear differential operator theory and functional analysis techniques to above mentioned requirements.

This study is related to the articles [1]-[3].

Key Words: Spectral problem, Fourier series expansion, Uniform convergence.

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Optimal Tilt Angle Determination Of Photovoltaic Panels

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ABSTRACT

Due to the energy problem that has arisen in our country and in the world, we have been searching for new energy sources. And thus the use of solar energy has become more important. Because solar energy is an energy source that will never run out. In addition to being an unlimited source of energy, it is clean energy, easy to store and its cost is quite low. One way to make use of solar energy is to generate electricity using the solar panel system. In order to obtain high efficiency from these solar panels, it is necessary to adjust the panels to the optimum tilt angle. This tilt angle varies according to geographical location and seasons.

Correlation procedures are required to obtain insolation values on a tilted surface from horizontal radiation. Because most published meteorological data give a global radiation on a horizontal surface. The monthly average daily total radiation on tilted surface is dependent on the direct beam, diffuse and ground reflected components. So, the incident total radiation on tilted surface at a tilt angle from the horizontal is given by [1].

$$H_T = H_B + H_D + H_R ,$$

where H_T , H_B , H_D , H_R respectively the monthly average daily total, beam, diffuse and reflected radiation on a tilted surface in.

In this paper, we calculated the optimum slope using these parameters. According to calculating optimum slope angle, the amount of energy which was obtained solar radiation is provided increasing.

Key Words: Solar energy, tilted angle, radiation.

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Oscillation Criteria For Second-Order Matrix Differential Systems With Damping

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ABSTRACT

In this paper, we are concerned with the second order matrix differential systems with damping of the form

$$(r(t)P(t)\Psi(X(t))K(X'(t)))'+p(t)R(t)\Psi(X(t))K(X'(t))+Q(t)F(X'(t))G(X(t))=0 \quad (1)$$

where $t \geq t_0 \geq 0$ *and* r, p, P, Ψ, K, R,Q and G satisfy the following conditions:

1) $r \in C^1([t_0,\infty);(0,\infty))$, $p \in C([t_0,\infty);(-\infty,\infty))$;

2) $P(t) = P^T(t) > 0$, $Q(t) \geq 0$,$R(t) = R^T(t) > 0$ for $t \geq t_o$, P, Q and R are $n \times n$ matrices real valued continuous functions on the interval $[t_0,\infty)$, and $P(t)$ and $R(t)$ are commutative. By A^Twe mean the transpose of the matrix A;

3) Ψ, K, G ,$F \in C^1(\mathbb{R}^{n^2};\mathbb{R}^{n^2})$ and $\Psi^{-1}(X(t))$, $K^{-1}(X'(t))$ and $G^{-1}(X(t))$ exist and $F(X') \geq 0$ for all real matrix $X \neq 0$.

We call a matrix function solution $X(t) \in C^2([t_o,\infty),\mathbb{R}^{n^2})$ of Eq. (1) is prepared nontrivial if $detX(t) \neq 0$ for at least one $t \in [t_0,\infty)$ and $X(t)$ satisfies the some properties. A prepared solution $X(t)$ of Eq. (1) is called oscillatory if $detX(t)$has arbitrarily large zeros; otherwise, it is called nonoscillatory..

Motivated by the idea of Li and Agarwal [1], Yang and Tang [2] and Yang [3], we establish the oscillation theorems of Kamenev type by using the generalized averaging technique and positive linear functionals. Our results make use of the oscillatory properties of the damping term and some of the them extend and generalized the main results given in [1,2,3].

Key Words: Matrix differential system; Oscillation; Generalized averaging technique; Kamenev type oscillation.

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Oscillation Criteria For a Class of Second-Order Nonlinear Differential Equations With Damping

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ABSTRACT

Consider the second-order nonlinear forced di_erential equation for $t \geq t_0$,

$$\left(r(t)k_1(x,x')\right)' + p(t)k_2(x,x')x' + q(t)f(x) = e(t)\,, \tag{1}$$

Where $p,q \in C([t_0,\infty),\mathbb{R}), r \in C^1\left([t_0,\infty),(0,\infty)\right), f,e \in C(\mathbb{R},\mathbb{R}), k_1 \in C^1(\mathbb{R}^2,\mathbb{R})$, and $k_2 \in C(\mathbb{R}^2,\mathbb{R})$. We restrict our attention to solutions of Eq.(1) which exists on $[t_0,\infty)$. As usual, such a solution, $x(t)$, is said to be oscillatory if it has arbitrarily zeros for all $t_0 \geq 0$, otherwise, it is called nonoscillatory.

In the last decades, there has been an increasing interest in obtaining sufficient conditions for the oscillation and nonoscillation of solutions for Eq.(1).

Many results are established for the particular cases of Eq.(1) for $t \geq t_0$, for example, the general nonlinear differential equation with damping for $t \geq t_0$

$$(r(t)k(x(t),x'(t))x'(t))' + p(t)k(x,x')x' + q(t)f(x) = 0, \tag{2}$$

has been considered recently by Ayanlar and Tiryaki [1].

The more general nonlinear differential equation with damping

$$(r(t)k_1(x(t),x'(t))x'(t))' + p(t)k_2(x,x')x' + q(t)f(x) = 0 \tag{3}$$

has been first studied by Rogovchenko and Rogovchenko [3], which coincides our main equation with $e(t) = 0$. Later, Rogovchenko's study [3] has been extended by Tiryaki and Zafer [4]. They obtained several oscillation criteria for solution of Eq.(3) under some relationships between the functions k_1 and k_2.

In 2006, Zhao and Meng [5] obtained some oscillation results for the non linear differential equation Eq.(3). They established new oscillation criteria which are extension and generalization of some known results by using the Riccati technique and the kernel functions of Philos' type under the following assumptations

(A_1) $p(t) > 0$ for all $t \geq t_0, xf(x) > 0$ for all $x \neq 0$;

(A_2) $vk_1(u,v) \geq \beta_1 |k_1(u,v)|^{\frac{\alpha+1}{\alpha}}$ for some $\beta_1 > 0$, and $\forall (u,v) \in \mathbb{R}^2$;

(A_3) $vk_2(u,v) f^{\frac{1}{\alpha}}(u) \geq \beta_2 |k_1(u,v)|^{\frac{\alpha+1}{\alpha}}$ for some $\beta_2 > 0$, and $\forall (u,v) \in \mathbb{R}^2$;

(A_4) $f'(x)$ exists and $\frac{f'(x)}{|f(x)|^{\frac{\alpha-1}{\alpha}}} \geq \beta_3 > 0$ for some constant β_3 and $\forall x \in \mathbb{R}\backslash\{0\}$;

or

(A_5) $q(t) \geq 0$ $for\ all\ t \geq t_0$, f satisfies $\frac{f(x)}{x} > L$ for for some constant L and $\forall x \neq 0$;

(A_6) $k_1(u,v) \geq \beta_4 |k_1(u,v)|^{\frac{\alpha+1}{\alpha}} u^{\frac{\alpha-1}{\alpha}}$ for some positive constant β_4 and $\forall v \in \mathbb{R}\backslash\{0\}$ and $\forall u \in \mathbb{R}$;

(A_7) $vu^{\frac{1}{\alpha}} k_2(u,v) \geq \beta_5 |k_1(u,v)|^{\frac{\alpha+1}{\alpha}}$ for some positive constant $\beta_5 > 0, \forall (u,v) \in \mathbb{R}^2$.

In 2007, Çakmak and Tiryaki [2] shoved that the proof given by Zhao and Meng [22] are inaccurate when $x(t) < 0$ for $t \geq t_0$, because if we take $x(t) < 0$ then by assumptations (A_1) and (A_5) $f(x)$ becomes negative, therefore (A_3), (A_6), and (A_7) are not satisfies. Thus, although, Zhao and Meng's study [22] is very interesting and well-organized, there are some important mistakes.

Therefore, Çakmak and Tiryaki [2] suggested to change the conditions (A_3), (A_6), and (A_7) by replacing with (A_3a), (A_6a), and (A_7a) such that

(A_3a) $vk_2(u,v)|f(u)|^{\frac{1}{\alpha}} \geq \beta_2 |k_1(u,v)|^{\frac{\alpha+1}{\alpha}}$ for some $\beta_2 > 0$, and all $(u,v) \in \mathbb{R}^2$;

(A_6a) $vk_1(u,v) \geq \beta_4 |k_1(u,v)|^{\frac{\alpha+1}{\alpha}} |u|^{\frac{\alpha-1}{\alpha}}$ for some positive constant β_4 and for all $v \in \mathbb{R}\backslash\{0\}$ and all $u \in \mathbb{R}$;

(A_7a) $v|u|^{\frac{1}{\alpha}} k_2(u,v) \geq \beta_5 |k_1(u,v)|^{\frac{\alpha+1}{\alpha}}$ for some positive constant $\beta_5 > 0$, and all $(u,v) \in \mathbb{R}^2$.

In 2008, Huang and Meng [6], take into considerations of Çakmak and Tiryaki's paper [2] and obtained some oscillation results for the nonlinear equation Eq.(3). But in 2011, Shang and Qin [7] showed that if Huang and Meng's conditions are taken into consideration, oscillatory solutions of the given equation in Example 5.2 in [6] does not exist. Thus it seems that the conditions (A_3a), (A_5a), and (A_7a) still need reconsideration.

Motivated by this fact, in this paper, we will investigate the oscillatory behavior of second-order nonlinear forced differential equation Eq.(1) by revising the conditions $(A_3 a)$, $(A_5 a)$, and $(A_7 a)$ to overcome the diffculties that we mentioned above.

Key Words: Differential Equations, Oscillation, Damping.

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Oscillation of a Third Order Nonlinear Differential Equation with Piecewise Constant Arguments

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ABSTRACT

Since the early 1980's, differential equations with piecewise constant arguments (DEPCA) have attracted great deal of attention of researchers in science. Such equations appear in diverse areas such as engineering, physics and mathematics. They are closely related to difference and differential equations. Therefore, they are stated as hybrid dynamical systems. The qualitative works such as oscillation and convergence of solutions of (DEPCA) have been studied in some papers. But a few of them is about third order nonlinear differential equations with piecewise constant arguments. So, we consider the following type third order nonlinear differential equation with piecewise constant arguments

$$(r_2(t)(r_1(t)x'(t))')' + p(t)x'(t) + f(t, x([t-1])) + g(t, x([\mathrm{t}])) = 0, \tag{1}$$

where $t \geq 0, r_1(t), r_2(t)$ are continuous on $[0,\infty)$ with $r_1(t), r_2(t) > 0$ and $r_1'(t) \geq 0,$ $p(t)$ is continuously differentiable on $[0,\infty)$ with $p(t) \geq 0$ and $[.]$ denotes the greatest integer function. Throughout this study, we make the following assumptions:

(A_1) $q_1(t), q_2(t) > 0$ and $q_1(t), q_2(t)$ are continuous on $[0,\infty)$,

(A_2) $\varphi_1(x)$ and $\varphi_2(x)$ are continuously differentiable, $\phi_1(x)$, $\phi_2(x)$ are nondecreasing on $(-\infty,\infty)$, $x\phi_2(x) \geq x\phi_1(x)$ and for $x \neq 0$

$$\frac{\phi_1(x)}{x} \geq K_1 > 0,\ \frac{\phi_2(x)}{x} \geq K_2 > 0,$$

(A_3) $|f(t,x)| \geq q_1(t)|\phi_1(x)|$ and $|g(t,x)| \geq q_2(t)|\phi_2(x)|$ for $x \neq 0, t \geq 0,$

(A_4) $K_1 q_1(t) + K_2 q_2(t) - p'(t) \geq 0$ on $[0,\infty)$,

(A5) $xf(t,x) \geq 0,\ \ xg(t,x) \geq 0.$

In addition to above assumptions we also need

$$R_1(t,T) \to \infty \text{ as } t \to \infty,$$

$$R_2(t,T) \to \infty \text{ as } t \to \infty,$$

here

$$R_i(t,T) = \int_T^t \frac{ds}{r_i(s)},\ i = 1,2.$$

Under the assumption that the following second order differential equation

$$r_1(t)(r_2(t)y'(t))' + p(t)y(t) = 0$$

is nonoscillatory, we obtain some conditions that guarantee that every solution $x(t)$ of Eq. (1) oscillates or converges to zero. Also, examples are given to illustrate the relevance of the results.

Key Words: Third order differential equation, Piecewise constant arguments, Oscillation, Asymptotic behavior.

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Rayleigh Quaotient And Rayleigh-Ritz Formula For Multi-Interval Boundary-Value Problems

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ABSTRACT

Many-interval boundary-value problems form an important part of the spectral theory of differential operators (see, [2,6,8] and references, cited therein). This type of spectral problems arise in many problems of physics, such as heat and mass transfer problems, vibrating string problems when the string loaded additionally with point masses, thermal conduction problems for thin laminated plate and in varied assortment of physical interaction problems (see, for example, [1,5-7]). The solution of such type problems of physics are involved in investigation of the eigenvalues and the expansion of an arbitrary function in terms of eigenfunction of a corresponding Sturm-Liouville problem with supplementary transmission conditions at the points of interaction. Multi-interval boundary-value-transmission problems cannot be treated with the usual techniques within the standard framework of Sturm-Liouville theory. In the present work we shall investigate some computational aspects of the eigenvalues of one multi-interval boundary-value-transmission problem for Sturm-Liouville equation. It is well-known that any eigenvalue can be related to its eigenfunction by the Rayleigh quotient. But this quotient cannot be used to explicitly determine the eigenvalue in the case when the corresponding eigenfunction is not known. However, some useful results can be obtained from the Rayleigh quotient without solving the differential equation. For instance, it can be quite useful in estimating the eigenvalues. The main goal of this study is to establish such properties of the problem under consideration, as uniform convergeness of the eigenfunction

expansion, Parseval's equality, Rayleigh-Ritz formula and monotonicity of the eigenvalues.

Key Words: Multi-interval boundary-value problems, transmission conditions, eigenvalue, Rayleigh quotient.

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Recent Advances on Weighted Rellich-Type Inequalities on the Heisenberg Group

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ABSTRACT

Inequalities involving integrals of a function and its derivatives have become very effective and powerful tools for studying a wide range of problems in various branches of mathematics as well as in other areas of science. Since from its discovery Hardy and Rellich type inequalities have evoked the interest of many mathematicians, and large number of papers have appeared which deal with new proofs, various extensions, refinements, and generalizations. And also many new developments are still forthcoming.

In this talk, we shall present some weighted Rellich type inequalities on the Heisenberg group H^n. In particular, we would like to exhibit some sufficient conditions that imply the Rellich inequality without assuming a priori symmetric hypotheses on the weights. More precisely, given two nonnegative functions a and b if there exists a positive supersolution ϑ of the given Kohn's sub-Laplacian Δ_{H^n} such that

$$\Delta_{H^n}\left(a(z,l)\left|\Delta_{H^n}\vartheta\right|^{p-2}\Delta_{H^n}\vartheta\right)\geq b(z,l)\,\vartheta^{p-1}$$

almost everywhere in H^n, then for every $u\in C_0^\infty(H^n)$ there holds

$$\int_{H^n} a(z,l)\left|\Delta_{H^n}u(z,l)\right|^p dzdl \geq \int_{H^n} b(z,l)\left|u(z,l)\right|^p dzdl,$$

where $p>1$. A generic point in H^n is defined by $(z,l)=(x,y,l)\in H^n$, where $z=(x,y)\in \mathbb{R}^n\times\mathbb{R}^n$ and $l\in\mathbb{R}$. Here, $dzdl$ denotes the Lebesgue measure on $\mathbb{R}^{2n+1}$

and $\Delta_{H^n}=\sum_{i=1}^{n}X_i^2+Y_i^2$ is the Kohn's sub-Laplacian on H^n, where $X_i=\frac{\partial}{\partial x_i}+2y_i\frac{\partial}{\partial l}$ and $Y_i=\frac{\partial}{\partial y_i}-2x_i\frac{\partial}{\partial l}$, $i=1,2,...$ are left invariant vector fields.

We also illustrate how our approach systematically yields previously known weighted Rellich type inequalities and also enables us to obtain new ones on some special domains Ω in H^n.

Key Words: Heisenberg group, weighted Rellich inequality, $p-$biharmonic equations.

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Reconstruction Of The Sturm-Liouville Operator With Discontinuities From Spectral Data

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ABSTRACT

We consider the following Sturm-Liouville boundary value problem

$$-y''(x) + q(x)\,y(x) = \lambda^2 y(x),\ x\epsilon[0,\pi]\backslash\alpha \quad (1)$$

$$y'(0) - hy(0) = 0,\ y(\pi) + Hy(\pi) = 0 \quad (2)$$

with discontinuous conditions

$$y(d+0) = \alpha y(d-0),\ y'(d+0) = \alpha y'(d-0) + \beta y(d-0) \quad (3)$$

Here λ is the spectral parameter, the real-valued function $q(.)\epsilon L_2[0,\pi]$ and h, H, $\alpha(>0)$, β and $\alpha \in \left(\frac{\pi}{2},\pi\right)$ are real numbers. Moreover, h and H are allowed to be infinity, and then the corresponding boundary condition is interpreted as Dirichlet one.

Boundary value problems with discontinuous points inside the interval arise in mathematics, mechanics, radio electronics, geophysics and other field of science and technology. Such problems are connected with discontinuous material properties (see, for example, $[1-3]$).

In this aspect, the studies of Gelfand, Levitan and Marchenko include basic investigations related to the integral representations of solutions to various direct and inverse problems for the Sturm-Liouville differential operators.

Inverse spectral problems were studied for the second-order differential operators on a finite interval with discontinuity conditions inside the interval. Uniqueness theorem was proved and necessary and sufficient conditions for the solvability of the inverse problem were given and also a procedure for the solution of inverse problem was obtained $[4]$.

In [4] solution of inverse problem was reduced to the existence of solution of infinite equations system in Banach space which is called main equation. As different from [3] and [5], in this study solution of inverse problem is reduced to the solution of the Gelfand-Levitan-Marchenko (GLM) type main integral equation which is used for solution of inverse problem in classical case. Item 3 deals with the solution of the inverse problem. We prove existence and uniqueness of the solution of the GLM integral equation and give a procedure for the solvability of classical inverse problem for impulsive differential operators. Also, necessary and sufficient conditions for existence of solution of inverse problem are mentioned in terms of given sequences.

Key Words: Sturm-Liouville equation, Integral equation, Expansion formula.

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Reduction of Nonlinear Partial Differantial Equations via Optimal Systems

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ABSTRACT

In Lie' s approach, continuous groups are determined by infinitesimal generators, and when an infinitesimal generators are given, the group transformations are found by solving the Lie equations, which is guaranteed only in a small neighbourhood of regular points as in point transformations. We regard as differential equations for which we have calculated the r-parametric maximal symmetry group. For every s-parametric subgroup H one is able to find a family of similarity solutions. The assumption is that s ≥ min{r, n}, s, r, n ϵ IN. n; is the number of independent variables of equation and r is the order of derivatives. Since in many cases there exists an infinite number of such subgroups it is impossible to calculate all similarity solutions relative to s-parametric subgroups. In this set there are similarity solutions which result

from other similarity solutions of the same set applying a transformation of the symmetry group. It would be profitable to have a minimal list of similarity solutions such that with these elements one can get all other similarity solutions

via transformation. Such a minimal list is called an optimal system and their elements are essentially different similarity solutions.

In this study, it will be discussed symmetry reductions of nonlinear partial differential equations by using optimal

systems. Via this optimal systems, it will be done Lie symmetry analysis of nonlinear partial differential equations and it will be constituted optimal system of Lie symmetries. And then according to this optimal system, it will be obtained symmetry reductions of nonlinear partial differential equations.

Keywords: Optimal System, Adjoint Representation, Symmetry

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Reliability Analysis for a System with Gradual Degradation by Using Asymptotic Methods

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ABSTRACT

In this study, a mechanical system with imperfect maintenance and gradual degradation is considered. It is assumed that at initial time the system has z>0 available resource and when the system is in operating for ξ_1 time units, resource is decreasing continuously and gradually by $c\xi_1$. When the system is not in operating mode, a given maintenance policy is applied and general resource of the system is increasing by a random amount (ζ_1). The resource of the system will reach $z - c\xi_1 + \zeta_1$ level at the end of the first period. We assume that $\zeta_1 < c\xi_1$ since the maintenance policy is imperfect. The subsequent periods will proceed similarly and eventually when the total available resource reaches zero, the system will be replaced the process continuous in a similar manner. The total amount resource of the mechanical system, which is changed as above mentioned, expressed by a stochastic process X(t). This process is called a semi-continuous process, in the literature. The main purpose of this study is to investigate the reliability parameters of the system by using asymptotic methods. For this aim, the process X(t) and certain boundary functional (τ_1) of the process are defined mathematically. The boundary functional (τ_1) is the first time in which the process reaches to level zero. Next, exact expressions for the expected value, variance, high order moments and moment generating function of the boundary functional (τ_1) are obtained. Afterwards, two-term asymptotic expansions for the probability and numerical characteristics of the boundary functional (τ_1) are derived.

Key Words: Gradual degradation, Imperfect maintenance, Semi-continuous process, Reliability analysis, Asymptotic methods.

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Restrictive Pade Approximation Solution of the Convection-Diffusion Equation

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ABSTRACT

In this study,we apply a new implicit method of high accuracy and the number of linear systems which to be solved are smaller than that for many famous known implicit methods of small step length. Therefore our required machine time is less than that for the other methods.

The convection-diffusion equation applies to problems in such areas a mass transport, momentum transport, energy transport and neutron transport. The problem is solving the convection-diffusion equation by a method related to the Restrictive Pade Approximation(RPA) is considered. This method will exhibit several advantageous features. For example the accuracy has not been lost when the value of the exact solution is sufficiently large the absolute error is sufficiently small whenever the exact solution is relatively large.The choice of time step length k is sufficiently large compared with that can be used for the classical schemes,this allows us to have the solution at high level of time. Restrictive pade approximation for parabolic partial differential equation and partial difference equation is a new technique done by İsmail and Elbarbary. In addition they studied numerical solution of the convection-diffusion equation using restrictive Taylor approximation [1].

The advantage is that it has the exact value at certain r. This method will exhibit several advantages for example highly accurate,fast and good results, etc. The absolutely error is still very small.The computed results are compared with the exact solution and the other methods.

Key Words: Restrictive Pade Approximation, Convection-Diffusion Equation, Finite Difference

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Semiparametric Wavelet Estimation in the Presence of Measurement Error

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ABSTRACT

Wavelet theory provides a unified framework for a number of techniques which had been developed independently for various signal processing applications [1]. Wavelets can be used to extract information from many different kinds of data as a mathematical tool, such as audio signals and images. Signal processing is an enabling technology that encompasses the fundamental theory, applications, algorithms, and implementations of processing or transferring information contained in many different physical, symbolic, or abstract formats broadly designated as signals [2]. Because image processing methods mostly deal with the image as two-dimensional signals and use standard signal processing techniques to improve it, this study has also an important place in image processing which uses mathematical operations for processing any form of signal processing.

In this study, we introduced the wavelet approach to estimate semiparametric errors in variables regression model. In this model it is assumed that the independent variable of a nonparamtric function of the semiparametric model has measurement error which has a known distribution. We also used Monte-Carlo simulation method to show that the resulting rates are comparable to no measurement error case.

The expanded model to a multidimensional case is still open. If it is achieved, mentioned model can cover image processing.

Key Words: Semiparametric regression, wavelet estimation, errors in variables.

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Separated Flow Structures in a Z-Shaped Cavity

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ABSTRACT

In this study, separated flow structures in a Z-shaped cavity with the lids moving in the same directions are obtained by a numerical and methods from dynamical systems. The flow is governed by Stokes or Navier-Stokes equations with two control parameters h_1 and h_2 which are related with to the heights of the Z-shaped cavity. This gives rise to a boundary value problem for the stream function. As h_1 and h_2 are varied, they showed that changes in flow pattern arose directly from stagnation point bifurcations. The flow structure is transformed as stagnation point changes from a centre to a saddle and vice versa. The purpose of this study is to construct a detailed control space diagram, including bifurcation curves, for Z shaped cavity.

The aim of this study is to present highly accurate benchmark results for lid-driven cavity flow in the corner and investigate local behaviour of the streamlines at the singular point. It is shown that dividing streamline near the re-entrant corner provides the key to understanding vortex generation in the Z-shaped cavity. We have obtained a set of codimension-one bifurcation curves in the (h_1, h_2) parameter space. The codimesion of a bifurcation is the smallest number of parameters needed to describe the bifurcation. The mechanism for eddy generation via the development of side eddies, as h_1 and h_2 are increased, is examined for S=1 and a number of different flow bifurcations and corresponding flow structures occur.

Key Words: Flow structures, Stokes flow, Matching, Least-squares approximations

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Solutions of an anisotropic Kirchhoff problem involving variable exponent

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ABSTRACT

In this study, we deal with a general class of anisotropic Kirchhoff problems. By means of the Mountain Pass theorem, we obtain the existence nontrivial weak solutions in appropriate anisotropic variable exponent Sobolev spaces.

We study the existence of solutions of anisotropic Kirchhoff problem involving $\vec{p}(.)-$Laplace type operator,

$$\begin{cases} -M\left(\int_\Omega \sum_{i=1}^{N} A_i\left(x,\partial_{x_i}u\right)dx\right)\sum_{i=1}^{N}\partial_{x_i}a_i\left(x,\partial_{x_i}u\right)=\lambda|u|^{q(x)-2}u, x\in\Omega \\ u=0, x\in\partial\Omega \end{cases}$$

where $\Omega\subset \mathrm{R}^N\ (N\geq 3)$ is a bounded with smoothy boundary, λ is a positive parameter and p_i, q are continuous functions on $\overline{\Omega}$ such that $2\leq p_i(x)<N$ and $q(x)>1$ for any $x\in\overline{\Omega}$ and $i\in\{1,...,N\}$. $a_i:\Omega x\mathrm{R}\to\mathrm{R}$ is Carathéodory such that $a_i(x,\xi)$ is the continuous derivative with respect to ξ of the mapping $A_i:\Omega x\mathrm{R}\to\mathrm{R}$, $A_i=A_i(x,\xi)$, i.e., $a_i(x,\xi)=\frac{\partial}{\partial\xi}A_i(x,\xi)$. Moreover, Kirchhoff function M is a continuous function.

The differential operator $\sum_{i=1}^{N}\partial_{x_i}a_i\left(x,\partial_{x_i}u\right)$ that appears in above problem is called the anisotropic variable exponent $\vec{p}(.)-$Laplace operator. This differential operator is an extension of the well-known $p(.)-$Laplace operator given by $\sum_{i=1}^{N}\partial_{x_i}\left(|\partial x_i u|^{p(x)-2}\partial_{x_i}u\right)$, i.e., $p(.)-$Laplace operator, obtained in the case when for each $i \in \{1,...,N\}$ we have $p_i = p$. This differential operator is a natural generalization of the isotropic $p-$ Laplace operator $\Delta_p u := div\left(|\nabla u|^{p-2}|\nabla u|\right)$, where $p > 1$ is a real constant. The main difference between them is that $p-$ Laplace operator $(p-1)-$homogenous. This causes many problems, some classical theories, such as the theory of Sobolev spaces, is not applicable. For the papers involving the $p-$ Laplace operator we refer to references [1,2,3].

Key Words: Anisotropic variable exponent Lebesgue-Sobolev spaces, variational approach, $\vec{p}(.)-$Laplace type operator, Dirichlet problem, Mountain Pass theorem.

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Solutions of Kirchhoff problem in anisotropic variable exponent spaces

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ABSTRACT

In this paper, we study Kirchhoff problem involving an anisotropic operator with variable exponent Sobolev spaces on a smoothy bounded $\Omega \subset \mathrm{R}^N$. We prove the existence and multiplicity of nontrivial weak solutions by using as main tools the variational approach and Fountain theorem.

We are to analyze the existence of solutions of anisotropic Kirchhoff problem involving $\vec{p}(.)-$Laplace type operator

$$\begin{cases} -M\left(\int_\Omega \sum_{i=1}^{N} A_i\left(x, \partial_{x_i} u\right)dx\right)\sum_{i=1}^{N} \partial_{x_i} a_i\left(x, \partial_{x_i} u\right) = \lambda |u|^{q(x)-2} u, x \in \Omega \\ u = 0, x \in \partial\Omega \end{cases}$$

where $\Omega \subset \mathrm{R}^N (N \geq 3)$ is a bounded with smoothy boundary, λ is a positive parameter and p_i, q are continuous functions on $\overline{\Omega}$ such that $2 \leq p_i(x) < N$ and $q(x) > 1$ for any $x \in \overline{\Omega}$ and $i \in \{1,..., N\}$. $a_i : \Omega x \mathrm{R} \to \mathrm{R}$ is Carathéodory such that $a_i(x,\xi)$ is the continuous derivative with respect to ξ of the mapping $A_i : \Omega x \mathrm{R} \to \mathrm{R}$, $A_i = A_i(x,\xi)$, i.e., $a_i(x,\xi) = \frac{\partial}{\partial \xi} A_i(x,\xi)$. Moreover, Kirchhoff function M is a continuous function and $\vec{p}(.) = \left(p_1(.),..., p_N(.)\right)$ is a vector with variable components.

In the last decade, problems involving variable exponent growth conditions have been extensively studied. The large number of papers studying equations involving variable exponent growth conditions is motivated by the fact that this type of problems can appear in the modelling of stationary thermo-rheological viscous flows

of non-Newtonian fluids, in the electrorheological fluids or elastic mechanics and the mathematical description of the processes filtration of an ideal barotropic gas through a porous medium. At the same time, due to the development of the theory concerning the anisotropic variable exponent Sobolev space, many authors interested problems involving the $\vec{p}(.)-$Laplace type operator see for example [**1,2,3**]. Moreover, a new theory captured attention when it introduced the anisotropic space with variable exponent.

Key Words: Anisotropic variable exponent Lebesgue and Sobolev spaces, variational approach, critical point, existence of solution weak solutions, Fountain theorem.

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Some Ambarzumyan Type Theorems For Bessel Operator On A Finite Interval

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ABSTRACT

Consider the following eigenvalue problem

$$Ly = -y'' + \left\{ q(x) + \left(\left(l(l-1) \right) / \left(x^2 \right) \right) \right\} y = \lambda y,$$

such that

$$y'(1,\lambda) = 0 \text{ and } y'(a,\lambda) = 0,$$

where $l > 1$ is a integral number, $a > 1$ and $q \in L^2[1,a]$. This type problems arise when separation of variables is used for the study of radial Schrödinger operators $\Delta + q(x)$ on a ball in Euclidean space [2].

This eigenvalue problem arises in many fields such as mechanics, physics, electronics, geophysics, meteorology and other branches of sciences and there is a lot of literature on solving this problem Ambarzumyan's paper can be viewed as first and vital reference in the history of inverse spectral problems associated with Sturm-Liouville operators In 1929, he showed that for the Neumann boundary conditions $\left(\theta = \chi = (\pi / 2) \right)$, if the spectrum (collection of the eigenvalues) in Sturm-Liouville problem is $\{ \lambda_n = n^2 : n = 0,1,2,... \}$, then the potential function *q(x)* is zero almost everywhere on *[0,π]*. Ambarzumyan's theorem was extended to the second order differential systems of two dimensions, to Sturm-Liouville differential systems of any dimension, to the Sturm--Liouville equation (which is concerned only with Neumann boundary conditions) with general boundary conditions by imposing an additional condition on the potential function and to the multi-dimensional Dirac operator. In

addition, some different results of Ambarzumyan's theorem have been obtained in later years [1,2,3].

In this study, we deal with an inverse problem for Bessel operator on a finite interval. We present some results of the associated with Ambarzumyan's theorem by using spectrum and nodal points (zeros of eigenfunctions).

Key Words: Spectrum, Ambarzumyan Theorem, Bessel Operator, Nodal Points.

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Some Exact Solutions to the Toda Lattice Equation

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ABSTRACT

A general theory to solve nonlinear partial differential equations does not seem to exist. However, there are certain nonlinear partial differential equations, usually first order in time, for which the corresponding initial value problems can be solved by the inverse scattering transform method. Such nonlinear partial differential equations are sometimes referred to as integrable evolution equations. Some exact solutions to such equations may be available in terms of elementary functions, and such solutions are important to understand nonlinearity better and they may also be useful in testing accuracy of numerical methods to solve such nonlinear partial differential equations.

We have considered the Toda lattice equation

$$\ddot{u}_n = e^{u_{n-1}-u_n} - e^{u_n-u_{n+1}},\ n \in \mathrm{Z}$$

and obtained some exact solutions to that equation in terms of a triplet of constant matrices. To obtain such solutions in terms of a matrix triplet A, B, C we have improved an algorithm as in [1].

Here and overdot indicates the derivative with respect to t and the dagger denotes the matrix adjoint. The subscript, runs over all integers. We worked on to obtain formulas for certain exact solutions to those nonlinear partial differential equations, and such solutions are constructed in terms of a triplet of constant matrices A, B, C whose size are $p \times p,\ p \times 1$ and $1 \times p$ for any positive integer p.

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Key Words: Exact solutions; Toda Lattice equation.

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Some Results on Dynamic Network Flows

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ABSTRACT

Dynamic network flows (some called dynamic network flows) are important features in optimization problems arising in various real applications such as road or air traffic control, production/distribution systems and communication networks. This class of network flows was first introduced by Ford and Fulkerson. Computation of a maximum (minimum) flow, minimum cost flow and shortest path flow problems in a graph have been an important and well-studied problems, both in the fields of computer science and operations research. Many efficient algorithms have beendeveloped to solve this problem.

Traditionally, flows over time are solved in time-expanded networks that contain one copy of the original network for each discrete time point. While this method makes available the whole algorithmic toolbox developed for static flows, its main and often fatal drawback is the enormous size of the time-expanded network. However, the order of such algorithms are pseudo polynomial in time-expanded networks.

In this paper, some new results will be extended for the maximum (minimum) flow problem in dynamic networks. Also, we present polynomial time algorithms for solving them.

Key Words: (Dynamic) Network flows, Combinatorial optimization, Polynomial time algorithms

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Stability Analysis and Numerical Solution of a HIV/AIDS Model

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ABSTRACT

In this study, we consider a delayed HIV/AIDS epidemic model with treatment and vertical transmission introduced in [1-2-3]. This model which incorporates time delay during which all individuals born into the susceptible class, middle-age adult male through sexual intercourse with individuals become infected with HIV/AIDS.

The aim of this study is to examine for both the stability properties and the numerical solution of the differential equations system by providing conditions under which the equilibria of this system is stable[1-6]. On the other hand, the results of this stability analysis of differential equations system are supported by numerical solutions obtained from the well known fourth order Runge-Kutta method. In this case results are found with respect to different value of parameters in the model.

The model has been proposed is not completely new but has been built from previous [1,2,3] by using the rate of the newborns with infection is nearly zero and only middle-aged adult males carry the infection.

The model allows the sexually mature population is divided into four subclasses: the susceptibles (S), the asymptomatic infectives (I), the symptomatic infectives (J) and full-blown AIDS group (A). This study based on some infected individuals to move from the symptomatic phase to the asymptomatic phase; next generation of infected individuals may be infected. We assume that the susceptibles become HIV infected via sexual contacts with infectives. We also assume that a fraction of infected middle-aged adult males, who sustain treatment, joins the asymptomatic infective class while others, who do not sustain treatment, joins AIDS class.

The number of total population is denoted by N(t) at time t. We assume that μK is the recruitment rate of the population, μ is the death rate, c is the average number of contacts of an individual per unit of time, β and $b\beta$ are the probability of disease transmission per contact by an infective in the first stage and in the second stage, k_1 and k_2 are transfer rate from asymptomatic phase to the symptomatic phase and from the symptomatic phase to the AIDS case. α is transformation rate from the symptomatic phase to asymptomatic phase, d is the disease-related death rate of the AIDS cases, γ the rate of the middle-aged adult males become HIV infected via sexual contacts, p is the fraction of infected adult males joining the asymptomatic infective class. Also we assume remaining part $(1-p)$ of the infected adult males joins the AIDS class $(0 \le p \le 1)$. With these assumptions, we obtain the following system:

$$\begin{cases} \frac{dS}{dt} = \mu K - c\beta[I(t) + bJ(t)]S(t) - \mu S(t) \\ \frac{dI}{dt} = c\beta[I(t) + bJ(t)]S(t) - (\mu + k_1)I(t) + \alpha J(t) + \gamma p I(t-\tau)e^{-\mu\tau} \\ \frac{dJ}{dt} = k_1 I(t) - (\mu + k_2 + \alpha)J(t) \\ \frac{dA}{dt} = k_2 J(t) - (\mu + d)A(t) + (1-p)\gamma I(t-\tau)e^{-\mu\tau} \end{cases}$$

Mathematical analysis shows that the global dynamics of the spread of the HIV/AIDS are completely determined by the basic reproduction number R0 for this model. If R0<1 then disease free equilibrium is globally asymptotically stable, whereas the unique infected equilibrium is globally asymptotically stable if R0>1.

Key Words: HIV/AIDS Epidemic model, Basic reproduction number, Stability, Numerical method.

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Stability Analysis of Combustion Waves in Porous Media

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ABSTRACT

We study combustion waves that arise in a simplified, one-dimensional model of enhanced oil recovery using air injection. Identifying and understanding these waves help to maximize oil recovery. The existence of combustion waves for the system of three partial differential equations that give temperature, oxygen, and fuel balance laws have been studied extensively and proved by using phase plane analysis. Studying stability starts by determining the spectrum of the operator which we obtain from the linearization of the partial differential equation system about traveling wave. A weight function is required to stabilize the essential spectrum for certain waves. For the discrete spectrum, we perform a numerical computation of the Evans function for certain waves to show that there is no unstable discrete spectrum. The Evans function is an analytic function that can be used to locate the discrete spectrum of the differential operator.

One problem during the stability analysis is that the system is partially parabolic, the linearized operator has a vertical line in its spectrum so it is not a sectorial operator. This difficulty is typical for systems with no diffusion in some equations. We use recent results about partially parabolic systems to overcome this issue.

We also identify all possible generic wave sequences that solve boundary value problems. In addition, numerical simulations are presented for the generic wave sequences that we expect to occur for large time.

Key Words: Traveling wave, combustion wave, porous media, stability, spectrum.

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Stability Conditions Of A Class Of Linear Retarded Differential Systems

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ABSTRACT

Retarded differential equations are a type of differential equation in which the derivative of the unknown function at a certain time is given in terms of the values of the function at previous times. They are also called time-delay systems, delay differential equations or differential-difference equations. The characteristic equations of retarded differential equations are polynomials. These polynomials are exponential polynomials or quasi-polynomials. On the other hand, their stability has a wide range of applications in science and engineering. We know that for the retarded delay equations, an equation is asymptotically stable if and only if all roots of the associated characteristic equation have negative real parts or all solutions of the equation having limit zero as $t\to\infty$. Recently, many authors have studied the asymptotic stability of linear retarded differential equations or systems. System (1) is a class of linear retarded differential systems, if the highest derivative term does not have a delay. In this study, we give some new necessary and sufficient conditions for the asymptotic stability of a linear retarded differential system with two delays

$$x'(t) + (1-a)x(t) + A(x(t-k) + x(t-l)) = 0, \qquad t \geq 0, \tag{1}$$

where $a<1$ is a real number, A is a 2×2 real constant matrix, and k, l are positive numbers such that $k>l$.

Key Words: Differential equations, Differential System, Characteristic equation, Asymptotic stability

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Stability Criterion For Difference Equations Including Generalized Difference Operator

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ABSTRACT

Stability of solutions of linear difference equations requires analysis of root of characteristic equation of difference equations. In this study some necessary and sufficient conditions are given for the stability of some class of difference equations involving generalized difference operator. In our study firstly we will consider the asymptotic stability of the zero solution of the difference equation involving generalized difference of the form

$$\Delta_{l,a}^{m} y(n) + r\Delta_{l,a} y(n) + sy(n) = 0 \qquad (1)$$

with initial conditions

$$y(i) = \varphi_i,\ i = 0,1,2,\dots,ml-1$$

and later we will consider the asymptotic stability of the zero solution of the delay difference equation involving generalized difference of the form

$$\Delta_{l,a}^{m} y(n-l) + r\Delta_{l,a} y(n) + sy(n-l) = 0 \qquad (2)$$

with initial conditions

$$y(i) = \varphi_i,\ i = -l, -l+1, \dots, (m-1)l-1$$

where $a, r, s \in \mathbb{R}$ and $l, m, n \in \mathbb{N}$. The difference operator Δ, the generalized difference operator $\Delta_{l,a}$ and $\Delta_{l,a}^{m} y(n)$ are defined as

$$\Delta x(n) = x(n+1) - x(n)\ ,\ n \in \mathbb{N},$$

$$\Delta_{l,a} y(n) = y(n+l) - ay(n),\ l, n \in \mathbb{N},\ a \in \mathbb{R}\ ,$$

$$\Delta_{l,a}^{m} y(n) = \Delta_{l,a}(\Delta_{l,a}^{m-1} y(n)),\ l, n, m \in \mathbb{N},\ a \in \mathbb{R}$$

respectively. To investigate the stability of equation (1) we reduce this equations to the constant coefficient linear equation using the expansion for $\Delta_{l,a}^{m} y(n)0$. Schur-Cohn criteria is used to determine the roots of any polynomial to be inside the unit disk. The coefficient of corresponding characteristic polynomial of (1) are used in Schur-Cohn criteria to give necessary and sufficient conditions for the stability of equation (1). For equation (2) similar argument is done and some examples are given to verify the results obtained.

Key Words: Generalized Difference Operator, Schur-Cohn Criteria, Stability.

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Statistical Analysis of Wind Speed Data with Weibull and Gamma Distributions

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ABSTRACT

Energy consumption increases along with the technological developments and population growth. Energy requirement is an important criteria in development of countries [1]. In almost every aspect of daily life, fossil energy fuels are used. However, the increase in the use of fossil fuels poses a threat for people by way of air pollution, climate change and carbon emission [2]. In order to eliminate these problems, alternative energy resources such as wind, solar and geothermal energy are recommended. Unlike fossil fuels, most of these energy resources are limited. These energy resources reduce carbon emission [3, 4]. There are two important factors in obtaining wind power. First one is choosing the location where the wind system will be installed and the second one is determining the wind speed characteristics statistically. Determining the distribution of wind speed is quite effective in calculating the wind force [5-8].

The data used in this study is the monthly average wind speed values between 2012 and 2016 and it was officially obtained from Directorate of Bitlis Meteorology. Weibull and Gamma distribution functions were used in evaluation of the wind speed data with the program written in Matlab R2009a. Parameter estimations for distributions were obtained using Maximum Likelihood method. Kolmogorov-Smirnov Goodness of Fit test, coefficient of determination (R^2) and root-mean-square error (RMSE) were used for determining the best fitting model. In addition, wind speed values were obtained depending on the parameters of the distributions used. The results are calculated monthly and annually. Thus, a preliminary study was conducted to determine the wind speed potential in Bitlis.

When the obtained results are examined, it was seen that 2015 has the lowest wind speed with 2.9 m/s value and 2012 has the highest wind speed with 3.2 m/s value. Seasonally, it was determined that the autumn has the lowest (2.9 m/s) and spring has the highest (3.30 m/s) average wind speed. As a result of obtaining parameter estimations and average wind speeds for all two distributions, it was determined that average wind speed estimations are similar, but Gamma distribution has the lowest standard deviation with the average wind speed value (0.15 m/s) in August. In the goodness of fit test of average wind speed data for fitness to Weibull and Gamma distributions, Kolmogorov-Simirnov Goodness of Fit Test is used. Gamma distribution yielded lower values compared to the other distribution (0.1053). When the distributions are compared in terms of coefficient of determination (R^2) and root-mean-square error (RMSE) criteria, highest R^2 and lowest RMSE values are obtained for Gamma distribution.

As a conclusion, it is recommended that Gamma distribution is used in modelling the wind speed of Bitlis between 2012 and 2016.

Key Words: Maximum likelihood, Root mean square error, Wind speed distribution.

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The Analysis of AES Cryptography Algorithm by using GPU based High Performance Computing Clusters

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ABSTRACT

High performance computing clusters become more efficient for solving computational problems by accelerating general purpose GPU systems. AES is one of the well known and robust cryptography algorithm. Thus, breaking AES keys in timely manner is a non-polynomial hard problem. GPU systems offer CUDA parallel programming model to process complex problems like AES.

In this study, we programmed AES in GPU based HPC cluster for computation. We compare execution time for flat computers, parallel CPUs and parallel GPU systems. We are expecting better result from CUDA coded AES algorithm.

In conclusion, when computationally intensive problems considered, GPU based systems performs well.

Key Words: AES Cryptography, High Performance Computing, CUDA.

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The Direct and Inverse Spectral Problem for Sturm-Liouville Operator with Discontinuous Coefficient

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ABSTRACT

We consider the problem of heat conduction in a rod which is composed of materials with different densities. At the initial time, let the temperature be given arbitrarily. Also, let the temperature at the endpoints of the rod be different from zero. In this case the heat flow in non-homogenous rod is expressed with the following boundary value problem:

$$\rho(x)\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+q(x)u, \quad 0<x<\pi,\ t>0$$

$$u\big|_{x=0}=0, \quad \left.\frac{\partial u}{\partial x}\right|_{x=\pi}=0, \quad t>0$$

where $u(x,t)$ denotes the temperature at the point x of the rod at the time t; $\rho(x)$ defines the density of the material and is a piecewise-continuous function. Applying the method of separation of variables to this problem, we get the spectral problem for Sturm-Liouville equation:

$$-y''+q(x)y=\lambda^2\rho(x)y, \quad 0<x<\pi, \tag{1}$$

$$y(0)=y'(\pi)=0. \tag{2}$$

The boundary value problem (1)-(2) is encountered also when the separation of variables method is applied to solve the vibrating wire problem [2]. The spectral problems with discontinuous coefficient on the bounded interval were investigated in [1, 3, 4, 5, 7]. For $\rho(x)\equiv 1$, the certain solutions of direct and inverse scattering problem of spectral analysis for Sturm-Liouville problem were given in [6, 8].

In [1] it is proved that the solution $\varphi(x,\lambda)$ of the equation (1) with initial conditions $\varphi(0,\lambda)=1$ and $\varphi'(0,\lambda)=0$ can be represented as

$$\varphi(x,\lambda)=\varphi_0(x,\lambda)+\int_0^{\mu^+(x)} A(x,t)\cos\lambda t\, dt\,, \tag{3}$$

where $A(x,t)$ belongs to the space $L_2(0,\pi)$ for each fixed $x\in[0,\pi]$ and is related to the coefficient $q(x)$ in equation (1) by the formula:

$$\frac{d}{dx}A(x,\mu^+(x))=\frac{1}{4\sqrt{\rho(x)}}\left(1+\frac{1}{\sqrt{\rho(x)}}\right)q(x);$$

$$\varphi_0(x,\lambda)=\frac{1}{2}\left(1+\frac{1}{\sqrt{\rho(x)}}\right)\cos\lambda\mu^+(x)+\frac{1}{2}\left(1-\frac{1}{\sqrt{\rho(x)}}\right)\cos\lambda\mu^-(x)$$

is the solution of (1) when $q(x)\equiv 0$; and

$\mu^{\pm}(x)=\pm\, x\,\sqrt{\rho(x)}+a\left(1\mp\sqrt{\rho(x)}\right)$.

We give our main result in the following theorem.

Theorem 1: For each fixed $x\in[0,\pi]$, the kernel $A(x,t)$ from the representation (3) satisfies the following linear functional integral equation

$$\frac{2}{1+\sqrt{\rho(t)}}\,A(x,\,\mu^+(t))+\frac{1-\sqrt{\rho(2a-t)}}{1+\sqrt{\rho(2a-t)}}\,A(x,\,2a-t)+$$
$$F(x,t)+\int_0^{\mu^+(x)} A(x,\,\xi)\,F_0(\xi,t)\,d\xi=0,\qquad 0<t<x$$

where

$$F_0(x,t)=\sum_{n=1}^{\infty}\left(\frac{\varphi_0(t,\,\lambda_n)\cos\lambda_n x}{\alpha_n}-\frac{\varphi_0(t,\,\lambda_n^0)\cos\lambda_n^0\, x}{\alpha_n^0}\right);$$

$$F(x,t)=\frac{1}{2}\left(1+\frac{1}{\sqrt{\rho(x)}}\right)F_0(\mu^+(x),\,t)+\frac{1}{2}\left(1-\frac{1}{\sqrt{\rho(x)}}\right)F_0(\mu^-(x),\,t);$$

$\{\lambda_n^0\}^2$ are eigenvalues, and α_n^0 are norming constants of the boundary value problem (1)-(2) when $q(x)\equiv 0$.

Key Words: Discontinuous Sturm-Liouville operator, Main equation, Inverse problem.

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The Efficiency Investigation of Turkish Manufacturing Industry by Multi-Period Two-Stage Data Envelopment Analysis

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ABSTRACT

Competition, the effects of which can be perceived deeply even in small market economies as a result of disappearing economic boundaries, can be turned into advantage by using sources efficiently and productively. Business enterprises need some kind of performance measurement systems to adapt themselves to increasing competition and changing environmental conditions, taking right actions with the right data by making accurate strategies and deciding the stage of the targets which is constructed according to their goals in the application.

Data Envelopment Analysis (DEA) is a statistical technique involving mathematical programming approach and applied to evaluate the relative efficiency of decision making units (DMU). This analysis is also a nonparametric measurement technique used to calculate the efficiency levels of companies, based on the determination of a large number of input and output variables. Recent years have been witnessing improvements in this technique. With these improvements, two-stage measurement methods have been developed to detect events or inefficiencies in two stages, rather than single input-output or multi input-output models.

The efficiency of decision processes, which can be divided into two stages has been measured for the whole process as well as for each stage independently by using the conventional data envelopment analysis (DEA) methodology to identify the causes of inefficiency. Conventional two-stage Data Envelopment Analysis (DEA) models measure the overall performance of a production system composed of two stages in a specified period of time, where variations in different periods are ignored.

In this study, it was aimed to measure the operational and financial activities of 90 companies in the manufacturing industry sector traded in the Stock Exchange Istanbul (BIST) with a multi-period two-stage data envelopment analysis (DEA). The efficiency of the companies was compared using financial statement data of the years 2013-2015 published in Public Disclosure Platform. The first stage inputs of this study are the number of employees, total assets and capital receipts, the outputs of first stage are revenue and operating expenses. The second stage inputs are revenue and operating expenses, the second stage outputs are net profit margin, operating profitability, return on assets and profitability of equity.

According to the analysis results, in the aggregated model including 2013-2015 period, 5 of firms in the first stage, 6 of the firms in the second stage and only 1 of the firms in the overall stage are efficient. On the other hand, in the multi-period model, none of the firms are efficient in both stages.

Key Words: Efficiency, data envelopment analysis, two-stage system.

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The Exact Solutions of Nonlinear Partial Differential Equation Using Four Different Techniques

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ABSTRACT

Many physical systems have a chaotic structure due to its nature. It is usually very difficult to understand these chaotic structures which present the non-linear differential equations. One of the these types of equations is Korteweg-de Vries (KdV) equation [1] which plays an important role in mathematical physics. The other one is the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli (BLMP) equation which is a different form of the KdV equation given bellow [2] :

$$U_{yt} + U_{xxxy} - 3U_{xx}U_y - 3U_xU_{xy} = 0$$

where $U = U(x, y, t), U_t = \frac{\partial U}{\partial t}, U_x = \frac{\partial U}{\partial x}, U_y = \frac{\partial U}{\partial y}, ...,$ etc. The solution of Cauchy problem for the BLMP equation was improved by using an inverse scattering scheme in Boiti et al. [3,4]. Recently, some exact solutions of the BLMPE equation were obtained by Arbabi and Najafi using the semi-inverse variational principle given in Ref. [5].

In this study, our aim is to present the exact solutions of the non-linear (2+1)-dimensional BLMP equation. Therefore, we have found the exact solutions of the BLMP nonlinear partial differential equation using four different techniques, direct integration, (G'/G)-expansion method [6], different form of the (G'/G)-expansion method [7], and two-variable (G'/G, 1/G)-expansion method [8]. Our solutions are reduced to the well-known solutions found in literature assigning the some special values to the constants appeared in the analytic solutions. Moreover, we have obtained, for the first time, the new exact solutions of the equation mentioned above.

Key Words: Boiti-Leon-Manna-Pempinelli equation, exact solution, (G'/G)-expansion methods.

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The impact of the Allee effect and global and local behaviour of the a nonlinear discrete-time population model

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ABSTRACT

In this article, we analyze the local and global dynamic behavior discrete-time population model analytically with and without Allee effect; and compare the results obtained. Also, we examine the impact of the Allee effect on both local behavior and global behavior of the population dynamic model. The theoretical results are confirmed with numerical simulation by using Maple and Matlab program.

To examine the dynamics of the population requires stability analysis. For local asymptotic stability, solutions must approach an equilibrium point under initial conditions close to the equilibrium point. In global asymptotic stability, solutions must approach to an equilibrium point under all initial conditions.

Former studies demonstrate that the Allee effects play an important role in the stability analysis of equilibrium points of a population dynamics model. The positive equilibrium point of the model which is subject to an Allee effect can become either destabilization or stabilization. Namely, the local stability of a positive equilibrium point can be changed from stable case to unstable case or vice versa. It is also possible that even if the model is stable at an equilibrium point, to reach its equilibrium point may take much longer time. This case has been used to mean "Allee effect decreases the local stability of the equilibrium point".

Consequently, this paper focused on the global and local stability analysis of the first-order discrete population models with and without the Allee effect. Firstly, local asymptotic stability conditions were investigated for the equilibrium points of both models. Secondly, global stability of the equilibrium points of the models was also evaluated. Finally, we compared the global and local stability of the equilibrium points of this two models. We conclude that Allee effect decreases the local stability and global stability of the equilibrium points of the population model.

Key Words: Stability, Allee effect, Discrete-time model

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The Mathematical Modeling of Pathogen and Host by Of Immune System's Holling Type 2 Response

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ABSTRACT

In this study, a mathematical model has been proposed to examine the dynamics between the pathogen infected to host and the host's immune system response given due this pathogen. The model is a system composed of two differential equations which represents the cells of host's immune system and the pathogen's burden by considering basic mechanisms of these. The model is

$$\begin{aligned} \frac{dS}{dt} &= \beta_S S\left(1-\frac{S}{K}\right) - \mu_s S - \frac{c_S S}{1+a_S S}B \\ \frac{dB}{dt} &= \frac{kS}{1+k_m S}B - \delta B + h \qquad (1) \\ &\beta_S, K, \mu_s, c_S, a_S, k, k_m, \delta, h > 0 \end{aligned}$$

where let us denote by S the population sizes of pathogen at time t, respectively and by B population sizes of immune cells (T-cells or Anti cells) at time t. The model proposed in this study is an improved version of model in [7]. In this sense, It is accepted that pathogen (S) reproduct by logistics rules and has rate of the natural death.

Qualitative analysis for system (1) found out the positive equilibrium points being possible in addition to the infection-free equilibrium point. The stability and bifurcation analysis of this equilibrium points was performed. Our mathematical model used Holling type-2 response is found as a useful tool that can be utilized in predicting the timing and expansion of infection and possible reinfection processes in an individual as depend on the parameters used. Also, the results of analysis were supported by numerical simulations.

Key Words: Ordinary differential equations systems, equilibrium point, immune system, holling response.

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The Positive Solutions Of The Exponential Type Difference Equations

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ABSTRACT

In recent years, studying of nonlinear difference equations have gained a great importance. Most of the recent applications of these equations have appeared in many scientific areas such as biology, physics, engineering, ecology and economics. Recently, the behavior of positive solutions for exponential type difference equation has attracted great attention of many authors. That is because exponential type difference equations have many applications in population dynamics. In [1], in particular, El-Metwally et al. studied the boundedness, the asymptotic behavior, the periodic character and global stability of difference equation

$$x_{n+1} = \alpha + \beta x_{n-1} e^{-x_n}, \ n = 0,1,2,...$$

where the parameters α and β are positive numbers and the initial conditions x_{-1} and x_0 are arbitrary non-negative numbers. Aboutaleb et al. [2] investigated the global asymptotic stability of the recursive sequence

$$x_{n+1} = \frac{\alpha - \beta x_n}{\gamma + x_{n-1}}, \ n = 0,1,2,...$$

where the parameters α, β and γ are non-negative numbers. Ozturk et al. [3] investigated the convergence, the boundedness and the periodic character of the positive solutions of the difference equation

$$y_{n+1} = \frac{\alpha + \beta e^{-y_n}}{\gamma + y_{n-1}}, \ n = 0,1,2,...$$

where the parameters α , β and γ are positive numbers and the initial conditions y_{-1} and y_0 are arbitrary non-negative numbers.

Motivated by all above mentioned work, in this study we investigate the convergence, the boundedness and the periodic character of the positive solutions of the difference equation

$$x_{n+1} = \frac{\alpha + \beta e^{-x_n}}{\gamma + x_{n-2}}, \quad n = 0,1,2,...$$

where the parameters α , β and γ are positive real numbers and the initial conditions x_{-2} , x_{-1} and x_0 are arbitrary non-negative real numbers.

Key Words: Difference equations, Boundedness, Periodicity, Asymptotic Behavior

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The Solution of Multiplicative Non-Homogeneous Linear Differential Equations

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ABSTRACT

Michael Grossman and Robert Katz indicated in their work [1] that infinitely many calculi can be generated independently. They also introduced a few of them namely geometric, anageometric, biogeometric and etc. This calculus also called non-Newtonian calculus. In non-Newtonian calculus, differentiation and integration are based on non-Newtonian operations instead of classical operations.

Geometric calculus, which depends on the division and multiplication operations instead of subtraction and addition operations for calculating differential and integral, was later named as multiplicative calculus by D. Stanley [2]. Then, some study with regard to multiplicative calculus is given by D. Campell [3]. The theoretical background of multiplicative calculus was given by Bashirov et al in [4]. Besides, solutions of homogeneous differential equations with constant exponentials in multiplicative analysis are obtained in [5]. Recent studies on multiplicative analysis [6-11] showed that some science and engineering problems can be solved in a more practical way by using this analysis.

In this study, taking particular solutions of non-homogeneous differential equations with constant coefficients in classical analysis as a basis, particular solutions of non-homogeneous differential equations in multiplicative analysis are obtained by using three methods namely operator method, the method of undetermined exponentials and the method of variation of parameters with constant exponentials.

Key Words: Multiplicative non-homogeneous linear differential equations, multiplicative derivative, the operator method, the method of undetermined exponentials, the method of variation of parameters with constant exponentials.

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Three Dimensional Flight of Multirotor Formations

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ABSTRACT

In this paper, the main aim is modelling and control of cohesive motion of autonomous unmanned aerial vehicle (UAV) groups in the three dimensions considering real flight models. Cohesive motion is a team work that gives lots of opportunities such as decreasing energy consumption and time. So the formation flight concept is currently so popular and have many scientific and operational application fields. Decision of the vehicle formation shape is related with keeping distances constant between each vehicle should be the most convenient way for communication and preventing from crashes. This phenomenon provides tracking of the desired orbit or path during the motion. For these requirements reveal some problems, which are motion forms and shapes, hierarchy of communication between agents, controlling and designing of the tracking orbit, in the formation flight [1]. For the sake of simplicity, first we use first order velocity dynamics, then we extend the modelling work to full linear model of the quadrotor vehicle considering both lateral and longitudinal dynamics [2]. In this study, we also represent our cohesive cooperative flight in the leader-follower structure. Finally, control algorithms, vehicle dynamics are modelled within the MATLAB/Simulink environment in order for testing and calibrating the performance of the formation flight.

Key Words: Cohesive Motion, Cooperative Control, Leader-Follower based Formation, Autonomous Vehicles, Unmanned Aerial Vehicles.

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Trigonometric B-spline Galerkin Method for the Numerical Solution of the KdV Equation

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ABSTRACT

We consider the Korteweg-de Vries (KdV) equation

$$u_t + \varepsilon u u_x + \mu u_{xxx} = 0, \tag{10}$$

where $u = u(x,t)$ is an unknown function of two independent variables. In the equation, ε and μ are positive constants and the subscripts x and t denote space and time derivatives, respectively. Appropriate boundary conditions over the timeinterval $[0,T]$ will be selected from the boundary conditions

$$\begin{aligned} &u(a,t) = u(b,t) = 0, \\ &u_x(a,t) = u_x(b,t) = 0, \ t \geq 0 \\ &u_{xx}(a,t) = u_{xx}(b,t) = 0 \end{aligned} \tag{11}$$

to model the analytical conditions that $u \to \infty$ as $x \to \pm\infty$. The initial condition over the space interval $[a,b]$ is given as follows

$$u(x,0) = f(x).$$

KDV equation is a fundamental mathematical model for describing the theory of water waves in shallow channels [1]. These water waves are known as solitons, which are stable and do not disperse with time and they are not deformed after collision with other solitons. The terms uu_x and u_{xxx} in the KdV equation represent the nonlinear convection and the dispersion effects, respectively and soliton waves are generated as a result of the balance between these terms. Various numerical methods, including finite element, differential quadrature, spline approximation, have

been used for the KdV equation by many researcher [2-6] so far. In this study, KdV equation is solved numerically by using the trigonometric quintic B-spline Galerkin finite element method. The Crank–Nicolson scheme has been used for the time integration and trigonometric quintic B-spline functions have been used for the space integration. The accuracy of the method is illustrated by studying single soliton propagation and the interaction of two solitons. We conclude from the test problems that the proposed numerical method is a useful approach for numerical solution of the KdV equation.

Key Words: Korteweg-de Vries, Soliton, Trigonometric Quintic B-spline

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Understanding the Impact of Linear, Quadratic and Exponential Functions on High-Performance Computing Clusters using Serial and Parallel Computations

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ABSTRACT

Parallel computing allows us to carry out many calculations simultaneously. However, problems for parallel computing should be split into smaller ones to be solved at the same time whereas serial computing does not require splitting. The main reasons to consider parallel computing might be counted as: (i) to save time by distributing tasks and executing these simultaneously; (ii) solve big data problems by distributing data; (iii) to take advantage of computer resources, and (iv) to scale up to clusters and cloud computing. Systematically, computationally intensive programs and models might be run on clusters, clouds, and grids since they offer multicore processors. However, it is critical to understand the impact of linear, quadratic, and exponential functions on clusters, clouds, and grids since they differ in terms of their forms. A linear function take the form $y = mx + b$ and change at a constant rate per unit interval while an exponential function is in the form $y = a^x$ and changes by a common ratio over equal intervals. On the other hand, a quadratic function take the form $y=x^2 + mx – n$ and does not change constantly. In this study, we construct, compare, and interpret linear, quadratic, and exponential functions using a high-performance computing cluster in terms of serial and parallel computation to understand the impact of multicore processors. MATLAB Distributed Computing Server was used to execute linear, quadratic, and exponential functions in a High-Performance Computing Cluster (H-HPCC) in Harran University.

The H-HPCC allows scientists and engineers to solve complex, compute-intensive and data intensive problems with its high network performance, fast storage, large amounts of memory and very high compute capabilities.

Key Words: high-performance computing, exponential, linear and quadratic computation, parallel programming

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With Kendall Distribution Function Archimedean Copula Parameter Estimation

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ABSTRACT

In the literature, up to now, it is common that for Gumbel, Clayton and Frank calculated Kendall Distribution function $K(u)$ and to the extent those applications have been made. Kendall distribution functions show stochastic orderings of random vectors. The aid of Kendall distribution function is selected suitable copula function for using data set. For dependence structures of the data sets calculated Kendall Tau and Spearman Rho values which are nonparametric. Based on this method, parameters of copula are obtained. In this paper, we are made Kendall Distribution function which obtained with the help of generator function of Archimedean copula calculation for Ali Mikhail Haq and Joe and in relation that simulation study. We generated dependent generalized pareto distribution (gp(3,3,3)). For dependency between these variables, we used Archimedean copula. In connection with this, we define basic properties of copulas and nonparametric methods Kendall Tau, Spearman Rho are given. In this study, to explain the relationship between the variables, five Archimedean copula families are used; Gumbel, Clayton, Frank Joe and Ali Mikhail Haq. We are obtained nonparametric estimation of parameters of these copula families with the help of Kendall Tau. With Kendall distribution function values, the suitable Archimedean copula family for this data set.

Key Words: Copula Function, Archimedean Copula, Kendall Distribution Function

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3-DOF Testbench Validating Real-Time Control Algorithms Designed for Aerial and Spatial Operations

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ABSTRACT

This paper is aiming to detail designing of aerial testbed compatible for all aerospace control operations in three degree of freedom. Validating and verifying the sensitivity of the aerial and spatial control implementations are extremely difficult [1]. Aforementioned in-house testbed design is capable of implementing all rotational and hover actions for aerospace operations [2,3]. The main frame includes rotatable two rings working with gyroscopic principles. The unmanned aerial vehicle (UAV) is fitting on specifically designed additional surface mounted over innermost third ring.

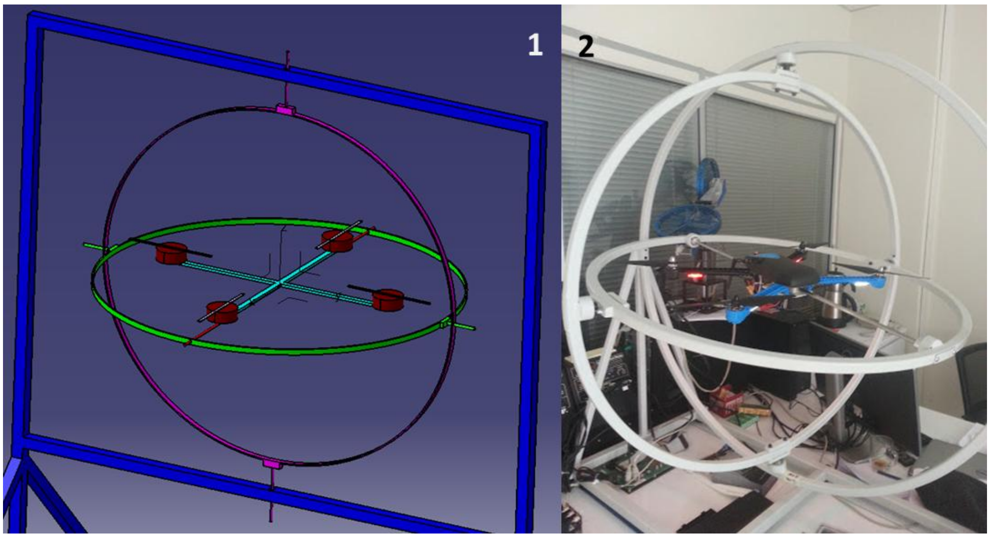

Figure 1: 3-DOF Aerial Testbench **1)** Catia Drawing, **2)** Testbed Calibrating Quadcopter Control Gains

The outer ring, middle, and innermost ring is used for testing and calibrating real-time controller gains for yaw, roll, and pitch actions respectively. The gimballed platform has orthogonal rings covers three dimensional rotations for the UAV over fixed frame. The main frame is the heaviest part fixing the whole platform on the ground. In this study, quadcopter is mounted on the testbench as it is shown in Figure 1 implementing real-time control algorithms and tuning the control gains and maps. We take the advantage of Hardware in the Loop (HiL) design and V-Cycle methodology as detailed in [4]. We are also capable of test the functional software in the Processor in the Loop cycles with the help of Pixhawk PX4 controllers [5]. As a result, we are able to tune the vehicle model parameters and maps, obtain actuator dynamics, calibrate attitude control gains by means of real-time testbed. In conclusion, this platform enables us model based calibration and control design of aerial vehicles.

Key Words: Experimental Testbed, Aerial and Spatial Control Design, Hardware in the Loop Testing, Real-Time Simulation

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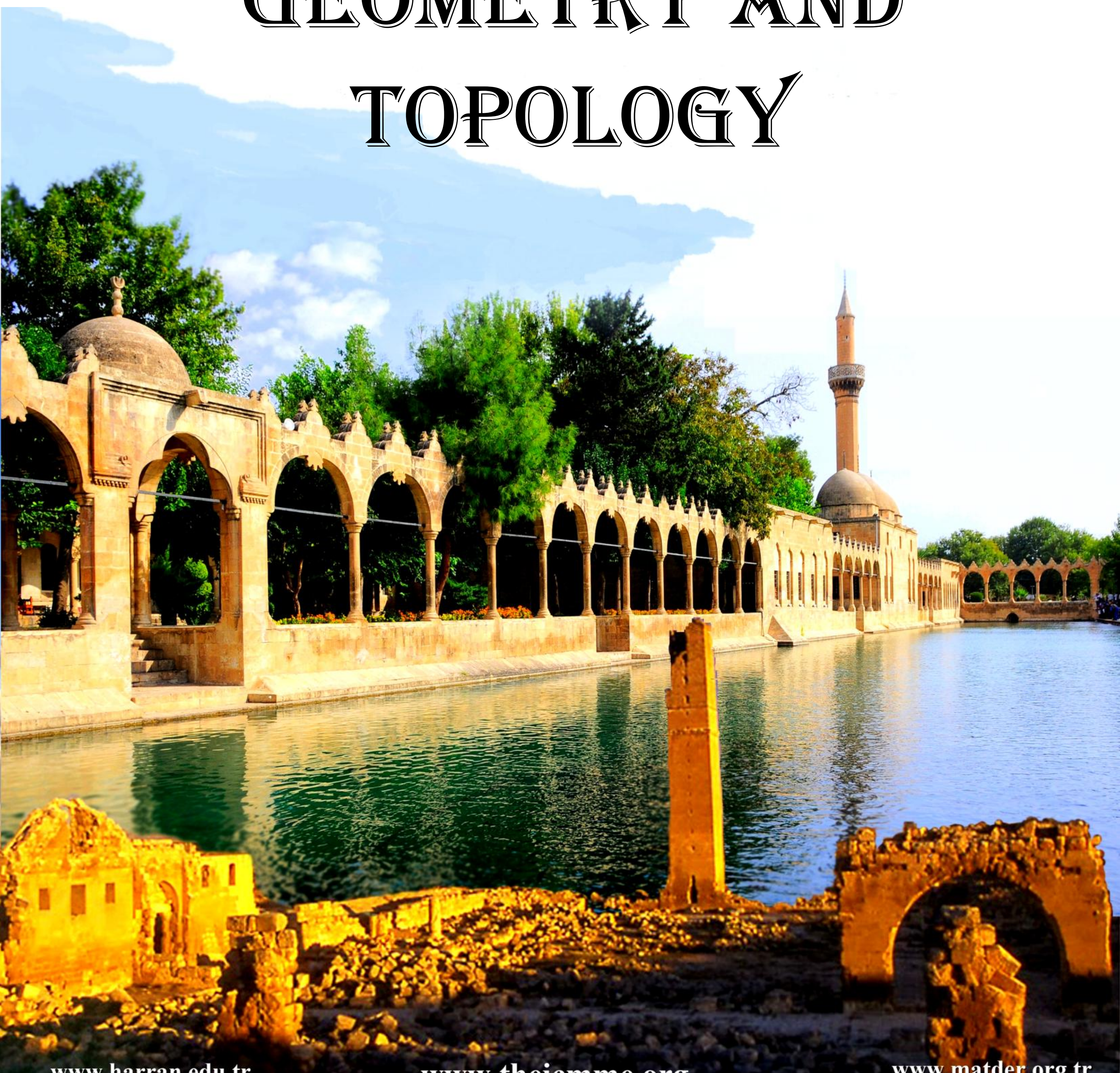

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A Fixed Fuzzy Point Theorem in Hausdorff Non-Archimedean Fuzzy Metric Spaces

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ABSTRACT

In 2004 Rodriguez-Lopez and Romaguera [4] introduced Hausdorff fuzzy metric on the set of the non-empty compact subsets of a given fuzzy metric space. Later, several authors proved some fixed point theorems for multivalued maps in fuzzy metric spaces. Heilpern [3] first introduced the concept of fuzzy mappings and proved a fixed point theorem for fuzzy contraction mappings. Many authors extended the result of Heilpern and proved fixed fuzzy point theorems (see [2,7,8]). Recently, Phiangsungnoen et al. [6] have studied fuzzy fixed point theory for fuzzy mappings in Hausdorff fuzzy metric spaces and then, Abbas et al. [1] have obtained fixed fuzzy points of fuzzy mappings in Hausdorff fuzzy metric spaces under generalized contractive conditions.

On the other hand, Mihet [5] introduced the concept of non-Archimedean fuzzy metric space and proved a fixed point theorem in this space for fuzzy φ-contractive mappings which enlarges the class of fuzzy contractive mappings.

In this study, we introduce Hausdorff non-Archimedean fuzzy metric space using fuzzy sets and establish the existence of fixed fuzzy point theorem for fuzzy mappings in Hausdorff non- Achimedean fuzzy metric space. Also we give an auxiliary example to support our theorem. As an application, we obtain a common fixed fuzzy point for a hybrid fuzzy pair. Our results can be applied in theoretical computer science.

Key Words: non-Archimedean fuzzy metric, fixed fuzzy point, Hausdorff metric.

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A Generalization of the Dual Fibonacci Quaternions

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ABSTRACT

Many mathematicians studied several problems and results involving the Fibonacci sequences. Fibonacci numbers are characterized by the fact that every number after the first two numbers is the sum of the two proceeding numbers,that is, the numbers have the form

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55,…

The real quaternions are the number system in 4-dimensional vector space which is extended by the complex numbers. William Rowan Hamilton in 1843 first introduced the number system. Then Clifford generalized the quaternions to bi-quaternions with his work in 1873 and provided to literature a useful tool for the analysis complex number. Generally, there has three types of quaternions called real, complex and dual quaternions. The family of dual quaternions is a Clifford algebra that can be used to represent spatial rigid body displacement in mathematics. Rigid motions in 3-dimensional space can be represented by dual quaternions of unit length this fact is used a theoretical kinematic and in applications to 3-dimensional computer graphics, robotics and computer vision.

The purpose of the paper is to construct a new representation of dual quaternions called bi-periodic dual Fibonacci quaternions. These quaternions are originated as a generalization of the known quaternions in literature such as dual Fibonacci quaternions, dual Pell quaternions and dual k-Fibonacci quaternions. Then, we give generating function, Binet formula and Catalan's identity in terms of these quaternions.

Key Words: Dual Fibonacci Quaternions, bi-periodic Fibonacci Quaternions, bi-periodic Dual Fibonacci Quaternions.

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A New Approach to Inextensible Flows of Curves with Ribbon Frame

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ABSTRACT

Developable surfaces are commonly used when manufacturing with materials that do not stretch or tear. commonly used when manufacturing with materials that do not stretch or tear. Any process manipulating fabric, paper, leather, sheet metal or plywood will bene.t from developable surface modeling techniques since these materials admit little distortion. In typical setups, the patterns of the product are first designed by trained individuals, with a computer performing a bending simulation to help forecast the manufactured result. The product is then fabricated by cutting out the patterns of the surface from a flat sheet of the respective material and bending these planar patterns to form the desired shape. Applications include modeling ship hulls, buildings, airplane wings, garments, ducts, automobile parts. Physically, inextensible curve and surface flows are characterized by the absence of any strain energy induced from themotion. Kwon investigated inextensible flows of curves and developable surfaces in R^3, [7,8].

A ribbon is a surface swept out by a line segment turning as it moves along a central curve. For narrow magnetic ribbons, for which the length of the line segment is much less than the length of the curve, the anisotropy induced by the magnetostatic interaction is biaxial, with hard axis normal to the ribbon and easy axis along the central curve. The micromagnetic energy of a narrow ribbon reduces to that of a one dimensional ferromagnetic wire, but with curvature, torsion and local anisotropy modified by the rate of turning. These general results are applied to two examples, namely a helicoid ribbon, for which the central curve is a straight line, and a Möbius ribbon, for which the central curve is a circle about which the line segment

executes a 180 twist. A repetitive crystal-like pattern is spontaneously formed upon the twisting of straight ribbons. Bohr and Markvorsen [3] gave a general description of developable ribbons using a ruled procedure where ribbons are uniquely described by two generating functions. This construction defines a differentiable frame, the ribbon frame, which does not have singular points, whereby we avoid the shortcomings of the Frenet–Serret frame. In this paper, we investigate inextensible flows of curves according to Ribbon frame in E^3. Using the Ribbon frame of the given curve, we present partial differential equations. We give some characterizations for curvatures of a curve in E^3.

Key Words: inextensible flows of curves, Euclidean space, Ribbon frame.

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A new characterization of Constant Angle Surface in Minkowski 3-space

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ABSTRACT

A submanifold M in Minkowski 3-space is said to be a constant angle surface whose tangent planes make a constant angle with a fixed vector field of the ambient surface. In other words, if there is a fixed vector k such that the unit normal vector of surface, N makes a constant hyperbolic angle with k. These surfaces can be considered as a generalization of the concept of helix, that is, curves whose tangent lines make a constant angle with a fixed vector of the ambient space. In this case, if U denotes the projection of the fixed direction k on the tangent plane of the surface M, then U is called as a principal direction of the surface M with corresponding principal curvature is zero. On the other hands the constant-angle condition can be also rewritten as a Hamilton–Jacobi equation correlating the surface and the direction field, according to some researcher.

In this talk, we will present a short survey on constant angle surfaces in Euclidean and semi-Euclidean spaces. Additively, we will give a new characterization for generalized constant angle surfaces in Minkowski 3-space. Also, we will present some examples of these surfaces in Minkowski 3-space.

Key Words: Constant angle surface, Space-like surface, Lorentzian surface, Minkowski space.

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A New Characterization of Bertrand curves Hyperbolic 3-Space and Minkowski 4-space

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ABSTRACT

In this study, firstly we review in detail the existing definitions and theorems about the concept of Bertrand curves in hyperbolic 3-space and four -dimensional Minkowski space. It is well known that an immersed curve in hyperbolic 3 -space is said to be a Bertrand curve if there exists another curve and a one-to-one correspondence between these curves which have common principal normal geodesics at corresponding points. After we define (1,3) -Bertrand curve with respect to the causal character of (1,3) -normal plane of non -degenerate special Frenet curves in four-dimensional Minkowski space. Then, we give the necessary and sufficient condition for the timelike (1,3) -Bertrand curve in four - dimensional Minkowski space which is obtained by a non -planar Bertrand curve with non-constant curvature in hyperbolic 3 -space. However, we give methods of obtaining Bertrand curve in hyperbolic 3-space by the spacelike or timelike (1,3) -Bertrand curve in four -dimensional Minkowski space. We get relationship between curvatures of these curves for the first time in this study. Finally, we show that a helix is also a Bertrand curve in hyperbolic 3- space. We draw images of the curve and its Bertrand mate in Poincare ball model of hyperbolic 3-space as an example.

Key Words: Bertrand curve, (1,3)-Bertrand curve, helix.

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A New Kinematical And Mathematical Modelling For 3d Printers

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ABSTRACT

A displacement in n-dimensional space is defined by the matrix-vector pair D = (A, d), where [A] is an $n \times n$ rotation matrix and d is an n-pure rotation and T = (I, d), called a pure translation.

One displacement can operate on another to yield a composite displacement. If we have $D_1 : F \rightarrow M_1$ and $D_2 : M_1 \rightarrow M_2$, then the composite displacement $D = D_1D_2 : F \rightarrow M_2$ exists. The formula for this composite displacement is obtained by substituting the coordinates obtained from the transformation $D_2 = (A_1, d_2)$ into (2.3) written for the transformation $D1 = (A_1, d_2)$. The result is

$$X = [A_1A_2]x + [A_1]d_2 + d_1.$$

Thus the composition displacement $D = D_1D_2 : F \rightarrow M_2$ is defined to be

$$D = D_1D_2 = (A_1, d_1)(A_2, d_2) = (A_2A_2, [A_1]d_2 + d_1).$$

The invers, D^{-1}, of a displacement $D = (A,d)$ is defined by inverting (1.1) to obtain :

$$x = [A^T]X - [A^T]d \ ,$$

Thus $D^{-1} = (A^T, -A^T d)$. Notice that $DD^{-1} = D^{-1}D = I$, where $I = (I, 0)$ is the identity displacement.

In 1955, Denavit and Hartenberg published a paper in the ASME Journal of Applied Mechanics that was later used to represent and model robots and to derive their equations of motion. This technique has become the standard way of representing robots and modeling their motions and thus is essential to learn. The Denavit-Hartenberg (D-H) model of representation is a very simple way of modeling robot links and joints that can be used for any robot configuration, regardless of its

sequence or complexity. It can also be used to represent transformations in any coordinates we have already discussed, such as Cartesian, cylindrical, spherical Euler, and RPY. Additionally, it can be used for representation of all-revolute articulated robots, SCARA robots, or any possible combinations of joints and links. Although the direct modeling of robots with the previous techniques are faster and more straightforward, the Denavit-Hartenberg representation has the added benefit that many techniques have been developed for use with its results, such as the calculation of Jacobeans, force analysis, etc.

A mechanism is a collection of links and joints. Every link pair are connected one joint.

The clasification of mechanism in technology two mechanisms are rather important. One of them is Stewart platform the other is Delta robot. The delta robot, a parallel arm robot, was invented in 1988 by Professor Reymond Clavel. In today, delta robot is fundamental of 3D printer. Every new design of 3D is very important. Every new design of 3D printer is in the need of a new mathematical and kinematical modelling.

In this articles we give a new kinematics and mathematics modelling for 3D printer.

Key words: 3D printer, mechanism, kinematic, modeling

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A New Method for Designing a Developable Surface Using Modified Orthogonal Frame

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ABSTRACT

A ruled surface is defined as a surface that through each point of it passes at least one straight line lying entirely on the surface." In other words, a ruled surface is a surface generated by a family of straight lines (for example, a conic surface). This special geometric property can be used to increase the productivity of machining ruled surfaces on five-axis machines. The basic idea of this type of machining is moving the cutter.s edge to follow the straight lines of the ruled surface.

A developable surface is a ruled surface having Gaussian curvature K=0 everywhere. Developable surfaces therefore include the cone, cylinder, elliptic cone, hyperbolic cylinder, and plane. Developable surfaces are widely utilized in geometric design and manufacturing systems because they can be developed onto a plane without stretching and tearing. Developable surfaces are widely used in design and manufacturing of materials that do not stretch or tear. Applications include modeling of ship hulls, apparel, ducts, automobile, and aircraft components. Products are first designed using developable surfaces in 3D space; then they are flattened and become a 'pattern' in a plane.

Line of curvature is one of the most important characteristic curves on a surface and it plays an important role in differential geometry. Line of curvature can guide the analysis of surfaces, widely used in geometric design, shape recognition, polygonization of surfaces and surface rendering. On the other hand, In ([8]), Sasai studied an orthogonal frame and obtained a formula which corresponds to the Frenet-Serret equation. In ([1]), the authors studied spherical curves according to modified orthogonal frame in the Euclidean 3-space. By utilizing the modified orthogonal frame, this paper proposes a new method to construct a developable surface possessing a given curve as the line of curvature of it. We investigate the necessary and sufficient conditions when the resulting developable surface is a cylinder, cone or tangent surface.

Key Words: Developable surfaces, cylinder surfaces, cone surfaces, tangent surface, modified orthogonal frame.

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A note on paracontact submersions

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ABSTRACT

The theory of Riemannian submersion was introduced by O'Neill and Gray in [5] and [1], respectively. Presently, there is an extensive literature on the Riemannian submersions with different conditions imposed on the total space and on the fibres. Semi-Riemannian submersions were introduced by O'Neill in his book [6]. Later, Riemannian submersions were considered between almost complex manifolds by Watson in [7] under the name of almost Hermitian submersion. He showed that if the total manifold is a Kahler manifold, the base manifold is also a Kahler manifold. Riemannian submersions between almost contact manifolds were studied by Chinea in [2] under the name of almost contact submersions. Since then, Riemannian submersions have been used as an effective tool to describe the structure of a Riemannian manifold equipped with a differentiable structure. In this paper, we define paracontact semi-Riemannian submersions between almost paracontact metric manifolds and study the geometry of such submersions. In this paper, we introduce the notion of paracontact semi-Riemannian submersions and give an example of paracontact semi-Riemannian submersion. Moreover, we investigate properties of O'Neill's tensors and show that such tensors have nice algebraic properties for paracontact semi-Riemannian submersions. We find the integrability of the horizontal distribution. We also find necessary and sufficient conditions for the fibres of a paracontact semi-Riemannian submersion to be totally geodesic.

Key Words: Almost paracontact metric manifold, semi-Riemannian submersion, paracontact semi-Riemannian submersion.

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A Note on Spacelike Curves in the Lightlike Cone

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ABSTRACT

Denote E_q^m as the m-dimensional pseudo-Euclidean space with the following metric

$$\tilde{G}(X,Y)=\langle X,Y\rangle=\sum_{i=1}^{m-q}x_i y_i-\sum_{j=m-q+1}^{m}x_j y_j$$

where $X=(x_1,x_2,...,x_m),Y=(y_1,y_2,...,y_m)\in E_q^m$, E_q^m is a flat pseudo Riemannian manifold of signature $(m-q,q)$.

Suppose that M is a submanifold of E_q^m. If the pseudo Riemannian metric $\tilde{G}$ (respectively, a Riemannian metric, a degenerate quadratic form) on M, then M is a timelike(respectively, spacelike, degenerate) submanifold of E_q^m.

Let c be a fixed point in E_q^m and r>0 be an arbitrary constant. The pseudo-Riemannian null cone (quadratic cone) is defined as follows

$$Q_q^n(c,r)=\left\{x\in E_q^{n+1}:\tilde{G}(x-c,x-c)=0\right\}.$$

It is known that $Q_q^n(c,r)$ is a degenerate hypersurface in E_q^{m+1}. The point c is the center of $Q_q^n(c)$. When c=0 and q=1, we denote $Q_1^n(0)$ by Q^n and call it the lightlike or null cone.

In terms of mathematics, curve theory has been an attraction for differential geometers and so it has been a widely studied topic. One of the most important tools used to analyse a curve is the Frenet frame. Many studies on curves using frenet frames have been done by many mathematicians, [2-8].

A helix in Euclidean 3-space IR^3 is a curve of constant slope, in other words, a curve whose tangent vector makes a constant angle with a fixed direction (called the axis). And also, it is known that a curve α is called a slant helix if the principal normal lines of α make a constant angle with a fixed direction [6].

In this paper, we study special spacelike curves according to asymptotic orthonormal frame in the lightlike cone. We give some characterizations of these curves.

Key Words: Asymptotic orthonormal frame, Frenet formulas, Lightlike cone.

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A problem: Finding the Place of $\sqrt[n]{a}$ on the Numerical Axis via Drawing

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ABSTRACT

The present study focuses on the issue of finding the place of the value of $\sqrt[n]{a}$, a non-routine drawing problem on the numerical axis via drawing (by the help of compass and ruler help) and on the analysis of the solution process. In order to be able to find the place of the value of $\sqrt[n]{a}$ on the numerical axis via drawing, primarily, it was necessary to find a scheme that binds a to a^n's for n € N. For the creation of this scheme, the path of metric connections in the right triangle was followed and a scheme that generated the values of a^n based on 1 and "a" was created. The method used for the creation of this scheme was termed as "steep turns" and the mathematical basis of method was justified. Afterwards, this scheme was expressed in a form that would bind a to $\sqrt[n]{a}$. By the use of this scheme, exact solutions were found for n = 2^k k € N in order to find the place of $\sqrt[n]{a}$ Bu on the numerical axis. In this solution too, metric connections were utilized for in the right triangle. In the case of k being greater in n = 2^k , the method should be used successively as many times as possible. No solution was found the other moods of k. Finding the place of $\sqrt[3]{a}$ in the unsolvable part has been known as the Delos Problem since the Middle Ages and no solution so far has been found.

The solution process was examined on the basis of the four-step process that Polya [1],[2] provided for problem solving and suggested some additions to the process steps. In these suggestions, the importance of mathematical studies of "emergence of problems" which were not included in the process steps for drawing problems were put forward and it was considered to be necessary to add such a step for uncommon problems. Furthermore, within the scope of "planning of the solution

strategy" and "evaluation of the solution", it was concluded that it would be appropriate to put some additional behaviours in the process or make changes in the expression of existing behaviours [3]. The results of the study can be expected to contribute to the literature of drawing problems and to the teaching of problem solving[4].

Key Words: Problem solving, Delos problem, problem solving process

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Action of R-Module Groupoids

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ABSTRACT

The concept of action is one of the most important tools in algebraic topology. The concept plays an important role in category theory especially in the study of groupoids. The concept of a groupoid action on a set was introduced by Ehresmann [5] and is fairly well-known. Subsequently the action of groupoids has been studied extensively in algebraic and differential topology. Many mathematicians studied various aspects of groupoid actions in sense of algebraic, topological and differentiable categories [1-4, 6].

The category GdOp(G) of the groupoid actions of G on sets and G×M action groupoid defined in [3]. Then topological version of this problem was introduced in [1]. Mucuk in [7, 8] constructed that the category GGdOp(G) of the group-groupoid actions of G on groups and the category RGdOp(R) of the ring-groupoid actions of R on rings. He also proved that there are action group-groupoid and action ring-groupoid. Topological versions of these categories and action groupoids proved in [2-4]. Geometric version of this problems introduced in [6].

Let R be a ring with identity 1_R. In this paper we deal with actions of R-module groupoid which is a new concept in the literature. After we give the definition of the action of a R-module groupoid on a R-module, we obtain some results related to action of R-Module groupoids. So we have a category RMGdOp(G_M) of actions of G_M on R-modules. We also prove that there is an action R-module groupoid G×M over R-module M.

Key Words: Groupod, action, R-module.

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An Introduction to Soft Cone Metric Spaces

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ABSTRACT

We introduce a concept of soft cone metric space via soft elements. Firstly, we define the concept of soft cone metric according to soft element and described some of its properties. Also, we give some examples of the soft cone metric spaces and relations between soft cone metrics and crisp cone metrics. Then, we investigate soft convergent sequence and soft Cauchy sequence in soft cone metric spaces. Finally, we prove Banach fixed point theorem in soft cone metric spaces.

Definition: Let X be a non-empty set and $\tilde{X}$ be absolute soft set. A mapping $d : SE(\tilde{X}) \times SE(\tilde{X}) \to SE(\tilde{E})$ is said to be a soft cone metric on $\tilde{X}$ if d satisfies the following axioms:

d1) $\Theta \tilde{\preceq} d(\tilde{x}, \tilde{y})$ for all $\tilde{x}, \tilde{y} \tilde{\in} \tilde{X}$ and $d(\tilde{x}, \tilde{y}) = \Theta$ if and only if $\tilde{x} = \tilde{y}$.

d2) $d(\tilde{x}, \tilde{y}) = d(\tilde{y}, \tilde{x})$ for all $\tilde{x}, \tilde{y} \tilde{\in} \tilde{X}$.

d3) $d(\tilde{x}, \tilde{y}) \tilde{\preceq} d(\tilde{x}, \tilde{z}) + d(\tilde{z}, \tilde{y})$ for all $\tilde{x}, \tilde{y}, \tilde{z} \tilde{\in} \tilde{X}$.

Then, the soft set $\tilde{X}$ with a soft cone metric d on $\tilde{X}$ is called a soft cone metric space and is denoted by $\left(\tilde{X}, d, A\right)$.

Definition: Let $\left(\tilde{X}, d, A\right)$ be a soft cone metric space and $T : \left(\tilde{X}, d, A\right) \to \left(\tilde{X}, d, A\right)$ be a mapping. If there exists a soft element $\tilde{x}_0 \tilde{\in} \tilde{X}$ such that $T\tilde{x}_0 = \tilde{x}_0$, then $\tilde{x}_0$ is called a fixed element of T.

Definition: Let $(\tilde{X},d,A)$ be a soft cone metric space and $T:(\tilde{X},d,A)\rightarrow(\tilde{X},d,A)$ be a mapping. For every $\tilde{x}_0 \tilde{\in} \tilde{X}$, we can construct the sequence $\{\tilde{x}_n\}$ of soft elements by choosing $\tilde{x}_0$ and continuing by:

$$\tilde{x}_1 = T\tilde{x}_0, \quad \tilde{x}_2 = T\tilde{x}_1 = T^2\tilde{x}_0, \quad \dots \quad \tilde{x}_n = T\tilde{x}_{n-1} = T^n\tilde{x}_0, \quad \dots$$

We say that the sequence $\{\tilde{x}_n\}$ is constructed by iteration method.

Definition: Let $(\tilde{X},d,A)$ be a soft cone metric space and $T:(\tilde{X},d,A)\rightarrow(\tilde{X},d,A)$ be a mapping. If there is a positive soft real number $\tilde{t}$ with $\overline{0} \tilde{\le} \tilde{t} \tilde{<} \overline{1}$ such that

$$d(T\tilde{x},T\tilde{y}) \tilde{\preceq} \tilde{t}d(\tilde{x},\tilde{y}), \ \forall \tilde{x},\tilde{y} \tilde{\in} \tilde{X},$$

then T is called contractive mapping in $\tilde{X}$.

Theorem: Let $(\tilde{X},d,A)$ be a complete soft cone metric space and $T:(\tilde{X},d,A)\rightarrow(\tilde{X},d,A)$ be a contractive mapping. Then T has a unique fixed soft element in $\tilde{X}$. For each $\tilde{x} \tilde{\in} \tilde{X}$, the iterative sequence $\{T^n\tilde{x}\}$ converges to the fixed soft element.

Key Words: Soft set, soft metric space, soft Banach space, soft cone metric space, fixed point

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Anti-Invariant Riemannian Submersions With Totally Umbilical Fibers

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ABSTRACT

Riemannian submersions between Riemannian manifolds were studied by O'Neill [4] and Gray [3]. Later such submersions were considered between manifolds with differentiable structures. In [7], Watson defined almost Hermitian submersions between almost Hermitian manifolds and he showed that the base manifold and each fibre have the same kind of structure as the total space, in most cases. Recently, the notion of anti- invariant submersions has been introduced by the second author in [6].

We note that Riemannian submersions have applications in Klauza-Klein theory and the theory of robotics. Altafini [1] used Riemannian submersion in redundant robots, it means that the robotic chain has more than six joints, and showed that the forward kinematic map from joint space to the workspace of the end effector is a Riemannian submersion. He also showed that there is a close relationship between inverse kinematic in robotics and the horizontal lift of vector fields in Riemannian submersions.

In this talk, we consider an anti-invariant Riemannian submersion from a Kaehler manifold onto a Riemannian manifold with umbilical fibers. We obtain a characterization for such maps. We also obtain a new inequality in terms of O'Neill's tensor field and give certain new results when the equality case is satisfied. Moreover, we check the existence of Riemannian submersions with umbilical fibers from complex space forms onto a Riemannian manifold.

Key Words: Riemannian submersion, Anti-invariant Riemannian submersion, umbilical fibers.

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Characterization of Helices According to Bishop Frame in Euclidean 3-Space

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ABSTRACT

There are many studies in literature about the space curves called generalized helices whose tangents make constant angle with the constant direction. Beside these, the slant helices whose principal normal and binormal vectors make constant angle with the constant direction, have been studied in several articles. The Bishop frame also called parallel transport frame has been defined by L.R. Bishop in 1975. He showed that it is possible to define orthonormal frame for C^2-curves which consist of three orthonormal vectors called the tangent, and the normal vectors N_1 and N_2. He gave the relations between the Frenet frame and parallel transport frame and also he gave the relations Frenet curvatures and new curvatures. There are many studies in literature about the classification of the space curve in both Euclidean space and in different space forms except that the curve is a generalized helice. The main problem to prepare this study is the question of "how can we classify the helices using new curvature?".

In this study, we characterized the generalized helices, cylindirical helice and N_1 and N_2 - helices according to the parallel transport frame also called Bishop frame whose tangents, N_1 and N_2 vectors of the space curve, make constant angles with the constant direction, respectively. We gave the position vectors in each cases.

Key Words: Generalized Helices, Parallel transport frame, Bishop curvatures, Natural curvatures.

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Chen Inequalities on Screen Homothetic Lightlike Hypersurfaces of a Lorentzian Space Form with Semi-Symmetric Metric Connection

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ABSTRACT

In degenerate submanifolds, M. Gülbahar, E. Kılıç and S. Keleş introduced *k*-Ricci curvature, *k*-scalar curvature, *k*-degenerate Ricci curvature, *k*-degenerate scalar curvature and they established some inequalities that characterize lightlike hypersurface of a Lorentzian manifold in [7]. However, as it is well known, since the sectional curvature and the induced Ricci curvature are not symmetric on lightlike manifolds, establishing Chen-like inequalities on lightlike submanifolds are more difficult than establishing such inequalities on non-degenerate submanifolds. Thus, due to above mentioned difficulties, they couldn't compute some Chen-like inequalities (Chen-Ricci inequality, Chen-inequality etc.). In [8], they established some inequalities involving k-Ricci curvature, k-scalar curvature, the screen scalar curvature on a screen homothetic lightlike hypersurface of a Lorentzian manifold.

In this paper, we introduce screen homothetic lightlike hypersurface of a Lorentzian space form with semi-symmetric metric connection. Since the sectional curvature and the screen Ricci curvature of screen homothetic lightlike hypersurface are symmetric therefore we are able to establish Chen's inequalities on screen homothetic lightlike hypersurface of a Lorentzian space form with semi-symmetric metric connection. We introduce k-Ricci curvature and k-scalar curvature on screen homothetic lightlike hypersurface of a Lorentzian space form with semi-symmetric metric connection. Using this curvatures, we establish some inequalities for screen homothetic lightlike hypersurface of a Lorentzian space form with semi-symmetric metric connection. Considering these inequalities, we obtain the relation between Ricci curvature and scalar curvature endowed with semi-symmetric metric connection and we give some characterizations using these inequalities.Also, we compute Chen-Ricci inequality and Chen inequality on a screen homothetic lightlike hypersurface of a Lorentzian space form with semi-symmetric metric connection. Moreover, we consider the equality cases of these inequalities.

Key Words: Chen inequality, screen homothetic hypersurfaces, semi-symmetric metric connection.

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Closedness in Reflexive Spaces

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ABSTRACT

Order theory is a branch of mathematics that studies various kinds of binary relations that capture the intuitive notion of a mathematical ordering. Orders appear everywhere – at least as far as mathematics and related areas, such as computer science, are concerned (see [1-4]).

Domain theory is a fast-growing branch in the interface between mathematics and computer science that studies special kinds of partially ordered sets commonly called domains. Consequently, domain theory can be considered as a branch of order theory. The primary motivation for the study of domains, which was initiated by Dana Scott in the late 1960s, was the search for a denotational semantics of the lambda calculus, especially for functional programming languages in computer science [2-4].

Baran, in [5], introduced local separation properties in set-based topological categories and then, they are generalized to point free definitions by using the generic element method of topos theory [6]. One of the other uses of local separation properties is to define the notion of (strong) closedness in set-based topological categories which are used in the notions of Hausdorffness [7], regular, completely regular, and normal objects [8].

Recall that a reflexive space (in [5]) is a pair (B,R), where B is a set and R is reflexive relation on B. A map $f: (B,R)\rightarrow(B_1,R_1)$ between reflexive spaces is said to be continous if aRb, then $f(a)R_1 f(b)$ for all $a,b \in B$.

In this paper, we give the characterization of closed and strongly closed subsets of reflexive space and investigate the relationships among these notions and the classical up-closed and down-closed subsets of reflexive space.

Key Words: Topological category, reflexive spaces, strong closedness, up (down)-closedness.

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Completeness of Soft Fuzzy Metric Spaces

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ABSTRACT

Lots of our traditional tools for formal modelling, reasoningand computing are crips. Crisp means dichotomous, that is yes-or-no type rather than more-or-less type. In traditional dual logic, for instance, a statement can be true or false and nothing in between. However, there are many complicated problems in economics, engineering, environment, social science, etc., that involve data which are not always all crips. So, we can not successfully use classical methods for some uncertainities in this problems.

There are three theories: theory of probability, theory of fuzzy sets and the interval mathematics. The most appropriate theory, for dealing with uncertainities is the theory of fuzzy sets introduced by Zadeh [7]. But the fuzzy set operations based on the arithmetic operation. For this reason, in 1999 Modolstov [5] showed a new mathematical tool for dealing with uncertainities which is free of the difficulties. This so-called soft set theory. A soft set is a parameterized family of subsets of the universal set. Research work in soft set theory have been progressing rapidly since Maji et al. [4] introduced several operations on soft sets and applied it to decision making problems. Then, Das and Samanta [3] initiated soft metric spaces and investigated some basic properties. For other some study see [1,2,6].

In this study, we introduce soft fuzzy metric spaces and define some topological structures. Completeness of soft fuzzy metric spaces is also investigated.

Key Words: Soft set, soft fuzzy metric, soft fuzzy completeness.

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Completion on the Category of Pro-*C* Crossed Square

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ABSTRACT

A profinite groups occur in problems relating to Number Theory, Commutative Algebra, Algebraic Geometry and Algebraic Topology. Although the category of profinite groups forms a natural extension of the category of finite groups, it carries a richer structure in that it has categorical objects and constructions which do not exist in finite case; e.g. projective limits and free products. The existence of such constructions in extended category leads to the definition of profinite analogues of the usual constructions of combinatorial group theory such as free groups and presentations of group by generators and relations.

The theory of crossed modules become a useful tool in combinatorial and cohomological group theory. Profinite crossed module is firstly defined by F.J.Kokers and T.Porter in [3]. In [4], they have also examine the Pro-C completion of crossed modules for C which is a full class of finite groups. The definition of crossed square is given in [2]. Pro-C completion of crossed square of groups is examined in [1].

In this work, we introduce subcrossed square, crossed ideal and also introduce the Pro-C analogues of these and Pro-C completion of crossed square of commutative k-algebras. We study the connection between crossed squares and Pro-C crossed square for C which is a full class of finite k-algebras. Also, we show that there is a Pro-C completion functor defined on the category of crossed squares. Furthermore we investigate the relationship between the Pro-C completion of (L,M,N,P) and the Pro-C completion of L, M, N, P as k-algebras.

Key Words: Crossed square, Pro-C crossed square, Pro-C completion.

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Conformal Riemannian Maps on Kaehler Manifolds

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ABSTRACT

Fischer introduced Riemannian maps between Riemannian manifolds in as a generalization of the notions of isometric immersions and Riemannian submersions, for isometric immersions and Riemannian submersions [1].

Let $F : (M; g_M) \rightarrow (N; g_N)$ be a smooth map between Riemannian manifolds such that $0 < \text{rank}F < \min\{m; n\}$, where dimM= m and dimN =n. Then we denote the kernel space of F_* by $\ker F_*$ and consider the orthogonal complementary space $H = (\ker F_*)^\perp$ to $\ker F_*$. Then the tangent bundle of M has the following decomposition:

$$TM = \ker F_* \text{©} H.$$

We denote range of F_* by rangeF_* and consider the orthogonal complementary space $(\text{range}F_*)^\perp$ to rangeF_* in the tangent bundle TN of N. Since rankF <min{m, n}, we always have $(\text{range}F_*)^\perp$. Thus tangent bundle TN of N has the following decomposition:

$$TN = (\text{range}F_*)^\perp \text{©} (\text{range}F_*).$$

Now, a smooth map $F : (M; g_M) \rightarrow (N; g_N)$ is called Riemannian map at $p \epsilon M$ if the horizontal restriction $F_{*p}: (\ker F_{*p})^\perp \rightarrow (\text{range}F_*)$ is a linear isometry between the inner product spaces $(\ker F_{*p})^\perp$ and $(\text{range}F_*)$. Therefore Fischer stated in [1] that a Riemannian map is a map which is as isometric as it can be. In other words, F_* satisfies the equation

$$g_N(F_*X, F_*Y) = g_M(X, Y)$$

for X, Y vector fields tangent to H. It follows that isometric immersions and Riemannian submersions are particular Riemannian maps with $\ker F_* = \{0\}$ and $(\text{range}F_*)^\perp = \{0\}$. It known that a Riemannian map is a subimmersion [1].

We note that there are many applications of conformal maps. Indeed conformal maps have been used in geometric modelling and computer vision which are widely used in medical imaging.

In this study, we investigate geometric structures for conformal Riemannian maps and introduce some conformal Riemannian maps from a Kaehler manifold (M, g_M, J) to a Riemannian manifold (N, g_N). We find integrability conditions for certain distributions defined on the total manifold and base manifold, and investigate the geometry of leaves.

Key Words: Riemannian maps, conformal maps, Riemannian submersions.

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Conformal Anti-Invariant $\xi^{\perp}-$Submersions

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ABSTRACT

Riemannian submersions between Riemannian manifolds were studied by O'Neill [10] and Gray [7]. The Riemannian submersions were considered between almost Hermitian manifolds by Watson under the name of almost Hermitian submersions. In this case, the Riemannian submersion is also an almost complex mapping and consequently the vertical and horizontal distribution are invariant with respect to the almost complex structure of the total manifold of the submersion. The study of anti-invariant Riemannian submersions from almost Hermitian manifolds were initiated by Şahin [11] and [12]. In this case, the fibres are anti-invariant with respect to the almost complex structure of the total manifold. Chinea defined almost contact Riemannian submersions between almost contact metric manifolds and examined the differential geometric properties of Riemannian submersions between almost contact metric manifolds. Recently, in [9], Lee defined anti-invariant $\xi^{\perp}-$Riemannian submersions from almost contact metric manifolds and then he studied the geometry of such maps. In this paper, as a generalization of anti-invariant $\xi^{\perp}-$Riemannian submersions, we introduce conformal anti-invariant $\xi^{\perp}-$submersions from almost contact metric manifolds onto Riemannian manifolds. We investigate the geometry of foliations which are arisen from the definition of a conformal submersion and find necessary and sufficient conditions for a conformal anti-invariant $\xi^{\perp}-$submersion to be totally geodesic and harmonic, respectively.

Key Words: Sasakian manifold, Riemannian submersion, anti-invariant $\xi^{\perp}-$Riemannian submersion, conformal anti-invariant $\xi^{\perp}-$submersion.

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Coproduct Object in the Category of Crossed Modules of Racks

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ABSTRACT

The theory of racks has strong connections with the theory of groups, especially with the theory of group conjugation. Unlike groups, racks are the non-associative algebraic structures. In the literature, racks are also called as automorphic sets, as crystals, as (left) distributive quasigroups and finally as racks (wracks) by Conway and Wraith.

A free rack on a set X is introduced by Fenn and Rourke in [3]. Moreover, they also introduced the free product of two racks. (Actually this is the coproduct object in the category of racks!)

A crossed module in the category of groups consists of a group homomorphism with a group action that satisfying two certain conditions (called Peiffer relations). Crossed modules are introduced by Whitehead as a model of homotopy 3-types and used to classify higher dimensional cohomology groups. Brown defined the coproduct object in the category of crossed modules of groups in [2] where they used free group and free product structures for the construction.

Crossed modules of racks are introduced by Crans and Wagemann in [1]. They generalize the notion of crossed modules of groups to that of crossed modules of racks. In this study, we define (and construct) the coproduct object in the category of crossed modules of racks, by using free racks.

Key Words: Coproduct, Crossed module, Free rack, Free product.

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Displacements with n-Control Points

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ABSTRACT

In the kinematics, some knowledges are fundamental. They are points, orbit, axis, etc. In inverse kinematics they are also important. The applications of kinematics in technology, social science, robotics, the orbit is important. The orbit of a moving point under a displacement give us important knowledge about the displacement. For example, suppose that a orbit of a moving point is on a sphere of radius r, then we can say that the displacement is only rotation of pole point is the centre of the sphere.

If an orbit is a continuous curve in three dimension space, then we can find n-control points controlled the curve. Furthermore, if we have n-points which they control an orbit and we have to define a displacement according to these points, then the main question is how we can define a displacement which we require. This article has the answer of this question.

For the answer for this questions, we used Bezier curve with n-control points. Through operation of control points, Bezier curve could be translated and rotated. A Bezier curve has fewer turning points, which are the points where the slope of the curve changes its sign, so that it is smoother than cubic splines and varies smoothly from the start point to the end point because of its continuous higher order derivatives. The Bezier curve passes through the start and final control points but, in generally, not the other ones, which define the start and the final orientation and the shape of the curve.

In this article we define a displacement with n-control points in 3-dimension space and give Matlab applications.

Key Words: Control points, orbit.

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Elementary Soft Topology

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ABSTRACT

Our main goal is to introduce a new soft topological space whose topology is defined by the elementary union and intersection operations over an initial universe with a fixed set of parameters, which is different from soft topological space due to both Shabir and Naz [2] and Hazra et al [3]. We call this new topology as elementary soft topology. We first give a new definition of topology, whose members are collections of soft elements, using ordinary union and intersection operations over an initial universe with a set of parameters. Secondly, we define the elementary soft topology using the elementary union and intersection operations although those operations are not distributive. Here, we give the relations between the elementary soft topology and the topology of collections of soft elements, and the Shabir and Naz's soft topology, and the Hazra et al.'s soft topology. We see that the elementary soft topology is different from both the topology of collections of soft elements and the Shabir and Naz's soft topology and the Hazra et al.'s soft topology. We also investigate the notions of soft open sets, soft closed sets, soft neighbourhoods of soft elements, soft interior elements, soft closure elements and their basic properties in elementary soft topological spaces. We work on soft basis and soft local basis in elementary soft topological spaces. Lastly, we describe in some detail soft continuous mappings over the elementary soft topological spaces via soft elements.

Key Words: Soft set, soft element, soft topology, soft interior, soft closure, soft basis, soft continuous mapping

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Factorable Graph Surfaces in Certain Cayley-Klein Spaces

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ABSTRACT

A Cayley-Klein n-space is a metric space constructed within a projective n-space with a quadratic hypersurface (so-called absolute figure) given by a quadratic form $x^TMx=0$, where M is a $(n+1)\times(n+1)$ symmetric matrix [4,8]. The Cayley-Klein construction provides models for the Euclidean, elliptic, hyperbolic, Galilean, isotropic and many other geometries.

In a Euclidean 3-space, a surface is said to be factorable if it is the graph of $u(x,y)=f(x)g(y)$. Besides to the Euclidean 3-space, a surface is said to be factorable in a Minkowskian 3-space if it is the graph of either $u(y,z)=f(y)g(z)$ or $u(x,y)=f(x)g(y)$. Minimal factorable surfaces in these ambient spaces were obtained in [5,6]. In addition, such graph surfaces were studied in terms of their position vectors and Laplace operators in [3].

This presentation concerns certain two Cayley-Klein geometries, that is, the pseudo-Galilean and isotropic geometries. Foundations of these areas were established in [7,8]. In a pseudo-Galilean 3-space, there exist two types of factorable surfaces arising from its absolute figure. Zero curvature factorable surfaces were provided in [1]. Still, it is an open problem to obtain such surfaces with nonzero curvature. We purpose to solve this problem. Furthermore, the factorable graph surfaces in an isotropic space were studied in [2] and this presentation revisits these concepts.

Key Words: Cayley-Klein geometry, Factorable graph, Mean curvature, Gaussian curvature.

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Fan-Gottesman Compactification and Approximation

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ABSTRACT

Many mathemathicians have studied the representation of topological spaces in terms of inverse systems of finite space. In 1961, Flachsmeyer [4] suggested one approach to the study of compact Hausdorff spaces, that is; to approximate the compact space by means of finite T_0-spaces. His work went largely unnoticed by topologists -possibly because compact Hausdorff spaces are in some sense the best in the field, whereas finite T_0 spaces are T_1 only if they are discrete. Today however, with computers widespread the use of 'large finite' objects is commonplace. Whenever you look at a computer screen, you are essentially looking at a finite topological space. Indeed, although the computer screen looks like a product of two intervals, much use is made of the fact that the pixels can be modelled as elements of the product of two finite sets. In [5] this situation is studied from a topological point of view. An interval in the Khalimsky line is an example of a finite connected ordered topological space (COTS); such spaces have frequently been used as finite models of intervals in the reals. R. D. Kopperman and his friend get a lot of results about this topic [5,6,7]. In 2009 R. D. Kopperman and R. G. Wilson showed that there is an inverse system arising from a directed collection of finite sets of open sets, which has the same limit, for each inverse system of finite T_0-space and continuous maps. Also they showed that the Wallman-type compactification of T_1-space are precisely the sets of closed points of an invese limit of finite T_0-space and continuous maps. In this paper, we will give Ky Fan and Noel Gottesman's method used in order to get a compactification of regular space and also, we will determine this compactification via open ultrafilters. We show that the Fan-Gottesman compactification of a $T_{3,5}$-space is

the sets of closed points of an inverse limit of finite $T_{3,5}$-spaces and continuous maps.

Key Words: Fan-Gottesman compactification, approximating family, inverse spectrum

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For Ideal Topological Spaces wI_*-closed Set

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ABSTRACT

A most of topologist worked on several versions of generalized closed sets in topological spaces. Also ideals play an important role in topology. The subject of ideals in topological spaces has been studied by Kuratowski [5]. And then the generalized closed sets in ideal topological spaces have been considerable interest among many mathematicians. Therefore many published works made on closed sets with used ideal topological spaces. Some of these are: *gI*-closed sets [4], *Iπg*-closed sets [7], *g*–closed sets [6], *πgs*-closed sets [1]. The goal of this paper is to introduce another generalized closed set in ideal topological spaces. We will define a wI_*-closed set, which is a new closed set in ideal topological spaces. Also we will investigate their properties and relationships between the others closed sets in ideal topological spaces. So firstly we will give definition of ideal topological spaces. After then we will define closed sets in point set topology and various closed sets in ideal topological spaces. An ideal *I* on a set *X* is a non-empty collection of subsets of *X* with heredity property which is also closed under finite unions. For definition of wI_*-closed sets we will define semi-I-open sets [2] in ideal topological spaces.

Key Words: Ideal topological spaces, semi-I-open sets, wI_*-closed sets.

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Fuzzy wI_*-closed Set in Fuzzy Ideal Topological Spaces

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ABSTRACT

The fundamental concept of a fuzzy set was introduced in Zadeh [5]. Afterwards fuzzy topology was defined by Chang and Lowen. Also Sarkar [3] and Yalvac [7] introduced fuzzy ideal, fuzzy local function, fuzzy set and fuzzy function. And then several fuzzy closed sets were obtained using the generalized closed sets in ideal topological spaces and fuzzy ideal. Therefore many published works made on fuzzy open sets with used fuzzy ideal topological spaces. Some of these are: fuzzy semi-*I*-open sets [4], fuzzy pre-I-open sets [1], fuzzy α-I-open sets [6], regular-*I*-closed sets [2]. The goal of this paper is to introduce another generalized fuzzy closed set in fuzzy ideal topological spaces. We will define a fuzzy wI_*-closed set, which is a new fuzzy closed set in fuzzy ideal topological spaces. Also we will investigate their properties and relationships between the others fuzzy closed sets in fuzzy ideal topological spaces. So firstly we will give definition of fuzzy ideal topological spaces. After then we will define fuzzy closed sets in fuzzy topology and various fuzzy open sets in fuzzy ideal topological spaces. With definitions of fuzzy set, fuzzy function, fuzzy topology and fuzzy ideal we will define fuzzy semi-I-open sets [2] in fuzzy ideal topological spaces. And then we will give definition of fuzzy wI_*-closed set.

Key Words: Fuzzy ideal topological spaces, fuzzy semi-I-open sets, fuzzy wI_*-closed sets.

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Generic Riemannian Submersions From Almost Product Riemannian Manifolds

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ABSTRACT

The theory of smooth maps between Riemannian manifolds has been widely studied in Riemannian geometry. Such maps are useful for comparing geometric structures between two manifolds. In this point of view, the study of Riemannian submersions between Riemannian manifolds was initiated by O'Neill [4] and Gray [3]. In [6], Bayram Şahin introduced the notion of semi-invariant Riemannian submersions as a generalization of anti-invariant Riemannian submersions [5]. As a generalization to semi-invariant Riemannian submersions, Ali and Fatima [1] introduced the notion of generic Riemannian submersions from almost Hermitian manifolds onto Riemannian manifolds. They showed that such submersions have rich geometric properties and they are useful for investigating the geometry of the total space. The present paper, we define and study generic Riemannian submersions from almost product Riemannian manifolds onto Riemannian manifolds. We survey main results of generic Riemannian submersions defined on almost product Riemannian manifolds. We give an example, obtain the integrability conditions for the horizontal distribution while it is noted that the vertical distribution is always integrable. We investigate the geometry of foliations of the two distributions which arise from the definition of a generic Riemannian submersion and find necessary and sufficient condition for a total manifold to be a generic product manifold. The decomposition theorems for the total manifold of generic Riemannian submersion are obtained. We also find necessary and sufficient conditions for a generic submersion to be totally geodesic.

Key Words: Almost product Riemannian manifold, Riemannian submersion, generic Riemannian submersion.

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Geometrical Modelling of Musical Work of “INDIA DANCE”

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ABSTRACT

The first academic studies on music and mathematics were started by Pythagoras, the Greek philosopher and mathematician, in the 6th century BC. Pythagoras used to define the frets in music by means of arithmetical processes, state that there is a relation between them and the numbers, and believe that the numbers compose the whole universe. Pythagoras found simple ratios between harmonic tones [1]. ,

Marin Mersenne (1588-1648), discovered the songs harmonics and defined the six harmonies. Moreover, he also defined the relation between the harmonic and non-harmonic, and published the harmony theory in his article “Treatise on Harmony” in 1720. At the beginning of the 18th century, Brook Taylor started to research vibration scales.

Brook found a differential equation representing a scale of vibration meeting the beginning conditions, and showed that the solution of such equation is sinus curve [2]. Daniel Bernoulli (1700-1782), Leonhard Euler (1707-1783), D'Alembert (1717-1783) are the mathematicians of the same period, and they made numerous studies on music and mathematics [3]. Mathematicians such as D’Alembert and Euler defined the vibration scale by means of a differential equation [4].

Jean Baptiste Fourier (1768-1830), defined vibration waves by using trigonometric functions. In 1869, the music theoretician Arthur von Oetting established the Harmonic Theory, which was a breakthrough in the music world of the 20th century [7]. In 1983, Mandelbrot defined the fractal theory which was a

breakthrough in geometry. Fractal geometry used in many science branches was used in music for the first time by Voss&Clarke, 1975; Campbell, 1986; Schroeder, 1987 [7].

Leonardo Fibonacci (1170-1250), made several studies on the theory that numbers called as Fibonacci Range or Numbers in theory and the Golden Ratio which is composed accordingly are used in natural sciences and music. The contemporary music composers who are known to have used Fibonacci Range in their works include I. Xenakis, L. Nono, E. Krenek and K. Stockhausen [8].

In this study, mathematical coding of "India Dance" work has been made according to duration of playing and note value. Geometrical shape of the musical work, "India Dance", by means of Rational Bezier curves of such mathematical coding Rational Bezier Curves, representing the right and left hand note data, have been drawn by using Maple13 programme. Note value of such mathematical model and duration of playing have been analysed in SPSS and linear regression model has been established.

Key Words: Statistical Modelling of the Musical Compositions, Mathematics and Music, Notation Musician programmes

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Geometry of Fractal Structures in Galilean Plane

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ABSTRACT

A fractal is a geometrical object which is made up smaller copies of itself. Self-similarity and fractal dimension are two important features of them. Fractals can be generated by iterated function systems which a collection of similarity transformations. Similarity transformations are the rigid motions: reflection, contraction, translation, and rotation. Applying this mappings to any structure with using deterministic algorithm we get fractal objects. Until now, Euclidean fractals have been studied mostly. But new types of geometries have been developed besides Euclidean geometry in the last two centuries. Galilean geometry is one of them. As the geometries change, the distance and scale change. Measuring is related to the metric and it is metric properties that shape an object. From this point of view, in this work we describe geometrical concept of the fractal structures in Galilean plane using Galilean metric. Also we obtain some known fractal structures such as Sierpinski gasket in Galilean plane using the similar idea in Euclidean plane. We called them Sierpinski-type fractals. Besides that we investigate the effects of rotation transformation to objects under the iterations in Galilean plane. We compare the objects created with the same rotation angle in both Euclidean and Galilean plane. Consequently this approach helped us to create Galilean self-similarity system.

Key Words: Fractal, iteration, Galilean transformation.

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Hard Lefshetz Theorem and Reidemeister Torsion of 3-manifolds

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ABSTRACT

Reidemeister torsion is a topological invariant. It was first introduced by K. Reidemeister in 1935 [2]. Using this combinatorial invariant of CW-complexes, K. Reidemeister classified 3−dimensional lense spaces. W. Franz [1] generalized Reidemeister torsion and classified the higher dimensional lense spaces; that is, S^{2n+1}/G, where G is a cyclic group acting isometrically and freely on the sphere S^{2n+1}. This topological invariant has many applications in several branches of mathematics and theoretical physics, such as topology, differential geometry, representation spaces, knot theory, Chern-Simon theory, 3-dimensional Seiberg-Witten theory, dynamical systems, theoretical physics and quantum fields theory.

E. Witten introduced the algebraic topological tool symplectic chain complex in [5], where using R-torsion and symplectic chain complex he computed the volume of several moduli spaces of representations from a surface to a compact gauge group.

The present abstract considers closed Kahler manifolds. Using Hard Lefschetz theorem and the symplectic chain complex method, it establishes a novel formula for computing the topological invariant Reidemeister torsion of closed Kahler manifolds. In applications, considering the three-holed-sphere-decomposition of orientable closed surfaces of genus at least 2, where two such three-holed-spheres are glued along only one common boundary circle, the present abstract also establishes a novel formula for computing Reidemeister torsion of compact orientable 3-manifolds

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with boundary finitely many closed orientable surfaces.

Key Words: Reidemeister torsion, Kahler manifolds, 3-manifolds.

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i(d)-Proper and i(d)-Admissible Topologies in the setting of Cech Closure Preordered Spaces

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ABSTRACT

In many branches of mathematics and computer science, assorted generalizations of topological spaces are used. One of this generalizations is done by using closure operator. Closure operators obtained from Kuratowski by omitting some axioms and they have numerous applications. Closure operators, which are, grounded, extensive and monotone were firstly studied by E.Cech [1]. Then, lots of researchers have studied it. In [2], Cech closure operators were employed for solving problems related to digital image processing and in [3], the relation between Cech closure space and structural configuration of proteins were studied.

The cooperation between topology and order was studied by Leopolda Nachbin [4] in the 1950's and he developed the theory of topological ordered spaces and it has an application in domain theory, mathematical economics and topology.

In [5], the notions of proper and admissible topologies were introduced on the sets of continuous functions between Cech closure spaces.

In this work, we put a preorder to Cech closure spaces and we will define Cech closure preordered spaces and we will generalize the proper and admissible topology notions by using preorder relation. To this end, we will give a more general definition of continuity between Cech closure preordered spaces and we will call them i(d)-continuity, then we will carry the concept of proper and admissible topology to the concept of i(d)- proper, i(d)-admissible topology.

Key Words: preorder, cech closure space, i(d)-continuous function

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Local PreT2 and T2 Objects In The Categorıes Of Cauchy Spaces

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ABSTRACT

Baran [1] defined separation properties first at a point p, i.e., locally (see [1; 2]), then they are generalized this to point free definitions by using the generic element, [3] p. 39, method of topos theory for an arbitrary topological category over sets. One reason for doing this is that, in general, objects in a topos may not have points, however they always have a generic point. The other reason is that the notions of "closedness" and "strong closedness" on arbitrary topological catogories is defined in terms of $T0$ and $T1$ at a point, p. 335 [1]. The notions of "closedness" and "strong closedness" in set based topological categories are introduced by Baran [1] and it is shown in [4], that these notions form an appropriate closure operatör in the sense of Dikranjan and Giuli in some well-known topological categories.

There are various generalization of the usual $T2$ (Hausdorff) axiom of topology to an arbitrary topological category defined in [5, 6]. In this paper, an explicit characterizations of each of the separation properties Pre T2, and $T2$ at a point p is given in the topological category of Cauchy spaces [7]. Moreover, specific relationships that arise among the various Ti, i = 0, 1, Pre $T2$, and $T2$ structures at p are examined in this category. Finally, we investigate the relationships between generalized separation properties ([7, 8]) and separation properties at a point p in this category [7].

Key Words: Topological category, Cauchy space, Cauchy map, separation, connectedness, compactness.

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Local Pre-Hausdorff Proximity Spaces

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ABSTRACT

Efremovich [3] introduced proximity spaces which were a generalization of metric spaces. He characterized the proximity relation “A is close to B” as a binary relation on subsets of a set X and showed that a topology can be introduced in a proximity space. This theory was improved by Smirnov. He showed which topological spaces admit a proximity relation compatible with the given topology. The first study of mappings of proximity spaces was performed by Katetov. A proximity mapping is just a function which preserves proximity of sets. It was shown that a proximity map between two proximity spaces is continuous with respect to the induced topologies. Hunsaker and Sharma [4] proved that Prox, the category of proximity spaces and proximity mappings, is a topological category over Set, the category of sets.

Baran, in [1], gave various generalizations of the usual separation axioms of topology and separation properties at a point p for an arbitrary topological category over sets. He defined separation properties first at a point p, i.e., locally, and then point free. The main objective of this paper is to characterize each of various notions of pre-Hausdorff and Hausdorff proximity spaces at a point. Furthermore, the relationships that arise among the various Ti, i=0,1,2 and PreT2 structures at a point are investigated in this category.

Key Words: Topological category, proximity space, pre-Hausdorff, Hausdorff.

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Local Pre-Hausdorff Relation Spaces

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ABSTRACT

In 1991, Baran [1] introduced two local pre-Hausdorff objects in a set-based topological category , denoted by PreT_2 at *p* and Pre$T_2^{'}$ at *p* in [1] or [2] which both are equivalent to the classical pre-Hausdorff axiom in the category of topological spaces, where a topological space is called pre-Hausdorff (PreT_2 at *p*) [1] or [2] for any point *x* distinct from *p*, if there is a neighbourhood of *x* missing *p* or there is a neighbourhood of *p* missing *x* , then the two points have disjoint neighbourhoods. It is shown, in [3], that pre-Hausdorff objects (PreT_2) are used to characterize the decidable objects, [4] p.162 in a topos, where an object *X* of □ , a topos, is said to be decidable if the diagonal $\Delta \subset X^2$ is a complemented subobject. Furthermore, it is proved in [5] that the image of a topos in a topological category by a geometric morphism [5] is a Pre$T_2^{'}$ object. In particular, Pre$T_2^{'}$ objects play a role in the general theory of geometric realizations, their associated interval and corresponding homotopy structures [5]. Moreover, If *X* is a finite set, then it is shown, in [5], that the distinct pre-Hausdorff topologies on *X* are in one-to-one correspondence with the distinct partitions on X. Another use of local pre-Hausdorff objects is to define various forms of each of local Hausdorff objects, local regular objects, and local normal objects in arbitrary topological categories, see [6-8].

Let *B* be a set and *p* ϵ *B*. Let $B \vee_p B$ be the wedge at *p* [1], i.e., two disjoint copies of *B* identified at *p.* A point x in $B \vee_p B$ will be denoted by x_1 (x_2) if x is in the first (resp. the second) component of $B \vee_p B$. Note that $p_1 = p_2$. The skewed p-axis map $S_p : B \vee_p B \to B^2$ is given by $S_p(x_1) = (x, x)$ and $S_p(x_2) = (p, x)$. The Principal p-axis map $A_p : B \vee_p B \to B^2$ is given by $A_p(x_1) = (x, p)$ and $A_p(x_2) = (p, x)$.

For a topological space *X*, it is shown, in [1], that the following are equivalent:

(1) A topological space *X* is PreT_2 at *p*,

(2) The induced topologies on $X \vee_p X$ from X^2 by the skewed p-axis map S_p and the principal p-axis map A_p agree.

(3) The induced topology on $X \vee_p X$ from X^2 by the skewed p-axis map S_p and the co-induced (final) topology on $X \vee_p X$ from X via the canonical injection maps i_1 and i_2 are the same.

In this paper, we characterize local PreT_2 and PreT_2' relation spaces and investigate the relationships between them.

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Key Words: Topological category, local pre-Hausdorff objects, relation spaces, ordered sets.

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Local T0 and T1 Objects In The Categorıes Of Cauchy Spaces

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ABSTRACT

Various generalizations of the usual separation properties of topology and for an arbitrary topological category over sets separation properties at a point p are given in [1]. Baran [1] defined separation properties first at a point p, i.e., locally (see [1; 2]) then they are generalized this to point free definitions by using the generic element, [3] p. 39, method of topos theory. One of the uses of local separation properties is to define the notions of closedness and strong closedness on arbitrary topological categories in set based topological categories. These notions are introduced by Baran [4] and they are used to generalize each of the notions of compactness, connectedness, Hausdorffness, and perfectness to arbitrary set based topological categories.

There are various generalization of the usual T0 and T1 axiom of topology to an arbitrary topological category defined in [5, 6]. In this paper, an explicit characterizations of each of the separation properties Ti, i = 0, 1, at a point *p* is given in the topological category of Cauchy spaces [7]. Moreover, specific relationships that arise among the various Ti, i = 0, 1 structures at *p* are examined in this category of Cauchy spaces. Finally, we investigate the relationships between generalized separation properties Ti, i = 0, 1 ([7, 8]) and separation properties at a point *p* in this category of Cauchy spaces [7].

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Key Words: Topological category, Cauchy space, Cauchy map, separation, connectedness, compactness.

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Local T_0 Approach Spaces

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ABSTRACT

In 1989, Robert Lowen introduced theory of approach spaces [1] which is based upon point-to-set distances rather than point-to-point distances. Approach spaces form a super category of the categories of topological spaces and metric spaces, and the main purpose for their introduction was to fill some gaps concerning categorical aspects of metrizable spaces. The categories TOP (category of topological spaces and continuous maps) and pq-MET(category of extended pseudo-quasi-metric spaces and non-expansive maps) were embedded in approach spaces. The reason for embedding was to view topological spaces and metric spaces as objects of the same type in terms of “distance between points and sets”. The category TOP is well behaved with respect to subspaces, products, quotients, coproducts. On the other hand, the category MET, of metric spaces and non-expansive maps, is stable only under the formation of subspaces and finite products. The main motivation to introduce approach spaces was to resolve the problem of infinite product and coproduct of metrizable topological spaces. There is another motivation for introducing approach spaces is to unify the theories of convergence, metric, uniformity and topological properties [1-2].

Over the years, various developments have been made in approach theory not only in mathematics but also in field of computer science including domain theory [2], convergence spaces, functional analysis, vector spaces, group theory, Index analysis [3] and probability theory [4].

In 1991, Baran [5] introduced local separation properties in set-based topological categories and then, they are generalized to point free definitions by using the generic element method of topos theory [6]. One of the uses of local T_0

property is to define the notion of local Hausdorff objects [7] in set-based topological categories.

The aim of this paper is to a characterize a T_0 distance-approach spaces at p and a T_0 gauge-approach spaces at p and investigate the relationships between them.

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Key Words: Topological category, local T_0 objects, distance-approach spaces, gauge-approach spaces.

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LocalT_0 Extended Quasi-Pseudo-Semi Metric Spaces

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ABSTRACT

In 1931, Wilson [1] introduced quasi-metric spaces (where the condition of symmetry is omitted) have a particular character and signifiance in the area of experimental psychology [3], biological studies and Quantum mechanics [4] from those sciences in which measurement plays an essential role. Quasi-metrics are common in real life. For example, given a set *X* of mountain villages, the typical walking times between elements of *X* form a quasi-metric because travel up hill takes longer than travel down hill.

It is well- known that metric structures behave badly with respect to the formation of infinite products and coproducts. As a remedy to this defect, in 1990, Adámek and Reiterman [5] defined extended pseudo-metric spaces (where an pseudo-metric is allowed to attain the value infinity) which are a generalization of metric spaces.

In 1991, Baran [6] introduced local T_0-axiom of topology to topological category which is used to define the notion of closed subobject of an object of a topological. One of the uses of localT_0 objects is to define various forms of local Hausdorff objects [6] in arbitrary topological categories. There are several ways to generalize the usual T_0-axiom of topology to topological categories [6-8] and the relationships among various forms of generalized T_0-axiom in topological categories have been investigated in [7-8].

Recall that an extended pseudo-quasi-semi metric space is a pair *(X,d),* where *X* is a set and $d: X\times X \to [0,\infty]$ is a function fulfills the pseudoreflexivity condition $d(x, x)= 0$ for all *x* in *X* . A map $f: (X,d) \to (Y,e)$ between extended pseudo-quasi-

semi metric spaces is said to be a non-expansive if it fulfills the property $e(f(x),f(y)) \leq d(x, y)$ for all x,y in X. The extended pseudo-quasi-semi-metric spaces, along with their corresponding non-expansive maps, are the best behaved of the metric spaces. One can take arbitrary products and coproducts and form quotient objects within it.

In this paper, we characterize local T_0 extended pseudo-quasi-semi metric spaces and compare them.

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Key Words: Topological category, local T_0 objects, pseudo-quasi-semi metric spaces, products.

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LocalT$_1$ Approach Spaces

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ABSTRACT

In 1989, Robert Lowen introduced theory of Approach spaces [1] which is based upon point-to-set distances rather than point-to-point distances. In [1] the categories TOP of topological spaces and continuous maps and pq-MET of extended pseudo-quasi-metric spaces and non-expansive maps were embedded in a common supercategory. The idea behind this embedding being that topological spaces and metric spaces can be viewed as objects of the same type, in the sense that they both can be described by a "distance between points and sets". Let be (X,d) extended pseudo-quasi-metric space, $x \in X$ and $A \subset X$. The distance is the usual one given by $(X,\delta) = \inf_{a\in A} d(x,a)$. Let (X,τ) be a topological space and $A \subset X$. The distance can be defined by $(X,\delta) = 0$ if $x \in \overline{A}$ and $(X,\delta) = \infty$ if $x \notin \overline{A}$. A notion of distance has been axiomatized in [1] in such a way as to generalize both the extended pseudo-quasi-metric and topological cases and resulted in the definition of the category AP of approach spaces and contractions.

It is well-known that metric structures behave badly with respect to the formation of initial structures or in particular, of infinite products. The main motivation to introduce approach spaces was to resolve the problem of infinite product and coproduct of metrizable topological spaces. There is another motivation for introducing approach spaces is to unify the theories of convergence, metric, uniformity and topological properties [1-2].

In 1991, Baran [3] introduced local separation properties in set-based topological categories and then, they are generalized to point free definitions by using the generic element method of topos theory [4-5]. One of the uses of local T_1 property is to define the notion of strong closedness in set-based topological categories [3] which are used in the notions of compactness [6]. The other use of local T_1 property is to define the notion of local regular and local normal objects [7] in set-based topological categories.

The aim of this paper is to a characterize a T_1 distance-approach spaces at p and a T_1 gauge-approach spaces at p and investigate the relationships between them.

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Key Words: Topological category, local T_1 objects, distance-approach spaces, pseudo-quasi-metric spaces.

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Magnetic Pseudo Null Curves in E_1^3

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ABSTRACT

A divergence-free vector field defines a magnetic field in a three-dimensional semi-Riemannian manifold M. It is known that, $V \in \chi(M^n)$ is a Killing vector field if and only if $L_V g = 0$ or, equivalently, $\nabla V(p)$ is a skew-symmetric operator in $T_p(M^n)$, at each point $p \in M^n$. It is clear that, any Killing vector field on (M^n, g) is divergence-free. In particular, if $n = 3$, then every Killing vector field defines a magnetic field that will be called *Killing magnetic field*.

If (M, g) is an n-dimensional semi-Riemannian manifold, then a *magnetic field* is a closed 2-form F on M and the *Lorentz force* Φ of the magnetic field F on (M, g) is defined to be a skew-symmetric operator given by

$$g(\Phi(X), Y) = F(X, Y), \ \forall X, Y \in \chi(M).$$

The *magnetic trajectories* of F are curves α on M that satisfy the *Lorentz equation*

$$\nabla_{\alpha'} \alpha' = \Phi(\alpha').$$

Note that, one can define on M the cross product of two vectors $X, Y \in \chi(M)$ as

$$g(X \times Y, Z) = dv_g(X, Y, Z), \ \forall Z \in \chi(M).$$

If V is a Killing vector field on M, let $F_V = \iota_V dv_g$ be the corresponding Killing magnetic field, where ι denotes the inner product. Then the Lorentz force of F_V is

$$\Phi(X) = V \times X.$$

Consequently, the Lorentz force equation can be written as

$$\nabla_{\alpha'} \alpha' = V \times \alpha'.$$

In this study, firstly we define the notions of T-magnetic, N-magnetic and B-magnetic pseudo null curves in Minkowski 3-space. Also, we obtain the magnetic

vector field V when the pseudo null curve is a T-magnetic, N-magnetic and B-magnetic trajectory of V and finally, we give an example for these magnetic curves.

Key Words: Magnetic curves, pseudo null curves, Killing vector fields, Lorentz force.

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Magnetic Trajectories in Sasakian Space

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ABSTRACT

A charged particle in a magnetic field experiences a force which called as Lorentz force. The Lorentz force, ϕ, on the particle is defined via the cross product in Riemannian manifold M by $\phi(X)=V\times X$, for all $X\in\chi(M)$. When a charged particle enters the magnetic field the velocity vector of the trajectory curve is expressed to the magnetic field and experience a force. So, charged particle follows a trajectory called magnetic curve with the influence of the force. The magnetic trajectory of a magnetic field, V is a curve which satisfies the Lorentz force equation $\nabla_{\gamma'}\gamma'=\phi(\gamma')=V\times\gamma'$.

Through this equation magnetic curves can found many useful applications in analytical chemistry, biochemistry, atmospheric science, geochemistry cycloton, proton, cancer therapy, and velocity selector. Moreover, the solutions of the Lorentz force equation are Kirchhoff elastic rods. This provides an amazing connection between two apparently unrelated physical models and, in particular, it ties the classical elastic theory with the Hall effect.

The aim of this study is to classify all magnetic curves in Sasakian space form $R^3(-3)$ related to the Killing magnetic field F_ξ. We find that they are Legendre circles, Legendre helices or circular helices in $R^3(-3)$. Finally, we give their illustrated examples and figures.

Key Words: Sasakian manifolds, magnetic vector fields, magnetic curve.

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New Type Slant Helices

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ABSTRACT

The curve theory has been one of the most studied subject because of having many application area from geometry to the various branch of science. Especially the characterizations on the curvature and torsion play important role to define special curve types such as so-called helices. The curves of this type has drawn great attention from mathematics to natural sciences and engineering. Helices appear naturally in structures of DNA, nanosprings. This magical structure seen also protein folds as three, four helix bundles dependent on the protein structures. And also widely used in engineering and architecture because of many advantageous features. By all means, these conspicuous properties of the helix arise interest also for those who studies visualizing response of materials, and computer designers. This striking features of helical structure first of all, motivates mathematicians to study on and encourage to define special type helices.

The concept of slant helix defined by Izumiya and Takeuchi [1] based on the property that the principal normal lines of a curve (with non-vanishing curvature) make constant angle with a fixed direction of the ambient space. In [2] authors extended slant helix to E^n and conclude that there are no slant helices with non-zero constant curvatures in the space E^4 and in [3] the authors studied slant helix concept to the k-type slant helix and they defined k-type slant helices.

After this lightening work many researchers have characterized this type of curves in various spaces and obtained different characterizations [3-5]. One may easily conclude that 0-type slant helices are general helices and 1-type slant helices correspond just slant helices.

In the present work, we have dealt with new type slant helices and focus on the characterizations of it.

Key Words: Helices, slant helices, Serret-Frenet formulas.

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On A Type Of α-Cosymplectic Manifolds

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ABSTRACT

A $(2n+1)$-dimensional differentiable manifold M of class C^{∞} is said to have an almost contact structure if the structural group of its tangent bundle reduces to $U(n)\times 1$, equivalently an almost contact structure is given by a triple (ϕ,ξ,η) satisfying certain conditions. Many different types of almost contact structures are defined in the literature.

In [3], Pokhariyal and Mishara have introduced new tensor fields, called W_2 and $E-$tensor fields, in a Riemanian manifold, and studied their properties. Then, Pokhariyal [2, 3] has studied some properties of this tensor fields in Sasakian manifolds. Recently, Matsumoto, Lanus and Mihai [4] have studied $P-$Sasakian manifolds admitting W_2 and $E-$tensor fields and De and Sarkar [5] have studied $P-$Sasakian manifolds admitting W_2 tensor field. The curvature tensor W_2 is defined by

$$W_2(X,Y,U,V)=R(X,Y,U,V)+\frac{1}{n-1}\left[g(\mathrm{X},\mathrm{U})\mathrm{S}(\mathrm{Y},\mathrm{V})-g(\mathrm{Y},\mathrm{U})\mathrm{S}(\mathrm{X},\mathrm{V})\right],$$

where S is a Ricci tensor of type $(0,2)$.

In [1], Yıldız and De have studied some curvature conditions on Kenmotsu manifolds. They have studied their geometric and relativistic properties in Kenmotsu manifolds satisfying $W_2=0$ and obtain W_2-semi-symmetric Kenmotsu manifolds. Also, they have classified Kenmotsu manifolds which satisfy $P.W_2=0$, $\tilde{Z}.W_2=0$, $C.W_2=0$ and $\tilde{C}.W_2=0$ where P is the projective

curvature tensor, $\tilde{Z}$ is the concircular curvature tensor, $\tilde{C}$ is the quasi-conformal curvature tensor and C is the conformal curvature tensor. In the present paper, we have studied some curvature conditions on $\alpha-$cosymplectic manifolds. We also have classified $\alpha-$cosymplectic manifolds which satisfy the conditions $P.W_2 = 0$, $\tilde{Z}.W_2 = 0$, $C.W_2 = 0$ and $\tilde{C}.W_2 = 0$.

Key Words: Contact Manifold, $\alpha-$Cosymplectic Manifold, W_2-curvature tensor.

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On Applications of Real Dicompactness to Hutton Spaces

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ABSTRACT

One of the theories defined to develop some complement-free concepts is the *texture theory*. There are so many results about the properties of textures and ditopological texture spaces in the topology literature.

Especially, in [3], we had presented several important relations occur between the properties of a T-lattice (lattice with a suitable translation operation) of bicontinuous real-valued point functions satisfying a compatibility condition and the ditopological properties of a ditopological texture space.

By using these relations, a natural counterpart for the notion of real compactness in the context of ditopological texture spaces was introduced under the name *real dicompactness* [2]. In fact, the notion of real dicompactnes is a generalization of the pointed notion of real compactness given in the literature for bitopological spaces. In particular, it was largely developed using the special point functions satisfying the compatibility condition and all details of this theory are investigated for the subcategories consisting of special types of ditopological texture spaces, as given in [1].

Hence, the principal aim of this work is to extend the real dicompactness to a large class of textures naturally and to adapt this extended theory to Hutton Spaces by taking into consideration the fact that ditopological texture spaces include Hutton Spaces as a particular case.

Key Words : Texture, Real Dicompactness, T-lattice, Hutton Space

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On Complex Sasakian Manifolds

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ABSTRACT

Sasakian manifolds are special classes of contact manifolds and they are studied by lots of geometers. Complex Sasakian manifolds are complex analogue of Sasakian manifolds and alongside normality condition the complex Sasakian manifolds have a global complex contact form. Kobayashi [5] proved that a complex contact form on a complex contact manifold is globally defined if and only if its first Chern class vanishes. In this way there are some restrictions on complex Sasakian manifolds with global complex contact form. Although complex projective space is an example of complex contact manifolds since it has not global complex contact form, is not complex Sasakian. On the other hand a well-known example of complex contact manifolds, complex Heisenberg group is an example of complex Sasakian manifolds. Complex Sasakian manifolds are firstly studied by Foreman [5] and then Fetcu [4] examined harmonic maps between complex Sasakian manifolds.

There are not enough studies on complex Sasakian manifolds. From this point of view one can see necessarily of working on this subject. In this work, we give the definition of complex Sasakian manifolds such as real Sasakian manifolds and we prove the necessary and sufficient condition for being complex Sasakian manifolds. Also we give curvature equalities and obtain some new results about them.

Key Words: Complex Sasakian, complex contact, global complex contact form.

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On Contact Pseudo-Slant Submanifolds of a Para- Sasakian Space Form

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ABSTRACT

In 1985, S. Kaneyuki and M. Konzai defined the almost paracontact structure on pseudo-Riemannian manifold M of dimension (2n+1) and constructed the almost paracomplex structure on $M^{2n+1}\times\mathbb{R}$. Recently, S. Zamkovoy studied paracontact metric manifolds and some remarkable subclasses like para-Sasakian manifolds. Especially, in the recent years, many authors have pointed out the importance of paracontact geometry, and in particular of para-Sasakian geometry, by several papers giving the relationships with the theory of para-Kaehler manifolds and its role in pseudo-Riemannian geometry and mathematical physics.

In 1990, B. Y. Chen introduced the notion of slant submanifolds, which is generalization of both the holomorphic and totally real submanifolds. After that many research articles have been published by different authors on the existence of these submanifolds in different ambient spaces. The slant submanifols of an almost contact metric manifolds were defined and studied by A. Lotta. After, these submanifolds were studied by J.L Cabrerizo et al. of Sasakian manifolds.

The notion of semi-slant submanifolds of an almost Hermitian manifold was introduced by N. Papagiuc . Cabrerizo et al. studied and characterized slant submanifolds of K- contact and Sasakian manifolds and gave several examples of such submanifolds. Cabrerizo et al. defined and studied bi-slant immersions in almost contact metric manifolds and simultaneously gave the notion of pseudo-slant submanifolds. Pseudo-slant submanifolds also have been studied by Khan at al. in After, U. C. De et al. studied and characterized pseudo-slant submanifolds of trans

Sasakian manifolds. Recently, in, Atçeken et al. studied slant and pseudo-slant submanifold in various manifolds.

In this paper, we study the geometry of the contact pseudo-slant submanifolds of a para-Sasakian space form. Necessary and sufficient conditions are given for a submanifold to be contact pseudo-slant submanifolds, contact pseudo-slant product, mixed geodesic and totally geodesic in para-Sasakian manifolds. Finally, we give some results for totally umbilical pseudo-slant submanifolds of para-Sasakian manifolds and para-Sasakian space forms.

Key Words: para-Sasakian manifold, para-Sasakian space form, slant submanifold, pseudo-slant submanifold.

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On *f*-Kenmotsu manifolds

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ABSTRACT

In this paper we study some semi-symmetry conditions and Ricci solitons on *3*-dimensional *f*-Kenmotsu manifolds with the Schouten-van Kampen connection. Firstly, we adapte the Schouten-van Kampen connection on 3-dimensional f-Kenmotsu manifolds and we give the curvature tensor, the projective curvature tensor and the conharmonical curvature tensor with respect to the Schouten-van Kampen connection on 3-dimensional f-Kenmotsu manifolds. We study projective symmetric 3-dimensional f-Kenmotsu manifolds with the Schouten-van Kampen connection and we obtain if the manifold is projective symmetric then the manifold is an Einstein manifold. Also, we consider conharmonical symmetric 3-dimensional f-Kenmotsu manifolds with the Schouten-van Kampen connection and we obtain if the manifold is conharmonical symmetric then the manifold is an Einstein manifold. Then we study Ricci semisymmetric 3-dimensional f-Kenmotsu manifolds with the Schouten-van Kampen connection and we prove that if a 3-dimensional f-Kenmotsu manifold is Ricci semisymmetric then it is an η-Einstein manifold. In the last, we study semisymmetric 3-dimensional f-Kenmotsu manifolds with the Schouten-van Kampen connection and we obtain if the manifold is semi symmetric then the manifold is an Einstein manifold. Also we study Ricci solitons on 3-dimensional f-Kenmotsu manifolds with the Schouten-van Kampen connection and we classify 3-dimensional f-Kenmotsu manifolds with the Schouten-van Kampen connection.

Keywords. Almost contact metric manifolds, the Schouten-van Kampen connection, semisymmetry, Ricci solitons.

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On Fuzzy Soft Topological Groups

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ABSTRACT

In 1968, Chang [1] introduced fuzzy topology by axiomatizing some properties of a collection fuzzy subsets. In 1976, Lowen [6] gave a new definition which is called fully stratified (or Lowen type) fuzzy topological space. In 1984, MA Ji Liang and YU Chun Hai [4] introduced fuzzy topological group by using fuzzy quasi-coincident and fuzzy Q-neighbourhood and they gave some notions related to their definition.

Afterwards, in 1999, Molodsov [2] proposed a new approach by introducing "soft set theory" for modeling uncertainties. Subsequently, several mathematical structures have been developed using fuzzy set theory and soft set theory or a combination of these two theories. In 2001, Maji et al. [5] worked on some mathematical aspects of fuzzy soft sets. In 2013, Roy and Samanta [8] studied fuzzy soft point and its Q-neighbourhood structure. In 2014, Nazmul and Samanta [7] introduced a notion of Lowen type fuzzy soft topology and studied some more properties of this fuzzy soft topological spaces. They defined fuzzy soft continuity and introduced fuzzy soft topological group by using continuous functions. Recently, we introduced fuzzy soft topological group by using fuzzy soft quasi-coincident and fuzzy soft Q-neighbourhood and gave a related example [3].

In this paper, we define the component of the fuzzy soft point and give the definition of the fuzzy soft continuous function with respect to Q-neighbourhood at any fuzzy soft point. Then, we introduce some notions about fuzzy soft topological group.

Key Words: Fuzzy soft quasi-coincident, Fuzzy soft Q-neighbourhood, Fuzzy soft topological groups

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On Globally Framed Almost f -Cosymplectic Manifolds

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ABSTRACT

The notion of an f -structure on a C^∞ manifold M was introduced by yano. It consist of a tensor field φ of type $(1,1)$ and rank $2n$ satisfying $\varphi^3+\varphi=0$. The existence of an f-structure is equivalent to a reduction of the structureal group of the tangent bundle to $U(n)\times O(s)$ ([2]). Blair studied various classes of such manifolds ([1]). The purpose of this paper is to study a new and wide subclass of framed manifolds which is admits an f -structure . Such manifolds are called globally framed almost f -cosymplectic manifolds . For some special cases of n and s, one obtains (almost) f -cosymplectic, (almost) C -manifolds, and (almost) kenmotsu f -manifolds and (almost) α - cosymplectic manifolds.

Firstly, we give the concept of globally framed almost f-cosymplectic manifolds and state general curvature properties. We derive several important formulas on globally framed almost f -cosymplectic manifolds. These formulas enable us to find the geometrical properties of globally framed almost f -cosymplectic manifolds with η -parallel tensors h_i and φh_i. We also examine the tensor fields τ_i's which are defined by

$$g(\tau_i X,Y)=(L_{\xi_i} g)(X,Y),$$

for arbitrary vector fields X,Y on M. Then we give some results on η -parallelity, cyclic parallelity, Codazzi condition and we get some classifications by using stated tensor conditions. Finally, we give an explicit example of globally framed almost f -cosymplectic manifolds.

Key Words: Framed manifold, Kenmotsu manifold, cosymplectic manifold

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On Inextensible Flows of Developable Surfaces Associated Focal Curve According to Ribbon Frame

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ABSTRACT

The analysis of space curves in differential geometry is a classical subject. For any unit speed curve, the focal curve is defined as the centres of the osculating spheres. According to Frenet frame $\{t(s), n(s), b(s)\}$ of unity speed curve $\gamma(s)$, the focal curve is given as follows,

$$C_\gamma(s) = (\gamma + c_1 n + c_2 b)(s)$$

where the coefficients are smooth functions that are called focal curvatures of γ. There have been several studies related to the focal curve using this definition Vergas, Arslan, Korpinar concerned with this isuse [1,2,3].

Any ribbon consists of two functions $w(s)$ and $\theta(s)$ are defined in interval s ∈ [0, L], where L is the intrinsic length of the ribbon under construction. A unit vector field $A(s)$ is defined by having the angle $\theta(s)$ to the center curve of the ribbon. On the other hand, $A(s)$ is the direction field for the Darboux vector $D(s)$ with the generating function $w(s)$. An orthonormal triple $\{e, f, g\}$ has the following differential system:

$$e' = wA \times e$$

$$f' = wA \times f$$

$$g' = wA \times g$$

This orthonormal triple defines a ribbon frame. According to this frame, the focal curve can be given as follows:

$$C_\gamma(s) = (\gamma + c_1 f + c_2 g)(s)$$

Curve design using splines is one of the most fundamental topics in CAGD. Inextensible flows of curves possess a beautiful shape preserving connection to their control polygon. They allow us the formulation of algorithms for processing. Kwon,

Park and Chi examined inextensible flows of curves and develople surfaces [6]. Korpınar, Turhan and Altay gave inextensible flows of develople surfaces associated focal curve [3]. Recently, Bohr and Markvosen examined the ribbon frame [4]. Giomi and Mahadevan gave develople ribbons [7].

In this paper, we study inextensible flows of focal curves with ribbon frame. We give some characterizations for curvature and torsion of focal curves associated with developable surfaces.

Key Words: Focal curve, inextensible flows, ribbon frame

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On Liftings of Crossed Modules

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ABSTRACT

A groupoid $\tilde{G}$ is called a covering of G if for a groupoid morphism $p:\tilde{G}\to G$ the restrictions of p to the stars of $\tilde{G}$ are bijections.Covering groupoids plays an important role in applications of groupoids [1,4]. For a fixed groupoid G the category of coverings of G and the category of groupoid actions of G on sets, i.e. functors from to Set category, are naturally equivalent. Internal categories in the category of groups or equivalently group objects in the category of small categories are called group-groupoids [6]. Recently, analogous equivalence has been proved for the group-groupoid case, that is, for a fixed group-groupoid G the category of covering groupoids of G and the category of group-groupoid actions of G on groups are naturally equivalent.

Crossed module notion has been defined by Whitehead in 1949 [8]. Crossed modules were the first example of higher dimensional algebra to be studied and has important role in many area of mathematics such as homotopy theory, (co)homology of groups, algebraic K-theory, combinatorial group theory and differential geometry. Thus crossed modules are one of the fundamental algebraic structures now. Brown-Spencer [6] and Loday [7] proved that the category of crossed modules and the category of group-groupoids are equivalent.

The main objective of this study is to define the corresponding notion of group-groupoid action in crossed modules, which will be called liftings of crossed modules. We also investigate some properties of lifting crossed modules. Further we prove the equivalence between the category of lifting crossed modules and the category of covering crossed modules for a fixed crossed module.

Key Words: Crossed module, covering, lifting, action groupoid.

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On Lorentzian Para-Sasakian Manifold

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ABSTRACT

Friedmann and Schouten [1] introduced the idea of semi-symmetric linear connection on a differentiable manifold in 1924. A linear connection $\widetilde{\nabla}$ in a n-dimensional differentiable manifold M is said to be semi-symmetric connection if its torsion $\widetilde{T}$ is of the form

$$\widetilde{T}(X,Y) = \eta(X)Y - \eta(Y)X,$$

where η 1- form. The connection $\widetilde{\nabla}$ is a metric connection if there is a Riemannian metric g in M such that $\widetilde{\nabla}g = 0$ otherwise it is non-metric. H. A Hayden [2] defined a semi-symmetric metric connection on a Riemann manifold and this was further developed by K. Yano [4] in 1930. Besides, Golab [6] defined and studied quarter-symmetric connection in a differentiable manifold with affine connection. After that various properties of quarter-symmetric metric connection have been studied by many geometers. Adati and Matsumoto defined Para-Sasakian and Special Para-Sasakian manifolds [7] which are special classes of an almost paracontact manifold introduced by Sato [3]. Para-Sasakian manifolds have been studied many others authors. M. M. Tripathi [5] proved the existence of a new connection and showed that in particular cases, this connection reduces to semi-symmetric connections; even some of them are not introduced so far. On the other hand, there is a class of almost paracontact metric manifolds, namely Lorentzian Para-Sasakian manifolds.

In this paper we investigate ξ–conharmonicly flat and ϕ-conharmonicly symmetric with respect to semi-symmetric non-metric connection in a Lorentzian Para-Sasakian manifold. Also we will find ξ–conharmonicly flat and ϕ-conharmonicly symmetric using the quarter-symmetric non-metric connection in a Lorentzian Para-Sasakian manifold.

Key Words: Lorentzian Para-Sasakian manifold, semi-symmetric non-metric connection, quarter-symmetric non-metric connection

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On Minimal Homothetical Surfaces

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ABSTRACT

Even in ancient times, people were interested in minimal surfaces. Now, the theory of minimal surface takes part in large research area of mathematics, physics, engineering, material sciences and robotics. Definition of minimal surfaces can be expressed in several equivalent ways. In this study, we make differential geometry approach which is having zero mean curvature. Minimal surfaces may also be known as surfaces of minimal surface area for given boundary conditions. The most popular classification of minimal surfaces is given by Lagrange's differential equation. A plane is a trivial minimal surface, but more with the help of the differential equation, determination which surface is minimal is so simple. On the other hand, the opposite problem namely obtaining the all solutions of the differential equation is still an open problem. Until now, just a few surfaces have been obtained as a minimal surface. The most famous examples of three dimensional Euclidean space are catenoids, helicoids, Enneper surface, Costa's surface and Scherk's surface. Because of the strict problem of minimality, the theory of minimal surfaces has initiated diversification to minimal submanifolds in ambient geometries. In this study, we focus on minimality condition for homothetical surfaces which is built by function product. Firstly, we give a summary about minimal homothetical hypersurfaces in Euclidean space. Then, the classification is expanded in other metric spaces.

Key Words: Minimal surface, homothetical surface, Lagrange's differential equation.

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On Mixed Fuzzy Soft Topological Spaces

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ABSTRACT

In 1965, Zadeh [5] introduced the notion of fuzzy sets. Afterwards, in 1999, Molodtsov [4] introduced the concept of soft set theory which is a completely new approach for modelling uncertainty. This new theory helps to solve problem in all areas. Maji, Biswas and Roy [6] defined and studied several basic notions of soft set theory.

Recently, researchers have contributed a lot towards fuzzification of soft set theory. In 2001, Maji et al. [7] introduced the concept of fuzzy soft set and some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan Law etc. Tanay and Kandemir [2] studied the definition of fuzzy soft topology over a subset of initial universe set while Roy and Samanta [8] gave the definition of fuzzy soft topology over the initial universe set. Tripathy and Ray [3] introduced and studied the concept of mixed fuzzy topological spaces and countability. Gezici and Yildiz [1] defined the complement of a fuzzy soft set which is different from the classical complement of a fuzzy soft set. By using the new complement definition they introduced mixed fuzzy soft topological spaces over a fuzzy soft set of initial universe set.

In this study we give some properties of mixed fuzzy soft topological spaces. Also we define the notions of fuzzy soft neighbourhood, Q-fuzzy soft neighbourhood over a mixed fuzzy soft topological spaces. We define first countable, second countable and Q-first countable spaces on this space.

Key Words: Fuzzy soft set, fuzzy soft topology, first countable space, second countable space.

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On Parallel Surfaces of Non-Developable Ruled Surfaces in Eucledian 3-Space

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ABSTRACT

A surface and another surface which have constant distance with the reference surface along its surface normal have a relationship between their parametric representations. Such surfaces are called parallel surface. So, there are infinite number of parallel surfaces because we choose the constant distance along the normal arbitrarily. A parallel surface can be regarded as the locus of point which are on the normals to M at a non-zero constant distance r from M.

We denote the Euclidean 3-space by E^3 and a regular parameter surface with the parameters u and v in E^3 by $X(u,v)$. A ruled surface defined by transformation

$$X(a,b): I \times E \rightarrow E^3$$
$$(u,v) \rightarrow X(a,b)(u,v) = a(u) + vb(u).$$

Here, $a: I \rightarrow E^3, b: I \rightarrow E^3\backslash\{0\}$ are differentiable transformations and I is an open interval. a is called as base curve of the ruled surface and b is the director curve. If $b(u)$ is a constant, ruled surface is called as a cylinder.

A developable surface is a ruled surface having Gaussian curvature $K = 0$ everywhere. Developable surfaces therefore include the cone, cylinder, elliptic cone, hyperbolic cylinder, and plane. We will refer the term non-developable, and by a non-developable surface we mean that a surface free of points of vanishing Gaussian curvature in a 3-dimensional Euclidean space. In this paper, we will take a non-

developable ruled surface $X(u,v) = a(u) + vb(u)$ in E^3 with $b^2(u) = 1$ and the parameter u is the arc length parameter of $b(u)$ as a unit spherical curve in E^3. Furthermore, we assume that the base curve $a(u)$ of the ruled surface $X(u,v)$ is the striction line of the surface, that means $a'(u).b'(u) = 0$. Our aim is to obtain parallel surface of $X(u,v)$ and is to examine the differential geometric properties between the surface $X(u,v)$ and the parallel surface of the surface $X(u,v)$.

Key Words: Parallel surface, ruled surface, non-developable ruled surface, Gauss curvature, mean curvature, striction line.

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On Screen Conformal Half-Lightlike Submanifolds

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ABSTRACT

The geometry of submanifolds is one of the most topic of interest. In general, the induced metric of a Riemannian or a semi-Riemannian manifold does not have to be non-degenerate. If the induced metric is degenerate (lightlike) therefore there is a natural existence of lightlike submanifolds and which the local and global geometry is completely different than non-degenerate case since normal vector bundle intersects with the tangent bundle. Because of this anomaly, Duggal and Bejancu [1] introduced a non-degenerate screen distribution to construct a nonintersecting lightlike transversal vector bundle of the tangent bundle. Also, in the same study, they defined a subclass called half-lightlike submanifolds. Then, many authors studied on geometry of half-lightlike submanifolds .

In this study, the geometry of screen conformal half-lightlike submanifold M of a semi-Riemannian manifold was investigated. Firstly, some properties of a half-lightlike submanifold of a semi-Riemannian manifold whose shape operator is conformal were given. Besides, it was shown that any screen distribution S(TM) of M is integrable and there exists a relation with geometry of M and the non-degenerate geometry of a leaf of S(TM). Then, screen conformal half-lightlike submanifolds of a semi-Riemannian space form with a Killing co-screen distribution were studied and some characterization theorems were given.

Key Words: Half-lightlike submanifold, screen conformal submanifold, semi-Riemannian manifold.

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On Semi-Slant Submanifolds of a Sasakian Space Form

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ABSTRACT

In this paper, we study semi-slant submanifolds of a Sasakian manifold. We review basic formulas and definitions for a Sasakian manifold and their submanifolds. After, we recall the definition and some basic results of a semi-slant submanifold of almost contact metric manifold. Finally, we will give same results for totally umbilical semi-slant submanifold in a Sasakian manifold and Sasakian space form $\tilde{M}(c)$.

The differential geometry of slant submanifolds has shown an increasing development since B.-Y. Chen defined slant immersions in complex geometry as a natural generalization of both holomorphic immersions and totally real immersions. Many authors have studied such slant immersions in almost Hermitian manifolds. A. Lotta has introduced the notion of slant immersion of a Riemannian manifold into an almost contact metric manifold. We have studied and characterized slant submanifolds of *K*-contact and Sasakian manifolds.

Recently, in N. Papaghiuc has introduced a class of submanifolds in an almost Hermitian manifold, called the semi-slant submanifolds, such that the class of proper *CR*-submanifolds and the class of slant submanifolds appear as particular cases in the class of semi-slant submanifolds.

Let $\tilde{R}$ be the curvature tensor of the connection $\tilde{\nabla}$. The sectional curvature of a $\varphi-$section is called a $\varphi-$holomorphic sectional curvature. A Sasakian manifold with constant $\varphi-$holomorphic sectional curvature c is said to be a Saakian space form and it is denoted by $\tilde{M}(c)$. The curvature tensor $\tilde{R}$ e of a Sasakian space form $\tilde{M}(c)$ is given by

$$\tilde{R}(X,Y)Z=\frac{(c+3)}{4}\{g(Y,Z)X-g(X,Z)Y\}+\frac{(c-1)}{4}\{\eta(X)\eta(Z)Y-\eta(Y)\eta(Z)X$$
$$+\eta(Y)g(X,Z)\xi-\eta(X)g(Y,Z)\xi+g(X,\varphi Z)\varphi Y-g(Y,\varphi Z)\varphi X+2g(X,\varphi Y)\varphi Z\}$$

for any $X,Y\in\Gamma(T\tilde{M})$.

We will study semi-slant submanifolds in a Sasakian space form, give some characterization and submanifold is characterized.

Corollary 1. Let M be a semi-slant submanifold of a Sasakian manifold of a Sasakian space form $\tilde{M}(c)$ with flat normal connection such that $c\neq 1$. If $TA_V=A_VT$ for any vector V normal to M, then M is either an invariant or generic submanifold of $\tilde{M}(c)$.

Corollary 2. Let M be a semi-slant submanifold of a Sasakian manifold of a Sasakian space form $\tilde{M}(c)$. Then the Ricci tensor S of M is given by

$$S(X,W)=\left\{\frac{(c+3)}{4}(2p+2q)+\right\}g(X,W)$$
$$+\frac{(c-1)}{4}\left\{(1-2q-2p)\eta(X)\eta(W)+(3\cos^2\theta-1)g(X,W)\right\}$$
$$+(2p+2q+1)g(h(X,W),H)-\sum_{m=1}^{2p+2q}g(h(e_m,W),h(X,e_m))$$

Corollary 3. Let M be a semi-slant submanifold of a Sasakian manifold of a Sasakian space form $\tilde{M}(c)$. Then scalar curvature ρ of M is given by

$$\rho=\left\{\frac{(c+3)}{4}(2p+2q)+\frac{(c-1)}{4}\right\}(2p+q+1)$$
$$+\frac{(c-1)}{4}\left\{(3\cos^2\theta(2p+2q+1)+(-4p-4q)\right\}-(2p+2q+1)^2\|H\|^2+\|h\|^2.$$

Key Words: Sasakian manifold, Sasakian space form, slant submanifold, semi-slant submanifold.

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On Soft Topological Categories

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ABSTRACT

Soft set theory was proposed by Molodtsov[1] in 1999 as a new mathematical approach for modeling vagueness and uncertainties inherent in the problems of physical science, biological science, engineering, economics, social science, medical science, etc. This theory has gone through a rapid development process and has found a wide application area. Especially, it has attracted a great attention of the mathematicians and has been worked on with some mathematical aspects such as algebra and topology by many mathematicians in recent years. For example, the concept of the soft category is defined by combining soft set theory with category theory which is defined as a classification of mathematical structures. In the light of this definition, categorical studies have been made on soft set theory.

In this paper, we introduce the notion of a soft topological category as a natural consequence of the existence of topological category and soft category. Some examples of the soft topological categories are given. The properties of soft topological category are investigated and some important results are obtained. It is proved that the product of two soft topological categories is also a soft topological category. Also, the notion of topological functor is extended to the notion of soft topological functor. Finally, we present some examples about it.

Key Words: Soft set, soft group, soft topological group, category, functor, soft category, soft topological category.

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On Some Classes of Ruled Surfaces in Isotropic Space

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ABSTRACT

The isotropic space is a projective metric space with the absolute figure consisting of a plane Ω (absolute plane) and two complex conjugate lines l_1,l_2 in Ω. Up to the homogenous coordinates, Ω is given by $x_0=0$ and l_1,l_2 by $x_0=x_1\pm ix_2=0$. The metric of the space is introduced by the absolute figure. In terms of the affine coordinates $x= x_1/ x_0$, $y= x_2/ x_0$, $z= x_3/ x_0$, the metric is $ds^2=dx^2+dy^2$.

There are two types of ruled surfaces in the isotropic space with respect to the absolute figure (see [6]). Certain isometric and conformal mappings between these surfaces were investigated in [4]. In addition, the authors in [2] concerned the ruled surfaces generated by elliptic cylindrical curves and characterized such surfaces in terms of their curvatures and Laplace operator.

In the present talk, we deal with the ruled surfaces whose the director curve is Frenet vector field of the base curve. Therefore, three different classes of such surfaces appear, called tangent developable, principal normal surface, binormal surface. However, the binormal surface is not admissible (i.e. with isotropic tangent planes) and we thus consider a generalized cone with the vertex at the end point of the binormal vector instead of the mentioned surface. We obtain those surfaces in which the Gaussian and mean curvature vanish.

Key Words: Isotropic space, Ruled surface, Isotropic Gaussian curvature.

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On Some Special Curves

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ABSTRACT

The theory of curves is one of the main study areas in differential geometry. Especially, some special curves have been studied for a long time by mathematicians, physicians. Among all space curves, helices have the most popular role regarding their properties and applicability in other disciplines. Helices can be seen in DNA molecules, bacterial flagella, structure of proteins, carbon nanotubes, architecture, fractal structures, magnetic curves, etc. Because of the popularity of them, they have studied in both Euclidean geometry and non-Euclidean geometries. Some of other known examples of the special curves are Rectifying curves, Bertrand curves, Involute-Evolute curves, Mannheim curves, Salkowski and anti-Salkowski curves etc.

The aim of this paper is to characterize some curves with the help of their harmonic curvature functions. First of all, we define harmonic curvature function of an arbitrary curve and re-determine the position vectors helices in terms of their harmonic curvature functions in Galilean 3-space. Then, we investigate the relation between rectifying curves and Salkowski (or anti-Salkowski) curves in Galilean 3-space with the help of their harmonic curvature functions. Furthermore, the position vectors of them are obtained. In order to obtain this aim, we will use the serial approach of the curve with the third-order polynomial coefficients differential equations. Finally, we give some illustrated examples of helices and rectifying curves with some assumptions.

Key Words: Rectifying Curve, Harmonic Curvature, Salkowski Curves

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On The Area Of Hyperbolic Triangle In Terms Of Gudermann Angles

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ABSTRACT

Christoph Gudermann (March 25, 1798 – September 25, 1852) was, a student of Karl Friedrich Gauss. He also was the teacher of Karl Weierstrass, who was greatly influenced by Gudermann's course on elliptic functions in 1839–1840. Gudermannian function, or hyperbolic amplitude, is named after him.

Gudermannian function relates the circular functions and the hyperbolic functions without using complex numbers.

Gudermannian function or the hyperbolic amplitude function is given by the following integral

$$gd(x) = \int_0^x secht\ dt$$
$$= 2\arctan(e^x) - \frac{\pi}{2}$$

Lobachewsky function (Angle of Parallelism) is seen in page http://mathworld.wolfram.com/AngleofParallelism.html
as

$$\Pi(x)= \begin{cases} 2\arctan(e^{-x}), x \geq 0 \\ \pi - 2\arctan(e^x), x < 0 \end{cases}$$

Lobachewsky function is connected with Gudermannian by

$$gd(-x) = \Pi(x) - \frac{\pi}{2}.$$

Using the ordinary trigonometric circle and its *“hyperbolic”* counterpart the hyperbolic functions defined on arcs of a unit hyperbola in Lorentzian metric, which plays the same role of the unit circle in Euclidean metric. The hyperbolic sine and cosine functions satisfy indeed the identity

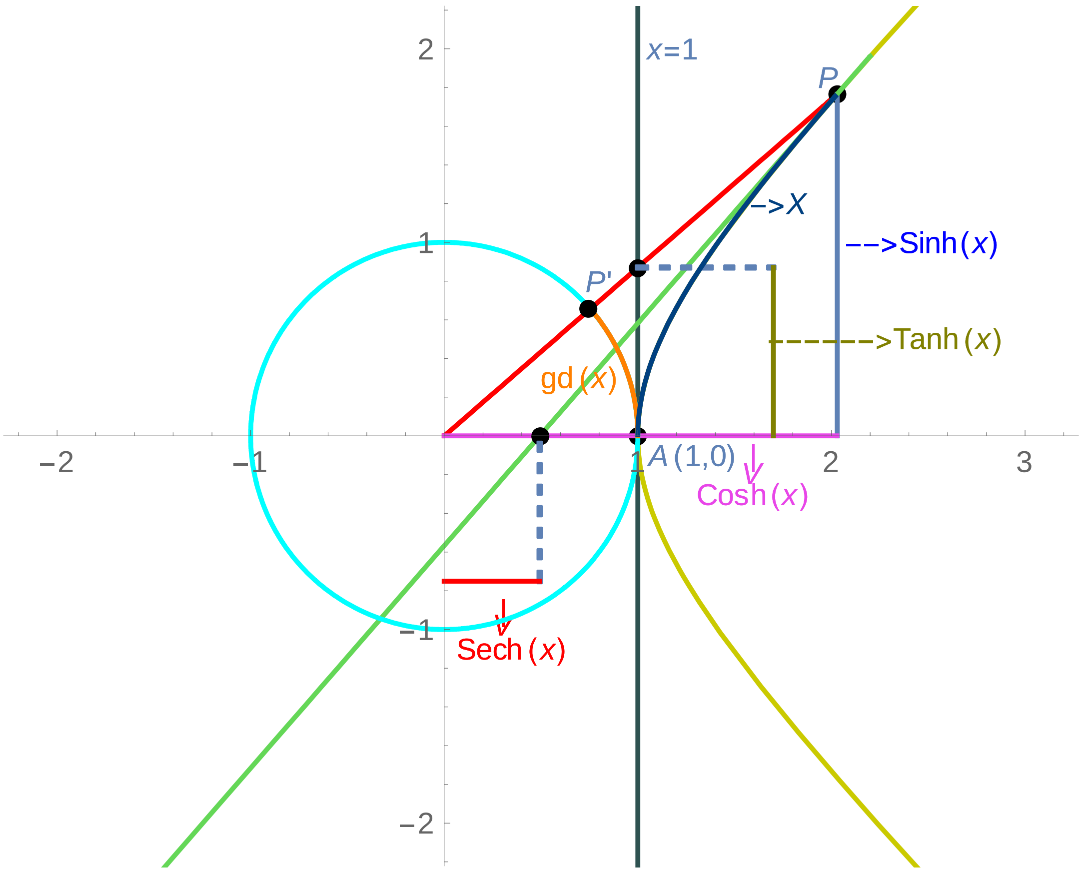
$$\cosh^2 x - \sinh^2 x = 1$$

and the argument which is the counterpart of the angle in ordinary trigonometry.

Measure of an hyperbolic angle is given by an area sector in Euclidean metric. Measure of an Euclidean angle is given by length of unit circular arc in Euclidean metric([1,3-5]).

Gudermann angle is defined as an ordinary angle, its measure is given by the length of unit circular arc ([1,3-5]). Guderman function is depend on unit hyperbole arc. The Inverse Guderman function depend on unit circular arc.

In that study, we consider the measure of an hyperbolic angle by an arc length of unit hyperbola in Lorentzian metric. After we present the characteristics of Gudermann functions, we give the area of a hyperbolic triangle interms of Gudermanian angles using formulas in [2] .



$Cosh(x), Sinh(x), Tanh(x), Sech(x), gd(x)$

Key Words: Gudermann, Inverse Gudermann function, Hyperbolic Angle

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On The Energy Of Elastica With The Help Of Null Curves In Minkowski Space

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ABSTRACT

The elastica caught the attention of many of the brightest minds in the history of mathematics, including Galileo, the Bernoullis, Euler, and others. It was present at the birth of many important fields, most notably the theory of elasticity, the calculus of variations, and the theory of elliptic integrals. The path traced by this curve illuminates a wide range of mathematical style, from the mechanics-based intuition of the early work, through a period of technical virtuosity in mathematical technique, to the present day where computational techniques dominate. There are many approaches to the elastica. The earliest (and most mathematically tractable), is as an equilibrium of moments, drawing on a fundamental principle of statics. Another approach, ultimately yielding the same equation for the curve, is as a minimum of bending energy in the elastic curve. A force based approach finds that normal, compression, and shear forces are also in equilibrium; this approach is useful when considering specific constraints on the endpoints, which are often intuitively expressed in terms of these forces. Later, the fundamental differential equation for the elastica was found to be equivalent to that for the motion of the simple pendulum. This formulation is most useful for appreciating the curve's periodicity, and also helps understand special values in the parameter space. In this paper, we investigate energy of elastic null curves in E^4_1. Firstly, it is obtained Frenet equations for null curves. Additionally, we give the energy of differerent type of lightlike curves defined in Minkowski space E^4_1. We also determine the connection both the energy of lightlike particle and bending energy of elastica functional.

Key Words: Elastica, Minkowski space, Energy.

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On the Evolution of Quaternionic Curves in Euclidean 4-space

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ABSTRACT

The time evolution of a curve or a surface generated by its corresponding flow. For this reason, curve and surface evolutions are usually called flows. The flow of a curve or a surface is said to be inextensible if arclength is preserved. Physically, inextensible curve and surface flows give rise to motion in which no strain energy is induced. The swinging motion of a cord of fixed length, for example, or of a piece of paper carried by the wind, can be described by inextensible curve and surface flows. Such motions arise quite naturally in a wide range of physical applications. For example, both Chirikjian and Burdick [1] and Mochiyama et al. [2] study the shape control of hyper-redundant, or snake-like, robots. Inextensible curve and surface flows also arise in the context of many problems in computer vision [3,4]. In a previous communication by the authors [5], the distinction between heat flows and inextensible flows of planar curves were elaborated in detail, and some examples of the latter were given. In [6,7], authors have studied flow concept differently.

Our aim is study concept of evolution as in [6,7] for quaternionic curve in the Euclidean four-dimensional space. Firstly, we introduce dynamic of the quaternionic curve. Then, we obtain the evolution equation of velocity (non-unit) of quaternionic curve. We give necessary and sufficient conditions for inextensible flows of quaternionic curve. Then, we express derivatives of the elements of Frenet frame of quaternionic curve with respect to time parameter. After that, we give the evolution equations for curvature of quaternionic curve. Further, we obtain integrability condition (zero curvature condition) for the considering model. Finally, we give new characterization for inextensible quaternionic curve flow by considering abelian matrices which are used in integrability conditions.

Key Words: Quaternion, evolution, inextensible flow.

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On The Exponential Transformations In Robotic Motions

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ABSTRACT

Kinematics is a branch of mechanics that does not contain the concepts of force and mass. That is, Kinematics just examines the time dependent change in the location of appoint or appoint system. Kinematic modeling which science of robotics is one of the most important analytical tools are used in many fields of science of robotics. In particular: Mechanisms, modeling of sensors and actuators, concurrent robot control applications, robot simulation and programming asynchronous transactions. Robot kinematic modeling of robots is one of the most important stages of the work of robotics. Cartesian plane, using conversion operators such as matrix or vectors, was called point conversion to this method is removed from the kinematic model. Further, If linear transformation vectors and the operators are used, then this method is called a linear transformation method. Using transformation Cartesian plane of vectors, Maxwell described the 4x4 homogeneous transformation matrixes. Denavit-Hartenberg, using the homogeneous transformation matrix, orientation and position of a coordinate system defined by the coordinate system to another. According to a coordinate system to another, transformation matrix can be expressed the movement of the screw. In this method expressed as a linear transformation takes both a displacement and rotation on the same axis. In this paper, it is investigated exponential transformations in robotic. Firstly, we are give some special exponential transformations in Lie group and Lie algebra. Then, it is given logarithmic functions of these groups.

Key Words: Robotic Motion, Lie Group and Lie Algebra, Exponential Transformations.

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On the M-Projective Curvature Tensor of a Normal Paracontact Metric Manifold

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ABSTRACT

In the present paper we have studied the curvature tensors of a normal paracontact metric manifold satisfying the conditions $R(\xi, X)W^{*}=0$, $W^{*}(\xi, X)R=0$, $W^{*}(\xi, X)W^{*}=0$, $W^{*}(\xi, X)\tilde{Z}=0$, $W^{*}(\xi, X)S=0$ and $W^{*}(\xi, X)\tilde{C}=0$ where R is Riemannian curvature tensor, S is Ricci tensor, W^{*} is M-projective curvature tensor, $\tilde{Z}$ is concircular curvature tensor and $\tilde{C}$ is quasi-conformal curvature tensor.

In 1971, G. P. Pokhariyal and R. S. Mishra defined a tensor field W^{*} on a Riemannian manifold as

$$W^{*}(X,Y)Z=R(X,Y)Z-\frac{1}{2(n-1)}[S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX-g(X,Z)QY]$$

for any $\forall X,Y,Z\in\chi(M)$. Such a tensor field W^{*} is known as M-projective curvature tensor, where S is Ricci tensor and Q is Ricci operator of M. R. J. Ojha showed that $M-$projective curvature tensor bridges the gap between the conformal curvature tensor, coharmonic curvature tensor and concircular curvature tensor.

Let M be $n-$dimensional differentiable manifold. If on M we have

$$\phi^{2}X=X-\eta(X)\xi,\quad \phi\xi=0,\quad \eta(\phi X)=0,\quad \eta(\xi)=1$$

and

$$g(\phi X,\phi Y)=g(X,Y)-\eta(X)\eta(Y),\quad \eta(X)=g(X,\xi),$$

for any vector fields X and Y on M, ξ is a contravariant vector and η is 1-form. Then, M is called almost paracontact metric manifold.

An almost paracontact metric manifold M is said to be normal if

$$(\nabla_X \phi)Y = -g(X,Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi$$

for any $\forall X,Y \in \chi(M)$.

A normal almost paracontact metric manifold M is said to have a constant c if and only if

$$\begin{aligned} R(X,Y)Z = & \frac{c+3}{4}\{g(Y,Z)X - g(X,Z)Y\} \\ & + \frac{c-1}{4}\{\eta(X)\eta(Y)Z - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi \\ & - g(Y,Z)\eta(X)\xi + g(\phi Y,Z)\phi X - g(\phi X,Z)\phi Y - 2g(\phi X,Y)\phi Z\} \end{aligned}$$

for any vector fields $X,Y,Z \in \chi(M)$.

Let M be $n-$dimensional Riemann manifold. The projective curvature tensor, the concircular curvature tensor and quasi-conformal curvature tensor are defined by, respectively,

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{n-1}\{S(Y,Z)X - S(X,Z)Y\},$$

$$\tilde{Z}(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}\left[g(Y,Z)X - g(X,Z)Y\right]$$

and

$$\begin{aligned} \tilde{C}(X,Y)Z = & aR(X,Y)Z + b\big[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX \\ & - g(X,Z)QY - \frac{r}{n}\left[\frac{a}{n-1} + 2b\right]\left[g(Y,Z)X - g(X,Z)Y\right] \end{aligned}$$

for any vector fields $X,Y,Z \in \chi(M)$, where a and b arbitrary constants, Q is Ricci operator, S is Ricci tensor and r is scalar curvature of manifold.

Key Words: M-projective curvature tensor, concircular curvature tensor, quasi conformal curvature tensor.

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Prolongations of Metallic Structure to The Tangent Bundle of Order 2

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ABSTRACT

Additional structures have an important role in the geometry of differentiable manifolds. Structures obtained with the support of a tensor field of type $(1,1)$ on a differentiable manifold have been studied by many researchers. A summary of the work of this obtained structure can be found [7].

In 1997 Spinadel [5] explored the metallic means. Metallic means family includes the golden mean, the silver mean, the bronze mean, the copper mean and many others.

In 2013, Hretcanu and Crasmareanu [1] defined the metallic structure, which is a new polynomial structure, by tensor field J of type $(1,1)$ which has structure polynomial, $Q(J) = J^2 - pJ - qI$, on a differentiable manifold. These numbers, denoted by

$$\sigma(p,q) = \frac{p + \sqrt{p^2 + 4q}}{2},$$

are also said (p,q) −metallic numbers. Later, many researchers [2-4] examined geometry of metallic structure on a differentiable manifold.

In this presentation, our aim is to focus on some applications in differential geometry of the metallic means family. A metallic structure on a differentiable manifold has been prolonged to the tangent bundle of order two of this manifold through the second lift. After, some essential definitions and theorems about the integrability and parallelism of this metallic structure on tangent bundle of order two were given. Finally, the metric, which was defined on the prolonged metallic structure, and its properties were investigated.

Key Words: Metallic structures, tangent bundle of order two, second lift.

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Pullback Crossed Modules in Modified Categories of Interest

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ABSTRACT

A crossed module in the category of groups consists of a group homomorphism (from G to H) with a group action of H on G which satisfies two Peiffer relations. Crossed modules are introduced by Whitehead as a model of homotopy 3-types and used to classify higher dimensional cohomology groups. Crossed modules also appear in the context of simplicial homotopy theory as they are equivalent to simplicial objects with Moore complex of length one.

Afterwards, this crossed module notion adapted to many algebraic structures as well as to modified categories of interest. Modified categories of interest [1] unify the notions of groups and various algebraic structures. In this work, we define pullback crossed modules in a modified category of interest. These are the crossed modules that obtained by a pullback diagram with extra crossed module structures on certain arrows. Remark that, these are not the pullback objects in the category of crossed modules of modified categories of interest.

This pullback crossed module structure unifies many corresponding results for the case of groups, commutative algebras, Lie algebras, etc. Moreover, one can also adapt this construction to other algebraic structures which are modified categories of interest. The main idea comes from [2] where the pullback crossed modules of groups are defined by Brown and Wensley.

Key Words: Modified category of interest, crossed module.

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Regular Objects at p in the Category of Proximity Spaces

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ABSTRACT

The notion of proximity on a set X was introduced by Efremovich [4]. He characterized the proximity relation "A is close to B" as a binary relation on subsets of a set X and defined the closure of a subset A of X to be the collection of all points of X "close" A. Thereby he showed that a topology (completely regular) can be introduced in a proximity space. He also showed that every completely regular space X can be turned into a proximity space by using Urysohn's function. The most extensive work on the theory of proximity spaces was done by Naimpally and Warrack [7]. All preliminary information on proximity spaces can be found in this source. Baran [1] gave various generalizations of the separation properties, the notion of closed and strongly closed points and subobjects of an object in an arbitrary topological category over sets. These generalizations are, for example, four notions of T3, each equivalent to the classical T3 notion for topological spaces.

Hunsaker and Sharma [5] showed that Prox, the category of proximity spaces and proximity mappings, is a topological category over Set, the category of sets. The main purpose of this paper is to characterize various notions of regular objects at a point in Prox.

Key Words: Topological category, proximity space, regular, separation.

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Representation Varieties of Free or Surface Group and Reidemeister Torsion

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ABSTRACT

Reidemeister torsion (R-torsion) is a topological invariant with many applications in topology, differential geometry, representation spaces, knot theory, Chern-Simon theory, 3-dimensional Seiberg-Witten theory, dynamical systems, theoretical physics and quantum fields theory. K. Reidemeister introduced this invariant in the paper [2], where he classified 3-dimensional lens spaces. In [1], W. Franz extended this invariant and classified the higher dimensional lense spaces; that is, S^{2n+1}/G, where G is a cyclic group acting isometrically and freely on the sphere S^{2n+1}.

Symplectic chain complex is an algebraic topological tool and introduced by E. Witten in 1991 [3]. By using R-torsion and the method of reel symplectic chain complex, he computed the volume of several moduli spaces Rep(Γ,G) of all conjugacy classes of homomorhisms from the fundamental group Γ of the Riemann surface Σ to the compact gauge group G=SU(2) or SO(3).

Let Σ_g be a closed orientable Riemann surface of genus at least 2, H_g be a handlebody of genus at least 2 and G be a complex Lie group with semi-simple Lie algebra. We consider in the present abstract the representation space Rep(Γ_g,G)=Hom(Γ_g,G)/G of conjugacy classes of homomorphisms from group Γ_g to Lie group G. Here, Γ_g is the fundamental group of Σ_g or H_g with genus $g \geq 2$. Moreover, we show that the topological invariant Reidemesiter torsion (R-torsion) of such representations is well-defined. Using the notion of symplectic chain complex, we also express R-torsion of such representations in terms of Atiyah-Bott-Goldman symplectic form for the Lie group G.

Key Words: Reidemeister torsion, representation varieties, symplectic form.

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Ricci Solitons and Gradient Ricci Solitons on (ε)- para Sasakian Manifolds

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ABSTRACT

The notion of Ricci soliton, a natural generalization of an Einstein metric (i.e. the Ricci tensor S is a constant multiple of g) was introduced by Hamilton [2]. A pseudo-Riemannian manifold (M, g) is called a Ricci soliton if it admits a smooth vector field V (potential vector field) on M such that

$$(L_V g)(X,Y) + 2S(X,Y) + 2\lambda g(X,Y) = 0,$$

where L_V denotes the Lie-derivative in the direction V, λ is a constant and X, Y are arbitrary vector fields on M. A Ricci soliton is said to be shrinking, steady or expanding according to λ being negative, zero or positive, respectively. It is obvious that a trivial Ricci soliton is an Einstein manifold with V zero or Killing.

The vector field V generates the Ricci soliton viewed as a special solution of the Ricci flow, and is called the generating vector field. A Ricci soliton is said to be a gradient Ricci soliton if the generating vector field V is the gradient of a potential function $-f$, that is $V = -\nabla f$.

If the manifold is Euclidean space, or more generally Ricci-flat, then Ricci flow leaves the metric unchanged. Conversely, any metric unchanged by Ricci flow is Ricci-flat. For a compact Einstein manifold, the metric is unchanged under normalized Ricci flow. Conversely, any metric unchanged by normalized Ricci flow is Einstein.

In [5] the authors studied (ε)- almost paracontact manifolds, and in particular, (ε)- para Sasakian manifolds. They gave basic definitions, some examples of (ε)-

almost paracontact manifolds and introduced the notion of an (ε)- para Sasakian structure.

In the present paper we study (ε)- para Sasakian manifolds admitting Ricci solitons and gradient Ricci solitons. We prove that if in an (ε)- para Sasakian manifold the metric is Ricci soliton, where potential vector field V is collinear with the characteristic vector field ξ, then V is a constant multiple of ξ provided $\lambda = \varepsilon(n-1)$ and M is an Einstein-like manifold. We also obtain some conditions for an Einstein-like (ε)- para Sasakian manifold with $V = \xi$ to be a Ricci soliton. Furthermore we investigate gradient Ricci soliton on (ε)- para Sasakian manifolds.

Key Words: Ricci soliton, gradient Ricci soliton, (ε)- para Sasakian manifold.

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Ricci Solitons on Complex Sasakian Manifolds

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ABSTRACT

Ricci solitons which are natural generalization of Einstein metrics were introduced by Hamilton [6]. A Ricci soliton is a triple (g, w, λ) with g a Riemannian metric, W a vector field, and λ a real scalar such that

$$L_W g + 2\,\rho + 2\lambda g = 0$$

where ρ is a Ricci tensor of M and L_w denotes the Lie derivative operator along the vector field V. The Ricci soliton is said to be shrinking, steady, and expanding accordingly as λ is negative, zero and positive, respectively .

Sasakian manifolds are special classes of contact manifolds and they are studied by lots of geometers. Ricci solitons on Sasakian manifolds were studied by He and Zhu [3]. Complex Sasakian manifolds are complex analogue of Sasakian manifolds and they have a global complex contact form. Foreman [5] studied on complex Sasakian manifolds and obtained some useful results and Fetcu [4] examined harmonic maps between complex Sasakian manifolds.

In this paper, we investigate Ricci solitons on complex Sasakian manifolds. We consider special vector fields in Ricci soliton equation which are very important for general relativity and we prove some new results about Ricci solitons on complex Sasakian manifolds. In addition, we obtain some results on complex Heisenberg group.

Key Words: Complex Sasakian manifolds, Ricci solitons.

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Semiparallel Submanifolds of a Normal Paracontact Metric Manifold

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ABSTRACT

The object of the present paper is to study invariant semiparallel and 2-semiparallel submanifolds of a normal paracontact metric manifold. We see that parallel submanifolds of a normal paracontact metric manifold are totally geodesic. Arslan K. and et. al. [1] defined and studied 2-semiparallel surfaces in space forms.

In modern the geometry of submanifolds has become a subject of growing interest its significant application in applied mathematics and physics. For instance, the notion of invariant submanifold is used to discuss properties of non-linear autonomous system. On the other hand, the notion of geodesic plays an important role in the theory of relativity. For totally geodesic submanifolds, the geodesics of the ambient manifolds remain geodesics in the submanifolds. Therefore, totally geodesic submanifolds are also very much important in physical sciences. The study of geometry of invariant submanifolds was initiated by Bejancu and Papaghuic. After then the invariant submanifolds have been studied by many geometers to different extent. Invariant submanifolds inherit almost all properties of the ambient manifolds.

Let M be a $(2n+1)-$dimensional manifold and ϕ,ξ and η be a tensor field of type $(1,1)$, a vector field 1-form on M, respectively. If ϕ,ξ and η satisfy the conditions

$$\phi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1,$$

for any vector field X on M, then M is said to be have an almost contact manifold. In addition, it called almost contact metric manifold if M has a Riemannian metric tensor such that

$$g(\phi X,\phi Y)=g(X,Y)-\eta(X)\eta(Y), \quad \eta(X)=g(X,\xi),$$

for any $X,Y\in\chi(X)$, where $\chi(X)$ denotes set of the differentiable vector fields on M. Furthermore, M is called a normal paracontact metric manifold if we have

$$\left(\bar{\nabla}_X\phi\right)Y=-g(X,Y)\xi-\eta(Y)X+2\eta(X)\eta(Y)\xi$$

and

$$\bar{\nabla}_X\xi=-\phi X,$$

for any $X,Y\in\chi(X)$, where $\bar{\nabla}$ denotes the Levi-Civita connection determined by g.

Key Words: Riemannian curvature tensor, Concircular curvature tensor, Invariant submanifold, semiparallel and 2-semiparallel.

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Sequentially Fréchet Spaces

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ABSTRACT

A subset of a topological space is called a sequentially open if every sequence convergent to a point in that subset is eventually in that subset. Clearly, every open set is sequentially open. Sequentially open sets are generalization of open sets. A sequential space is a topological space that satisfies a very weak axiom of countability, that is, every sequential open subset is open. Sequential spaces are the most general class of spaces for which sequences suffice to determine the topology.

Separation axioms are about the use of topological means to distinguish disjoint sets and distinct points. In a topological space, sometimes two points being distinct means nothing. However distinct points which are distinguishable by open sets are very interesting, since each separation axiom is a topological property. A topological space satisfying the T_1 axiom of separation is called a Fréchet space.

In this study we defined a new type of separation axiom using sequentially open sets instead of open sets which will be called sequentially T_1 axiom. Our aim is to define a new topological property which may be use to classify topological spaces. Topological spaces satisfying sequentially T_1 axiom will be called sequentially T_1 space or sequentially Fréchet space. Further we investigate some properties of sequentially T_1 spaces and place among other separation axioms.

Key Words: Separation axioms, Fréchet space, sequentially open set.

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Simplicial Leibniz - Rinehart algebras

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ABSTRACT

Let K is a fixed field, A is a commutative algebra, and L is a Leibniz - K algebra. A Leibniz - Rinehart algebra (equipped with an anchor morphism) is a Leibniz - K algebra with an A - module structure. One of the main example of Leibniz - Rinehart is the right noncommutative poisson algebra [1].

Leibniz - Rinehart algebras can be considered as an algebraic counterpart of Leibniz algebroids. It is clear that any Lie algebroid morphism is already a Leibniz algebroid morphism. Thus, we can say that the category of Lie algebroids is the full sub category of the category of Leibniz algebroids.

If we define the anchor morphism of a Leibniz - Rinehart algebra as the zero morphism, then it becomes a Leibniz - A algebra. Moreover if the Leibniz bracket has the antisymmetry property, then the Leibniz - Rinehart algebra becomes a Lie - Rinehart algebra. Therefore Leibniz - Rinehart algebra structure is the generalization of Lie - Rinehart algebra structure.

In this study, we define simplicial Leibniz - Rinehart algebras. By using this definition, we also discover some relations between the crossed modules of Leibniz - Rinehart algebras and simplicial Leibniz - Rinehart algebras.

Key Words: Leibniz - Rinehart algebra, simplicial Leibniz - Rinehart algebra, crossed module.

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Slant submersions from almost product Riemannian manifolds

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ABSTRACT

Riemannian submersions between almost Hermitian manifolds were studied by Watson in [4] under the name of holomorphic submersions. One of the main results of this notion is that vertical and horizontal distributions are invariant under almost complex structure. He showed that if the total manifold is a Kahler manifold, the base manifold is also a Kahler manifold. Recently, Şahin [3] introduced slant submersions from almost Hermitian manifolds to Riemannian manifolds. He showed that the geometry of slant submersions is quite different from holomorphic submersions. Indeed, although every holomorphic submersion is harmonic, slant submersions may not be harmonic. Let π be a Riemannian submersion from an almost product Riemannian manifold (M_1, g_1, F) onto a Riemannian manifold (M_2, g_2). If for any nonzero vector $X \in (\ker \pi_*)$, $p \in M_1$, the angle $\theta(X)$ between FX and the space $(\ker \pi_*)$ is a constant, i.e. it is independent of the choice of the point p $\in M_1$ and choice of the tangent vector X in $(\ker \pi_*)$, then we say that π is a slant submersion. In this case, the angle θ is called the slant angle of the slant submersion. We give the definition of slant Riemannian submersions and provide examples. We also investigate the geometry of leaves of the distributions. Finally, we give necessary and sufficient conditions for such submersions to be totally geodesic.

Key Words: Riemannian submersion, almost product Riemannian manifold, slant submersion.

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Soft Mappings and Elementary Operations over Soft Sets

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ABSTRACT

We give some properties of the elementary union, intersection and complement operations over soft sets. A lot of properties of the set operations and soft set operations are not valid for elementary operations in general. The most important fact is that the elementary union and intersection operations are not distributive. Also, De Morgan's laws are not provided using elementary operations over soft sets in general. We show that these properties are provided under some restrictions. In addition, we describe in some detail soft mappings over soft sets via soft elements and give relations between soft mappings and crisp mappings.

Proposition: For any soft sets $F, G \in S(\tilde{X})$, we have the following properties.

1. $F \text{Ò} G = F \tilde{\cup} G$.
2. $F \text{Ó} G \tilde{\subseteq} F \tilde{\cap} G$.
3. $F^{\square} \tilde{\subseteq} F^{C}$.
4. $F \text{Ò} F^{\square} \tilde{\subseteq} \tilde{X}$.
5. $F \text{Ó} F^{\square} = \Phi$.
6. If $F_i = SS(\mathcal{B}_i)$, $i \in I$, then $\underset{i\in I}{\text{Ò}} F_i = SS\left(\bigcup_{i\in I} \mathcal{B}_i\right)$.
7. If $F_i = SS(\mathcal{B}_i)$, $i \in I$, then $\underset{i\in I}{\text{Ó}} F_i \tilde{\supseteq} SS\left(\bigcap_{i\in I} \mathcal{B}_i\right)$.

Proposition: For any soft sets $F, G, H \in S(\tilde{X})$,

1. $(F \text{Ó} G) \text{Ò} H \tilde{\supseteq} (F \text{Ò} H) \text{Ó} (G \text{Ò} H)$.
2. $(F \text{Ò} G) \text{Ó} H \tilde{\supseteq} (F \text{Ó} H) \text{Ò} (G \text{Ó} H)$.

Proposition: For any soft sets $F, G \in S(\tilde{X})$,

1. If $F^{\square}\ \text{Ó}\, G^{\square} \neq \Phi$, $F^{\square} \neq \Phi$ and $G^{\square} \neq \Phi$, then $\left(F\ \text{Ò}\, G\right)^{\square} = F^{\square}\ \text{Ó}\, G^{\square}$.
2. If $F\ \text{Ó}\, G \neq \Phi$, $F^{\square} \neq \Phi$ and $G^{\square} \neq \Phi$, then $\left(F\ \text{Ó}\, G\right)^{\square} = F^{\square}\ \text{Ò}\, G^{\square}$.

Let X and Y be non-empty sets, A be a set of parameters. The mapping $f : SE(\tilde{X}) \to SE(\tilde{Y})$ is called a soft mapping from X to Y.

Theorem: Let X and Y be non-empty sets and A be a set of parameters. If the soft mapping $f : SE(\tilde{X}) \to SE(\tilde{Y})$ satisfies the condition:

(f_*) $\{f(\tilde{x})(\lambda) : \tilde{x}(\lambda) = \xi\}$ is a singleton set, $\forall \lambda \in A$, $\forall \tilde{x} \tilde{\in} \tilde{X}$ and $\forall \xi \in X$.

and if $f_\lambda : X \to Y$ is defined by $f_\lambda(\tilde{x}(\lambda)) = f(\tilde{x})(\lambda)$, $\forall \lambda \in A$ and $\forall \tilde{x} \tilde{\in} \tilde{X}$, then f_λ is a mapping from X to Y.

Key Words: Soft set, soft element, soft mapping

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Some Fixed Point Theorems On Partial Metric and M-Metric Spaces

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ABSTRACT

Fixed point theory has a very important place in the world of mathematics since it has many applications in various branches of mathematics. Partial metric spaces as defined has now found vast applications in topological structures in the study of computer science, information science and in biological sciences. Matthews [1] introduced the notion of partial metric space with the aim of providing a quantitative mathematical model suitable for program verification. He introduced the partial metric space as a generalization of ordinary metric space. After that lots of fixed point theorems on partial metric spaces is studied by many authors. Very recently, Asadi et al. introduced the concept of M-metric space and then obtained some fixed point theorems for single valued mappings on M-metric space. So that, they extended notion of partial metric spaces to M-metric space [4]. In this paper, we study some relations between partial metric spaces and M-metric spaces. To do this, firstly some notions like symmetric convergent and symmetric lower semi-continuous function are remembered. Then, Caristi Kirk's theorem is discussed on partial metric spaces and M-metric spaces comparatively. Finally, some examples which are convenient to theorems are given.

Key Words: Fixed point, Partial metric, M-metric space, symmetric convergence, symmetric lower semi-continuous, Caristi-Kirk Theorem.

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Some Functor Examples Between Fuzzy And Crisp Categories

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ABSTRACT

For each mathematical discipline we define at first objects and then admissible maps for describing the objects. This procedure is formalized by the concept 'category'.

A category C consists of

1) A class ICI of objects (which are denoted by A,B,C,...)
2) A class of pairwise disjoint sets $[A,B]_C$ for each pair (A,B) of objects (the members of $[A,B]_C$ are called morphisms from A to B), and
3) A composition of morphism, which provides associativity and exixtence of identities.

As well-known mathematical objects may be described by means of maps. There is an analogous description of categories via so called functors. A functor is a morphism of categories. Functors were first explicitly recognized in algebraic topology, where they arise naturally when geometric properties are described by means of algebraic invariants.

Category theory and functors provide a tool by which many parallel techniques used in several branches of mathematics can be linked and treated in a unified manner.

The aim of this study is to give some information about the topics of category, functor and fuzzy sets (A fuzzy set is a map A from a set X to unit interval [0,1]). Then try to find some functors from special fuzzy categories to related crisp categories.

Key Words: Category, functor, fuzzy sets.

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Some Properties Of Hypergeometric Meixner-Pollaczek Polynomials

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ABSTRACT

The present study is about the Meixner-Pollaczek polynomials. These are the polynomials known as Meixner polynomials of the second kind (see Chihara [1]) discovered by Meixner [3] and they were later studied by Pollaczek Chihara [5]. These polynomials are deal with some new properties of the Meixner-Pollaczek polynomials. First of all, the generating function of Meixner-Pollaczek polynomials have been derived using some features of Pochhammer symbol and the expansion of the hypergeometric functions. In order to show these theorems, some properties of summation formula have been used. These theorems would help to define new and different kind of properties of Meixner-Pollaczek polynomials. Later on some values of n such as 1, 2, 3,… were used and the corresponding expressions were obtained. Then, in accordance with this purpose, some results have been obtained by taking derivative of the generating functions with respect to different variables. After then some corollary and remarks have been presented by applying this generating functions to the some theorems. The relation of this polynomial to the integral is shown by some equations. The results obtained here include various families of multilinear and multilateral generating functions, miscellaneous properties and also some special cases for these polynomials. In [2] and [3] contain the similar study presented here.

Key Words: Meixner-Pollaczek polynomials, generating function, multilinear and multilateral generating function, recurrence relations.

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Some Properties of Soft Fuzzy Metric Spaces

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ABSTRACT

After the article of Molodtsov in 1999 [4] which initiated the concept of soft set theory to deal with the uncertainity problems, many researchers worked in the soft mathematics and its applications in various types.

The idea of soft topological spaces was first given by Shabir, Naz [6] and mapping beetween soft sets were described by Majumdar, Samanta [5]. After, Dus and Samanta introduced a notion of soft real sets and soft real numbers and gave some of their properties [1]. Later, they introduced a notion which is called soft metric and investigated some basic properties of soft metric spaces [2].

After that, Tantawy and Hassan examined that; since the soft real number is a parametrized collection real numbers, every properties of real numbers can be discussed in soft real numbers. And they introduced the operations on soft real numbers and defined countable soft real sets, uncountable soft real sets, soft supremum and soft infimum [7].

In this study we give a new notion which is called soft fuzzy metric spaces, by using soft points of soft sets and soft real numbers of soft real sets, as a generalization of soft metric spaces. For this notion we give a new definition soft t-norm for the triangle inequality of soft fuzzy metric and give some of its properties. Also, the structure of this new space is examined, some main properties and examples are given.

Key Words: Soft sets,Soft real sets, Soft real numbers and Soft fuzzy metric.

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Some Results on Ricci Solitons in Almost Paracontact Metric Manifolds

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ABSTRACT

In 1982, R.S. Hamilton introduced the notion of Ricci solitons as which are generalization of an Einstein metric [1]. On a semi-Riemannian manifold with semi-Riemannian metric g, a Ricci soliton is a triple (g,V,λ) such that

$$(\pounds_V g)(U,V)+2S(U,V)+2\lambda g(U,V)=0$$

where U,V vector fields on M , λ is a constant and $\pounds$ denotes the Lie derivative. It is said to be shrinking, steady or expanding according as

$$\lambda<0\ ,\ \lambda=0 \text{ or } \lambda>0.$$

In 1985, the concept of almost paracontact structure in a semi-Riemannian manifold was defined by S. Kaneyuki and M. Konzai [2] and then they characte-rized the almost paracomplex structure on M^{2n+1}. Later, S. Zamkovoy [3] investigated almost paracontact structure and some considerable subclasses (para-Sasakian manifolds). Especially, in the recent years many authors have pointed out the importance of paracontact geometry and in particular of para-Sasakian manifold by several papers [4, 5, 6].

In 1968, the concept of quasi-conformal curvature tensor was introduced by K. Yano and S. Sawaki [7]. Later, generalized quasi-conformal curvature tensor was investigated in [8].

Studying on Ricci solitons of an almost paracontact metric manifolds (in particular para-Sasakian manifolds) we obtain that

a) A Ricci soliton in a para-Sasakian manifold is expanding,

b) A Ricci soliton in a para-Sasakian manifold satisfying semi-symmetry condition $R(\xi, X) \cdot W = 0$ is expanding,

c) A Ricci soliton in a para-Sasakian manifold satisfying the condition $S(\xi, X) \cdot W = 0$ is either steady or shrinking,

where W is generalized quasi-conformal curvature tensor.

Key Words: Ricci soliton, paracontact manifolds, generalized quasi-conformal curvature tensor.

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Some Types of Lightlike Hypersurfaces in (ε)-para Sasakian Manifolds with Semi-Symmetric Non-Metric Connection

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ABSTRACT

The study of manifolds with indefinite metric is of interest from the stand point of physics and relativity. Manifolds with indefinite metrics have been studied by several authors. The idea of a semi-symmetric non-metric connection on a Riemannian manifold was introduced by Ageshe and Chafle [1]. They defined a linear connection on a Riemannian manifold admitting a semi-symmetric non-metric connection and studied some properties of the curvature tensor of a Riemannian manifold with respect to the semi-symmetric non-metric connection.

A linear connection $\bar{\nabla}$ on a Riemannian manifold (M^n, g) is called semi-symmetric, if its torsion tensor $\bar{T}$ satisfies

$$\bar{T}(X,Y) = \eta(Y)X - \eta(X)Y$$

where η is a non-zero 1-form associated with a vector field ξ defined by

$$\eta(X)=g(X,\xi).$$

Let M^n be an almost paracontact manifold equipped with an almost paracontact structure (ϕ,ξ,η) consisting of a tensor field ϕ of type (1,1), a vector field ξ and a 1 form η satisfying

$$\phi^2=I-\eta\otimes\xi,$$

$$\eta(\xi)=1,\ \phi(\xi)=0,\ \eta o\phi=0.$$

If g is a semi-Riemannian metric with index $(g) = \upsilon$ such that

$$g(\phi X,\phi Y)=g(X,Y)-\varepsilon\eta(X)\eta(Y),$$

where $\varepsilon=\pm1$, then M^n is called an (ε)- almost paracontact metric manifold equipped with an (ε)- almost paracontact structure (ϕ,ξ,η,g). In particular, if index(g) = 1, then

an (ε)- almost paracontact metric manifold will be called a Lorentzian almost paracontact manifold. If in case, the metric is positive definite, then an (ε)- almost paracontact metric manifold is the usual almost paracontact metric manifold .

In view of above equations, we have

$$g(X,\phi Y)=g(\phi X,Y)$$

and

$$g(X,\xi)=\varepsilon\eta(X),$$

for all $X,Y\in TM^n$. It follows that

$$g(\xi,\xi)=\varepsilon,$$

i.e. the structure vector field ξ is never lightlike. An (ε)- almost paracontact metric manifold (resp., a Lorentzian almost paracontact manifold) ($M^n,\phi,\xi,\eta,g,\varepsilon$) is said to be space like (ε)- almost paracontact metric manifold (respectively a space like Lorentzian almost paracontact manifold), if ε = 1 and M^n is said to be a time like (ε)- almost paracontact metric manifold (respectively a Lorentzian almost paracontact manifold), if ε = -1. An (ε) - almost paracontact metric structure is called an (ε)- para Sasakian structure if

$$(\nabla_X\phi)(Y)=-g(\phi X,\phi Y)\xi-\xi\eta(Y)\phi^2 X, \qquad X,Y \in TM^n$$

where ∇ is the Levi-Civita connection [5].

In this paper, we initiate the study of lightlike hypersurfaces of an (ε)-para Sasakian manifold admitting semi-symmetric non-metric connection which are tangent to the structure vector field. We investigate invariant lightlike hypersurfaces, screen semi-invariant lightlike hypersurfaces and radical transversal lightlike hypersurfaces with respect to semi-symmetric non-metric connection. We obtain integrability conditions for the distributions.

Key Words: (ε)-para Sasakian manifolds, lightlike hypersurface, semi-symmetric non-metric connection.

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T_0-Quasi-Metric Spaces With A Unique Convexity Structure

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ABSTRACT

Our observations and results about the convexity structures in the sense of Takahashi are presented for T_0-quasi-metric spaces, and focused on that in details as a systematic study in the asymmetric setting.

Accordingly, in this context, we prove that the generalized convexity structures from metric spaces to T_0 –quasi-metric spaces, naturally satisfy interesting additional conditions. Especially, generalized convexity structures desribed for T_0 –quasi-metric spaces occur in asymmetrically normed real vector spaces.

Following that, we discovered in the quasi-metric setting similarly to the metric theory one needs to consider convexity structures satisfying additional conditions in order to obtain strong results. For instance, for each T0-quasi-metric space (X, d) equipped with a convexity structure satisfying some specific conditions, one can also define a natural convexity structure on the set of nonempty doubly closed bounded convex subsets of X equipped with the Hausdorff–Bourbaki T0-quasi-metric. In addition, if a T_0-quasi-metric space has unique convexity structure, then it has additional nice properties,

Specifically, it will be proved that if W(x,y,β) is the unique convexity structure on a T_0-quasi- metric space (X,d) then the corresponding quasi-metric segment constructed by using any distinct points is isometrically embedded into the metric space (X,d), via the map β → W(x,y,β).

Key Words : Convexity Structure, T_0-quasi-metric, Asymmetrically normed vector space

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The Cubic Trigonometric Bezier Transition Curves with Two Shape Parameters

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ABSTRACT

The importance of trigonometric polynomials in different areas, such as electronics, medicine or computer graphics. It is surprising, however, that the corresponding curves have not received much attention in Computer Aided Geometric Design (CAGD) for a long time. The theory of Bezier curves uphold a key position in Computer Aided Geometric Design. Recently, trigonometric splines and polynomials have gained very much interest within CAGD, in particular curve design. Many new curves related with Bezier curves are introduced by many authors.

In this talk, we present a new type curve with two shape parameters namely Trigonometric Cubic Bezier Transition Curves. The curve is constructed based on new cubic trigonometric polynomials. The cubic trigonometric Bezier curve can be made closer to the cubic Bezier curve or nearer to the control polygon than cubic Bezier curve due to shape parameters. On the other hand, transition curves important in road design, highways design, railway design and in the orbit of satellite design. And it is important in manufacturing industries because of its use in the cutting paths for numerically controlled cutting machinery. Also they are suitable for the composition of G^2 blending curves. Therefore, it is important to find transition curves between line to line, line to circle, or circle to circle in different shapes like S-shape, J- shape or C- shape. Therefore in this talk finally, we find sufficient conditions for such curve due to be transition curves from line to line and circle to circle.

Key Words: Cubic trigonometric basis functions, Bezier curves, Transition curves .

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Totally Umbilical Semi-Invariant Submanifolds of a Locally Decomposable Riemannian Manifold

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ABSTRACT

The purpose of this present paper is to study totally umbilical semi-invariant submanifolds of a locally decomposable Riemannian manifold. First of all, we give the basic definitions, concepts and formulas in the theory of Riemannian and locally decomposable Riemannian manifolds which will be used throughout the paper. We also give some fundamental properties of semi-invariant submanifolds of a locally decomposable Riemannian manifold. We try to classify totally umbilical semi-invariant submanifolds. We get a primary classification theory for totally umbilical semi-invariant submanifolds. Besides, we obtain some special classification theorems of totally umbilical semi-invariant submanifolds by using dimensions of the invariant and anti-invariant distributions which are defined on a semi-invariant submanifold. Firstly, we investigate the condition that dimension of the anti-invariant distribution is greater than 1. In this case, we show that a totally umbilical semi-invariant submanifold is a semi-invariant product. Furthermore, we give some important results on totally umbilical semi-invariant submanifolds. After, we show that a totally umbilical semi-invariant submanifold is totally geodesic when the invariant distribution is parallel and dimension of anti-invariant distribution is equal to dimension of the normal bundle of the totally umbilical semi-invariant submanifold. Finally, we show that a proper totally umbilical semi-invariant submanifold which has non-zero mean curvature vector is an extrinsic sphere when dimension of the invariant distribution is greater than 2.

Key Words: locally decomposable Riemannian manifold, semi-invariant submanifold, totally umbilical semi-invariant submanifold, semi-invariant product, totally geodesic, extrinsic sphere.

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Tubular Surface Around A Spacelike Focal Curve In Lorentz 3-Space

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ABSTRACT

A canal surface is the envelope of a moving sphere with varying radius, defined by the trajectory C(t) (spine curve) of its center and a radius r(t) and it is parametrized through Frenet frame of the spine curve C(t). If the radius function r(t)=r is a constant, then the canal surface is called a tube or tubular surface [1]. The focal curve of an immersed smooth curve $\alpha: I \rightarrow E^3$ in Euclidean space consists of the centres of its osculating hyperspheres. This curve may be parametrised in terms of the Frenet frame of α (T,N,B), as $C=(\alpha+c_1N+c_2B)$, where the coefficients c_1, c_2 are smooth functions that we call the focal curvatures of α [7]. In this work, we initially gave characterization of canal and tubular surfaces around a timelike focal curves in Lorentz 3-Space, afterwards we investigated the curvatures of tubular surfaces around a timelike focal curves in Lorentz 3-Space.

We will obtain the tubular surface from the canal surface around a spacelike focal curves in Minkowski 3-space. A canal surface is de.ned as the envelope of a family of one parameter spheres. Alternatively, a canal surface is the envelope of a moving sphere with varying radius, de.ned by the trajectory C(t) of its center and a radius function r(t). This moving sphere S(t) touches the canal surface at a characteristic circle K(t). If the radius function r(t) = r is a constant, then the canal surface is called a tube or pipe surface [1].

Key Words: Canal Surfaces, Tubular Surfaces, Space Curve, Focal curvatures, Lorentz Space.

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Weakly ϕ-Ricci Symmetric Lightlike Hypersurfacesin Indefinite Kenmotsu Space Forms

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ABSTRACT

Notion of ϕ-symmetric has been studied by many authors. For example, T.Takahashi has introduced the notion of locally ϕ-symmetric Sasakian manifolds as a weaker notion of locally symmetric manifolds. Also, U.C. De has studied ϕ-symmetric Kenmotsu manifolds with several examples. Later, U.C. De and A. Sarkar have introduced the notion of ϕ-Ricci symmetric Sasakian manifolds and obtained some interesting results of this manifold. S.S. Shukla and M.K. Shukla have studied the notion of ϕ-Ricci symmetric in the context of Kenmotsu manifolds. Also, D.G. Prakasha, S.K. Hui and K. Vikas have introduced notion of weakly ϕ-Ricci symmetric of Kenmotsu manifold. Then, a Kenmotsu manifold M (n > 2) is said to be weakly ϕ-Ricci symmetric if the non-zero Ricci curvature Q of type (1;1) satisfies the condition

$$\phi^2(\nabla_X Q)(Y) = A(X)Q(Y) + B(Y)Q(X) + g(QX,Y)\rho,$$

where the vector fields X and Y on M, ρ is a vector field such that g(ρ,X) = $D(X)$, A and B are associated vector fields (not simultaneously zero). Also, a weakly ϕ-Ricci symmetric Kenmotsu manifold M (n > 2) is said to be locally ϕ-Ricci symmetric, if

$$\phi^2(\nabla Q) = 0.$$

In this study, we investigate weakly ϕ-Ricci symmetric lightlike hypersurfaces of indefinite Kenmotsu space form, tangent to the structure vector field. We obtain sufficient condition for a weakly ϕ-Ricci symmetric lightlike hypersurface to be ϕ-Einstein in indefinite Kenmotsu space form. Also, we give some results fort this hypersurfaces.

Key Words: Weakly ϕ-Ricci symmetric, Indefinite Kenmotsu space form, Lightlike hypersurface .

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Zero Dimensional Constant Filter Convergence Spaces

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ABSTRACT

Filters appear in order and lattice theory, but can also be found in topology whence they originate. Filters were introduced by Henri Cartan [1] in 1937 and subsequently used by Bourbaki as an alternative to the similar notion of a net developed in 1922 by E. H. Moore and H. L. Smith [4]. Filters occupy an important place in the characterization of several topological theories. In particular, the (ultra) filter monad has been used extensively as the fundamental tool to describe various topological theories by means of lax algebraic and categorical methods in [5].

Let A be a set, $F(A)$ set of all filters on A, and K be a subset of $F(A)$. A filter σ is proper if and only if σ does not contain the empty set, Ø and a filter σ is improper if and only if σ contains the empty set. (A, K) is called a constant filter convergence space if and only if K satisfies the following two conditions.

(1) $[x] \in K$ for each $x \in A$, where $[x] = \{B \subset A \mid x \in B\}$

(2) If $\beta \subset \alpha$ and $\alpha \in K$ implies $\beta \in K$ for any proper filter β on A

A map $f:(A, K) \to (B, L)$ between constant filter convergence spaces is called continuous if and only if $\alpha \in K$ implies $f(\alpha) \in L$ (where $f(\alpha)$ denotes the filter generated by $\{f(D) \mid D \in \alpha\}$). The category of constant filter convergence spaces and continuous maps is denoted by ConFCO which is introduced by Schwarz [6] in 1978. He showed that ConFCO is isomorphic to GRILL, the category of grill spaces, which is introduced by Robertson [5] in 1975.

Zero-dimensional spaces were defined by Sierpinski [7] in 1921, where a topological space is zero-dimensional if it has a base consisting of both open and closed sets [2].

In this paper, we characterize zero-dimensional constant filter convergence spaces and give some invariance properties of these spaces. Furthermore, in the realm of each of zero-dimensional constant filter convergence spaces we investigate the relationships among each of T_0, T_1, and T_2 constant filter convergence spaces.

Key Words: Topological category, filters, zero-dimensional spaces, constant filter convergence spaces, grill spaces.

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Zero-Dimensional Kent Convergence Spaces

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ABSTRACT

From the early days of Topology, several important concrete situations simply could not be captured in the setting of topological spaces. All these early examples had to do with convergence in terms of convergent sequences. Nets and filters are two different theories of convergence which both generalize the notion of convergence of sequences. In 1922, Moore and Smith [5] had introduced convergence of nets, and in 1937, Cartan [1] provided the notion of filter. In 1954, Kowalsky [4] gave a filter description of convergence. In 1964, Kent [3] introduced Kent convergence spaces (there it is called convergence functions) by further weakening of the convergence axioms. Kent convergence space is a generalization of a topological space based on the concept of convergence of filters.

Zero-dimensional spaces were defined by Sierpinski [6] in 1921 (a topological space is zero-dimensional if it has a base consisting of clopen (both open and closed) sets [2]), before dimension theory was originated. He used the concept of zero-dimensional spaces in order to characterize, among the hereditarily disconnected subspaces X of R^n the linearly orderable ones. In 1997, Stine [7] proved that a topological space (X,τ) is zero dimensional if and only if there exists a family of discrete spaces $(X_i,\tau_i), i\in I$ and a family of functions $f_i : X \to (X_i,\tau_i), i \in I$ such that τ is the topology induced on X by $(X_i,\tau_i), i\in I$ via f_i. In view of this, in [7] the notion of zero-dimensional object was introduced to arbitrary topological categories.

In this paper, we characterize zero-dimensional Kent convergence spaces and give some invariance properties of these spaces. Furthermore, we show that in the realm of each of zero-dimensional Kent convergence spaces each of T_0, T_1, and T_2 Kent convergence spaces are equivalent.

Key Words: Topological category, filters, zero-dimensional spaces, Kent convergence spaces.

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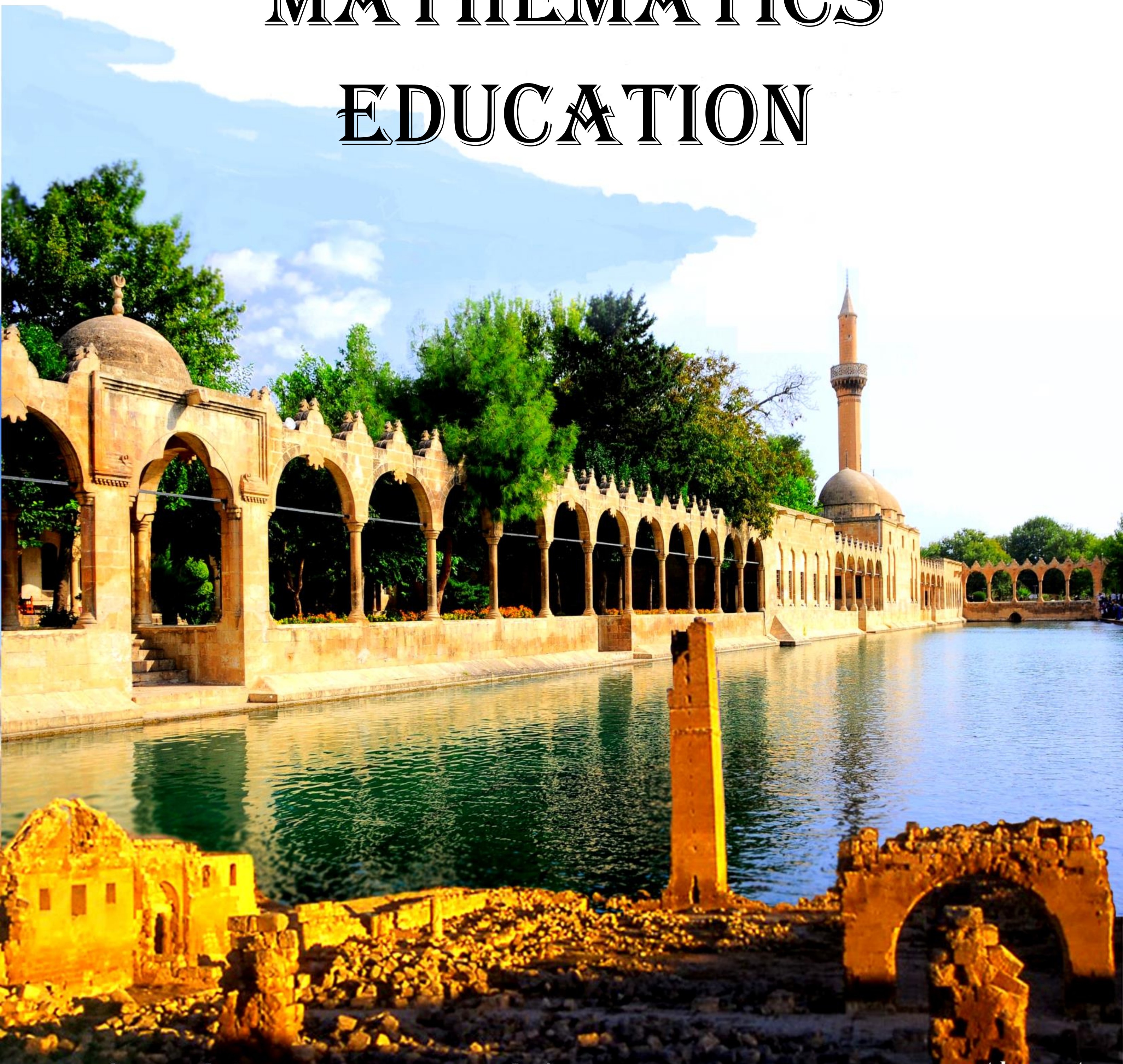
MATHEMATICS EDUCATION



A Brand New Approach To Sets In Mathematics Education*

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ABSTRACT

In this study, first of all, a definition of sets will be explained. In addition to the definition of the set known in mathematics, new approaches will be presented. Eleven rules about the sets definition model will be proposed. 1- The set header and its descriptive identity. 2- Define the set address. 3- To suit the purpose, define the set bounder. 4- To specify the time of the set. 5 - The status of the set's action. 6- The set group state. 7- The Set's Energy State. 8- If possible, positive, negative, and neutral set load must be specified. 9- If the set's female, male, and neutral gender must be specified. 10- If possible, Scientific and Local Names of the set must be specified. 11- If possible, the special case of the set must be specified. Later, the definitions of these rules will be presented with examples. Also belonging to the sets; Gender, burden, location, action, group, living and inanimate concepts will be defined. Examples of live and lifeless creature sets will be presented.

In the end, it is to gain the consciousness of understanding all the energy states in the nature, with set consciousness. In addition to this; Natural, artificial, virtual, objective, mental, light ... etc. examples of sets will be given.

Key Words: A Brand New Approach To Sets, Set Definition Rules, Mathematical Education

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A Study on Creative Problem Solving Skills of Fifth Grade Students

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ABSTRACT

Creative thinking and reasoning skills have become important in today's world, and their significance in terms of mathematics is undeniable [1]. Besides, it is believed that seeking an answer to the question of ''How creative our students can be in problem solving situations?'' will significantly contribute to mathematics education. The objective of the study is to determine Creative Problem Solving Skills (CPSK) of secondary school fifth grade students. The research group consists of fifth grade students (7 female, 12 male) studying at a state school in Samsun in the academic year of 2016-2016. Case study, which is one of the qualitative research patterns, has been used as the method of the study. 2 problems have been used as the data collection tool of the study, so as to reveal CPSK of students. The problems were developed by Ragsdale and Saylor [2] and revised by the researchers. The students were given 1 course hour in order to implement the problems. The retrieved data were evaluated by using the scoring system developed by Akgül and Kahveci [3].By this scoring system, which is based on the assumption that creativity consists of 3 dimensions, fluency, flexibility, and originality scores were calculated and evaluations were based on total and average scores.

According to the data obtained, CPSK scores of 58% of the students are below the average. Moreover, the Fluency score average of CPSK, which was analyzed in a 3-dimension manner, is 3,6. This average demonstrates that the Fluency score of 74% of the students are also below the average. In terms of the Fluency dimension, approximately 63% of the students are below the average of 2,2 point. At the Originality dimension, 63% of the students stand below the average of 4 point.

Examining the CPSK scores of male and female students in the research, the average of the female students ($\bar{X}$=25,25)is lower than that of the male students ($\bar{X}$=15,6).Consequently, it can be mentioned that CPSK skills of female students is higher. Furthermore, the CPSK scores of 75% of the male students and 29% of the female students seem to be below the average.

Key Words: Creativity, Creative Problem Solving, Creativity at Fifth Grade

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A Study On The Effects Of Educational Computer Games On 5th Grade Students' Success, Attitudes And Metacognitive Skills

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ABSTRACT

The aim of this study is to investigate the effects of the lesson, presented through the web of computer-aided educational mathematics games, to the success on fractions, attitude towards mathematics and educational computer games, and metacognitive skills of fifth grade students.

In this study, quantitative research design was used. The study is designed as a 'pretest - posttest' and a 'experimental-control group' study. A total of 34 students, attending at a private school in the district of Çankaya in Ankara, participated in the research in the academic year of 2014 - 2015. 5 weeks lessons were carried out both in the control group using traditional teaching method and the experimental group applying educational computer games on fractions.

Students' success on fractions was determined by the Mathematics Achievement Test developed by the first researcher and applied as a pretest and post-test. To determine the students' attitudes towards mathematics, Mathematics Attitude Scale (MEB, http://ogm.meb.gov.tr/mat_9.doc), and attitudes towards educational computer games, Attitude Scale Towards Educational Computer Games (Çankaya, S., Can, G. & Karamete, A., 2008) were applied. Moreover, Metacognition Questionnaire (Büyüköztürk, 2004) was applied to determine the students' metacognitive skills. Data were analyzed by Mann Whitney U-Test for independent samples and Wilcoxon Signed Rank Test for Paired Samples (Büyüköztürk, 2014)

According to the results, experimental group obtained higher scores on the achievement test, attitude scales and metacognition questionnaire than the control

group. Because of the increase of the scores of control group, there were no significant differences between the groups. Therefore, despite the increase in the motivation and the success of students in experimental group, no difference was observed.

Key Words: Mathematics, Fraction, Computer Aided Mathematics Education, Computer Aided Educational Games

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A Study on The Philosophical Thought of Elementary Mathematics Teacher Candidates

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ABSTRACT

Epistemological beliefs have been accepted as one of the basic building blocks of the science of philosophy. The concept of "epistemology", which is of interest to philosophers, has been in the interest of researchers working in educational psychology in recent years (Ozkan and Tekkaya, 2011). In the past century, mathematics has become an important issue for educators as well as the philosophers (Handal, 2009). The semi-experimental and absolute viewpoints, which have an undeniable significancy in terms of mathematical philosophy, are gathered under the heading of thoughts on the nature of mathematics (Bas et al., 2015). Absoluteism advocates that mathematical knowledge already exists in the ideal realm, and therefore it cannot be falsified, it is always true (Van De Walle et al., 2012). Semi-experimentalism advocates that mathematics knowledge stemmed from applied and practical experiences, it can be falsified but it is true until it is falsified, and as a mankind product, it always develops and changes. (Baki, 2008). In this study, it was aimed to investigate the change in the philosophical thoughts of the mathematics teachers about the nature of mathematics. Semi-experimental design was used in the study. The study included 140 teacher candidates who were studying at the fourth grade of the elementary mathematics department and taking Mathematics Philosophy course. “Determination of Philosophical Thoughts about the Nature of Mathematics Scale (MADİFDÖ)" - a 5-point likert scale including 25 items- developed by Sanalan et al. (2013) was used as a data collection tool. According to the results of the study, it has been seen that the participants tend towards a semi-experimental viewpoint from a mixed (absolute + semi-experimental) point of view.

Key Words: Mathematics philosophy, nature of mathematics, mathematics teacher candidate.

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Algebraic Expressions vs. Number Pattern Generalisations: Which one should be taught first, in mathematics classrooms?

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ABSTRACT

Attention to patterns is acknowledged in its importance as an introduction to algebra [4], hence constructing a conceptual understanding in students' minds is crucial. Because teaching the generalization of number patterns requires both to have specialized subject matter knowledge and purposefully constructed teaching approaches, it could be useful to use tasks within a pedagogy including the meaningful use of representations [1].

As mentioned before, pattern generalization is an introduction to algebra. However, most of mathematics teachers do not think so. According to recent studies and the interviews we made before, most of the teachers mentioned that pattern generalization should be taught after algebraic expressions [2].

Taken into consideration both the views, we conduct this study to see, which way of teaching cause learning most effectively. In this study, we observe and interview six primary mathematics teachers on the week they decided to start the unit of number pattern generalization and algebraic expressions. Some of the teachers decide to teach algebraic expressions first, and some of them decide on the contrary. We observe their lessons first, and take some notes on important parts. After the observation, we interview with the teachers about the lesson and the researchers' notes. After all the observations and interviews, we separate, code and re-organise the data as recommended [3].

As a result, we conclude that both of the methods can be useful in different classroom contexts. Each teacher has their own teaching aims and subject matter knowledge, that's why there are some useful techniques that may work or not.

Key Words: Algebraic expression, number pattern generalisation, representation.

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An Ethnographical Study On The Formation Of A School Culture (Bingol Province Example)

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ABSTRACT

The school is an important piece that enables the society to be complete with its mission, vision, and values. People are encultured based the characteristics of their environment, and they become the individuals that the culture shaped by acting as required by the culture. A significant part of spontaneous behavior and reactions are determined by the cultural environment (Sargut, 2001). Thus, an important characteristic of the culture is that it could be both learned and taught. The individual acquires the culture from older generations and peers. The process of socialization enables the individual to assimilate into the culture of the society (Fichter, 2002). The schools in Turkey are affected by the culture of their location, the values of the space and in turn affect the neighborhood. The objective of the study is to investigate in depth the process where the appointed administrators affect the school culture in a school that was transformed from a primary school to a junior high as a result of the implementation of the 4 + 4 + 4 education system in Bingöl province. The current study is an ethnographic study due to the use of cultural content in the description of the school and a case study since it is limited to a single school. For data collection purposes, unstructured observation, unstructured interviews and stories were used in the study. The data obtained were analyzed with content analysis. It was observed that the school administration was effective in creating the school culture, and in replacing the negative opinions about the school with the positive ones. It was also observed that the teachers experienced difficulties in adapting to the administration and changing the order they had established before the administration was appointed. Difficulties experienced by the teachers were mainly about keeping up the class schedules. This situation has led administrators to experience difficulties in

establishing order among teachers and caused misunderstandings. Administrators wanted to have conversations with teachers especially in mathematics classes and asked them to overcome the negative prejudices of the students towards the math class. Thus, it was concluded that building a positive school culture would be a long process, and problems could be experienced with long-term plans and reaching the educational and instructional goals set by the school.

Keywords: School, School Culture, Administration, Teacher.

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An Investigation of Eighth Grade Students' Skills in Different Types of Problem Posing

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ABSTRACT

One of the general aims of mathematics education is to develop students' problem solving skills (MEB, 2013). These skills help students to compete with problems that can face throughout their lives. Problem posing is considered as another aspect of problem solving. For these reasons; problem solving and problem posing are seen as an integral part of mathematics courses and activities (Altun, 2014; Kılıç, 2013). There are different types of problem posing. Free, semi-structured, and structured problem posing situations are among the most common types of problem posing in the related literature (Kar, 2014).

There are various researches that investigate the problem posing skills of middle school students (Gökkurt et al., 2015; Gür & Korkmaz, 2003; Tertemiz & Sulak, 2013; Turhan & Güven, 2014). However, it seems that there are limited number of studies that deeply investigate the skills of students for different types of problem posing.

The purpose of this study is to investigate the eighth grade students' skills in different types of problem posing. Regarding this purpose three questions below need to be answered.

1. How are the eighth grade students' skills in different types of problem posing?
2. What are the types of problems that eighth grade students posed in different types of problem posing?

3. What are the problem solving strategies eighth-grade students use to solve problems that they have developed for different types of problem posing?

In this study, case study which is one of a descriptive research methods was adopted because it was aimed to investigate the problem-posing skills of middle school students in depth. The study was conducted with a total of 166 eighth grade students in two cities in the spring term of 2015-2016 academic year.

The data collection tool is a "Problem Posing Test" which is included 6 open-ended questions about free, semi-structured and structured problem posing situations developed by researchers. In the process of development of problem posing test, the opinions of experts and middle school curriculum was taken into account. In the test, which was formed by selecting different topics, students were asked to pose two problems for each problem-posing type.

An analytical rubric was developed by researchers for the evaluation of the problems that students have posed. Rubric consists of seven criteria for problem posing skills. For the first sub-problem of the study, each criteria was interpreted by creating frequency and percentage tables. As a result of the analysis of the obtained data, it was observed that the students had generally low success in problem posing activities. In addition, it was determined that students had difficulties in solving the problems they have posed.

For the second sub-problem of the study, the problems students posed were classified as routine or non-routine. It was found that the majority of the problems students posed were routine problems.

Finally, it was determined that students mostly used “writing an equation” and “drawing a figure” strategies for the solutions of the problems they posed in different types of problem posing.

Key Words: Problem posing, Types of problem posing, Middle school students.

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Analysis of Attitudes about Mobile Learning of Prospective Teacher of Mathematics in Terms of Variable Various

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ABSTRACT

The concept of "mobile", which stands out with communication technologies, is used for wireless communication, mobility and portability. The mobile communication devices that supplies reaching, using and transacting the information rapidly are ranged as tablet, smart phone and MP3 player. The education activities that conducted with these devices are called mobile learning (Güneş, Işık & Çukurbaş, 2015). Attitude is a tendency against the specific object, situation, institution and conception (Tavşancıl, 2014).

The aim of this research is the examine the approaches of the prospective teacher of mathematics in terms of gender, class levels, owned mobile devices, the aim of using mobile phones and the total period of using a mobile phone actively in a day. The scanning type research maintained with 150 students studying at the department of teaching mathematics at a faculty of education of a university in 2016-2017 academic year spring semester. As a data collecting method used in the study to determine the attitudes of prospective teachers about mobile learning was used Mobile Learning Attitudes Scale and to explain some of the socio-demographical features, Personal Information Form was used. Descriptive statistics, t test, one way analysis of variance and LSD test were used in the analysis of the research data. As a result of the research, it is found that the attitudes of prospective teachers to the mobile learning is above mid-level. In accordance with findings the issue of giving more importance to forming a mobile learning environments at universities is suggestible

Key Words: Mobile Learning, Attitude, Mathematics Education, Math Mobile Applications

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Characterization of 2-dimensional hybrid cellular automata with exceptional family of two rules over ternary fields

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ABSTRACT

Recently, a spectacular study on two dimensional cellular automata (2D CA) has been observed in the literature. Most of the works are on two dimensional cellular automata with two states 0 and 1. These can be referred as binary two dimensional cellular automata. The states considered as the set of integers modulo 2, say $Z_2=\{0,1\}$ with addition and multiplication operation built on this set Z_2 becomes a finite field with two elements usually related as binary field. One dimensional (1-D)CA fundamentally is presented by Ulam and von Neumann [1]. Later, Complex behavior of 1-D CA rules were investigated by Stephanone Wolfram [2] and then Chattopdhyay et al.[3] studied the characterization of hybrid 2-dimensional CA with the help of matrix algebra.

A mathematical model using matrix algebra over GF(2) was reported for characterizing the behaviour of two dimensional nearest neighbourhood linear cellular automata with null and periodic boundary conditions for two dimensional case. Recently, in [4] the algebraic approach for characterizing two dimensional cellular automata is established for two dimensional cellular automata over a field with three elements. In the present work, the algebraic structure of two dimensional ternary hybrid cellular automata defined by new local rules is studied and some facts regarding their structures are established.

Key Words: 2-dimensional hybrid cellular automata,null boundary condition, ternary fields.

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Characterization of one dimensional cellular automata with intermediate boundary over the prime field of order p

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ABSTRACT

The study of Cellular Automata (CA) dates back to John von Neumann [1] in the early 50's. Von Neumann framed CA as a cellular space capable of self-reproduction. Since then, many researchers have taken interest in the study of CA for modelling the behaviour of complex systems.Wolfram et al. (1983) studied one-dimensional CA with the help of polynomial algebra.

One of the main problems while studying CA is the reversible property of them. Naturally if CA is reversible, then when the process can be reversed in order to understand the initial nature of CA. K. Morita [2] (2008) investigated properties of reversible cellular automata. Z. Cinkir et al. [3] (2011) was studied the reversibility problem for linear cellular automata with periodic boundary over the field with radius 2. Martin del Rey et al. [4] (2011) have studied the reversibility problem for LCA with null boundary defined by a rule matrix in the form of a pentadiagonal matrix over the binary field.

In this paper, we investigate one dimensional linear cellular automata with intermediate boundary condition via using matrix algebra built on $\mathbb{Z}_p$. We give an algorithm for determining the reversibility of this family of cellular automata. We also answer the reversibility question for some special subfamilies of these cellular automata. Finally, we present some applications of this family of cellular automata under the intermediate boundary condition.

Key Words: one dimensional cellular automata, intermediate boundary condition, matrix presentations.

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Comparison of Effectiveness Between The Mimio Classroom Teaching Materials and Commercial Materials in Mathematics Education

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ABSTRACT

Nowadays, the use of information and technology has been leading to the rapid spread of information, in education and its application has been led to new investments in learning and teaching. In the world, the expensive technological investments are made for increase success in mathematics subject. The environments in which instructional technologies exist provide a student-open and research environment [7]. Smarter boards than these technologies; owing to interactive applications ensure to the students to discover new information except what was learned before, allowing to the students to organize activities appropriate to the age groups, It is the most preferred technology in mathematics education because they have the ability to record some transactions to the students and provide them the possibility to watch them again later, to keep motivation and attention very high [8]. This article is a screening study showing the effectiveness of technology on education and the contributions of mimio material to learning and teaching.

The purpose of this study is to investigated whether there is any difference in the level of achievement between the instruction using concrete materials in the Mathematics of Vocational Mathematics course offered in Uludag University Vocational School of Technical Sciences, Food and Computer programs and the instruction using mimio classroom materials in which representations represented. The data which we use in this research; the students of the Food and Computer program who took the basic mathematics course explained using concrete materials in 2009-2010, 2010-2011, 2011-2012 academic years and The students of the Food and Computer program taking vocational mathematics course using Mimio Classroom teaching materials in 2012-2013, 2013-2014, 2014-2015 academic year. It is a general achievement score of over 100. The data were analyzed and evaluated in the SPSS 18.0 package program. As a result of this analysis, mathematics education described using mimio materials has shown success in both programs according to classical training.

Key Words: Mathematical Teaching, Mimio Teaching Materials, Concrete Teaching Materials.

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Comparison of Elementary School Mathematics Curricula with respect to Values Education

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ABSTRACT

The all aims of the school curricula is not only related to acquisition of course subject itself [1,2,3]. They can also include the development of students' characteristics such as values education. There are some studies on the value education related to mathematics [e.g. 4, 5, 6]

In 2017 elementary school mathematics curriculum value education is stated explicitly such as being scientific, freedom, sharing, patience, saving, equality, self-respect and tolerance. When this curriculum was announced, they emphasized on this value education [7]. Hence the purpose of this study is to compare the 2009 elementary school mathematics curricula with the 2017 curricula in terms of value education.

The present study was a qualitative study. The document analysis technique was used to analyse the data obtained from 2009 and 2017 elementary school mathematics curricula documents. The reliability and validity of the study were satisfied. For example, the curricula were coded by two researchers separately. These codes were compared and discussed to become consensus on them.

The results of the study reveal that each elementary school mathematics curricula has some explanations about the values education such as the value of mathematics in daily life and other disciplines, self-confidence in mathematics, aesthetic feelings, responsibility, fairness, flexibility, self-control and cooperation.

Key Words: Values education, mathematics curricula, elementary school

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Comparison of Spatial Visualization and Geometric Thinking Abilities of Preservice Teachers

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ABSTRACT

It is clearly seen that students are unsuccessful in geometry questions that were asked in a math examination when the results of international examinations were evaluated. The geometric and spatial thinkings are necessary for an every area of human life. The geometry, which leads to an improve the thinking, reasoning, and evidencing abilities of the student, is a part of mathematics. NCTM [1] emphasized that visualization, reasoning, and geometric modeling should be used for solving the geometric problems. It was determined that the visualization was more used in a geometric process instead of solving algebra problems when the previous studies that were investigated the importance of visualization in mathematics and geometry educations. Therefore, the necessary precautions have to be taken account in the teaching of mathematics and geometry. In addition, the level of preservice teacher's the spatial visualization ability needed for the geometry teaching is an important due to the fact that the basic of geometry as a branch of mathematics should be created in the elementary education. The spatial visualization was a sub-dimension of visual-spatial skill. It was the ability to determine how the artifact obtained by rotating or moving the object.

In this research, it was aimed to compare the spatial visulization and geometric thinking abilities of preservice teachers. This study is a descriptive survey model. This group consists of 85 preservice teachers. Purdue Spatial Visualization Ability and Van Hiele Geometric Thought Test were used for collecting data. The analysis of dataset has been carried out. After the completion of analyses, the findings obtained from these analyses will be explained in detail.

Key Words: Preservice Teachers, Spatial Visualization, Geometric thinking.

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Comparisons of Instructional Approaches of Elementray School Mathematics Curricula of 1998 and 2009

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ABSTRACT

There are many research studies evaluation of the elementary school mathematics curricula for 2009 which is revised version of the 2005 curricula [e.g. 1, 2, 3, 4]. There is a few study to compare this curriculum and the previous one (e.g. 5]. Most of the people including the researchers have been realizing that instructional approach of the 2009 curriculum was radically different from the 1998 elementary school mathematics curriculum since 2004. In the present study the purpose of the study is to compare them in terms of instructional approaches to reveal the facts. There is a need to change this great misunderstanding because the scientific reasons and respect for the 1998 curriculum development committee, the Turkish Republic of Ministry of National Education and this curriculum.

The present study was a qualitative study. The document analysis technique was used to analyse the data obtained from mathematics curricula documents approved by the Turkish Republic of Ministry of National Education in 1998 and 2009 [6,7,8]. The reliability and validity of the study were assessed. For example, the documents were coded by two researchers separately. These codes were compared and discussed to become consensus on them. The themes and their sub-categories were determined.

The results revealed that similarities between the instructional approaches of the elementary school mathematics curricula of 1998 and 2009 are related to skills, teaching techniques, technology use and assessment for improving the instruction.

Key Words: Mathematics curriculum, elementary school, instructional approach

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Conceptualization of Elements of Geometric Shapes in Technology-Enhanced Learning Environment

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ABSTRACT

It is necessary to provide students experiences and tasks to learn geometry by improving their geometric thinking and broadening their views of geometry content. To meet this necessity, the classrooms must become environments including opportunities for exploring and understanding geometric concepts through inductive and deductive reasoning, and valuable geometric activities (Henningsen & Stein, 1997). At the point of enhancing such environments for geometry, dynamic geometry software (DGS) is beneficial since it supports potentiality such as accessing easily many of the geometrical concepts and different representations of geometrical shapes in the processes of exploration, interpretation and sense making (Liang & Sedig, 2010). DGS provide many advantages and more flexible learning environments to examine the shapes than concrete materials by the dragging aspect. Hence, in the present study, the conceptualization of preservice mathematics teachers about the elements of geometric shapes was explored. Twenty preservice middle school mathematics teachers who were junior participated in the present study. They engaged in the activities about the elements of geometric shapes, their roles in constructions, drawings and definitions of the geometric shapes by DGS. The data collected through written worksheets were analysed by content analysis. In the the study, it was observed that the DGS enhanced the preservice teachers' conceptualization of elements of geometric shapes. They successfully understood and examined the roles of the elements. Also, by the dragging aspect of the program, they successfully and easily constructed and examine the roles of the elements in geometric shapes.

Key Words: Elements, geometric shapes, technology.

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Determining the Attitudes of the Teacher Candidates Towards Mathematics- The Example of Ondokuz Mayıs University

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ABSTRACT

Since the candidates of primary school teachers are well connected to mathematics because of their subject area, determining their attitude to this lesson is very important. The aim of this study is to determine the level of attitude towards mathematics of the 3rd class candidates who are trained in the department of primary school, mathematics, science, social sciences and preschool teaching. This study, with this aim, was carried out by 193 teacher candidates who receive education in Ondokuz Mayıs University in autumn term of 2016-2017 education year. 'Attitude Towards Mathematics Scale' with 20 items which was developed by Aşkar (1986) was used as data collection tool. Descriptive statistical techniques was used to determine the points of teacher candidates' attitude towards mathematics. Descriptive analysis is used to process data which don't need deep analysis (Şimşek and Yıldırım, 2016). At the end of the study, it was showed that the points of the teacher candidates' attitude towards mathematics were high except for teachers of primary social sciences. One of the most significant result of this study is that the candidate teachers of primary school mathematics have the highest attitude points. Also, another important result of our study is that the candidate teachers of social sciences have less attitude points than the other teacher candidates.

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Development of Mathematical Connection Self-Efficacy Scale

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ABSTRACT

Mathematical connection generally described in three categories. These are connection between mathematics and real world, between other disciplines and within mathematics (NCTM, 2000; Özgen, 2013a; Özgen, 2013b). In this study, it has been accepted that these three types of mathematical connection. The aim of this study is to develop a valid and reliable scale that measures the self-efficacy beliefs of mathematical connection of high school students. The draft form of the scale consisting of 36 items was applied to 378 high school students. In the analysis of the obtained data, item total correlation, explanatory and confirmatory factor analysis and reliability analyzes were performed according to related literature (Büyüköztürk, 2005; Kalaycı, 2014). As a result of the factor analysis, it was found that the scale had a 5 factor structure and the variance ratio explained by the whole scale was 52.34%. Factor names were given to the sub-factors of the scale as difficulty, using mathematics, connecting mathematics within itself, connecting with real world, and connecting with different disciplines. The fit of the 5-factor structure obtained from the exploratory factor analysis was examined by two-factor confirmatory factor analysis. It was determined that the fit index values obtained as a result of the confirmatory factor analysis applied were in agreement between the model and the observed data. It has been found that the proposed model is good or acceptable level. In addition, the Cronbach alpha internal consistency coefficient of the scale is 0.85. As a result of the statistical analyzes, a total of 22 items of Likert type and 6 negative ones were developed for the mathematical connection self-efficacy scale. According to the findings of this study, it is understood that the mathematical connection self-efficacy

scale can be used as a valid and reliable scale. It can be said that researches to be done with this scale can be used especially for high school students to examine, recognize and deduce the mathematical connection self-efficacy. It is also thought that this scale can be used by researchers who want to study the self-efficacy beliefs about the mathematical connection of the teacher and the teacher candidates in order to provide the conditions of validity and reliability.

Key Words: Mathematical connection, self-efficacy, scale development.

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Effect of Computer Aided Education on Students' Statistics Achievement and Attitude

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ABSTRACT

The purpose of this research is to investigate the effect of computer-assisted instruction on statistical teaching. To this aim, the impact on students will be examined using an educational software designed to teach basic statistical procedures on student attitudes and academic achievement. In the study, quasi-experimental design was used with pre-test-final test control group. The research population consists of students taking the measurement and assessment course in the fall semester of the Faculty of Education, 2014-2015 academic year, and the course includes basic statistical procedures. The sample of the research consists of 38 students determined by convenience sampling method. "Statistical Attitude Scale" (2014) developed by Yaşar and "Basic Statistics Operations Achievement Test" developed by the researcher were used for the collection of research data. The results of the success test and attitude scale were analyzed by SPSS package program and the findings were obtained. It has been determined that the educational software used in this information contributes positively to the academic achievements of the students but has no effect on the attitudes of the students towards the course. In conclusion, research findings indicate that computer-assisted education contributes to the academic achievement of students but does not have an effect on their attitudes.

Keywords: Computer Assisted Instruction, Instruction of Basic Statistical Operations, Instructional Software, Success, Attitude.

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Encountered Student Mistakes About Fractions and Its Possible Reasons

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ABSTRACT

The generation and improving of the fraction term takes long time in learning process. That is why one of the hardest topic in mathematic is fractions for students and teachers (Biber, Tuna and Aktaş, 2013). It is emphasized in the studies that teachers should use models and shapes on solving the operations related to the fractions. It is indicated that with this way the fraction term is materialized and the students are solving the operations related to the fractions more easily (Kocaoğlu and Yenilmez, 2010). Also it is recommended to the teachers that they should deal with the matter of fractions by considering the misunderstanding and mistakes of the students about fractions and prepare their lecture designs and presentations according to that. It is indicated that with this way the making mistake risk of the students can be minimalized by expelling the students from possible misunderstanding and mistakes (Soylu and Soylu, 2005). Within this scope, in this study it is aimed that to identify the misunderstanding and mistakes about fractions and operations with fractions of the 5th and 6th grade middle school students. In the study, one of the qualitative research pattern, case study model is used. The study is carried out with totally a hundred secondary school student being fifty-fifth and fifty-sixth class which they have different success level, by using maximum variation sampling method which belongs to purposeful sampling methods. The success status of the students is identified with their mathematics grade point averages and the information gathered from their mathematics teachers. To identifying the misunderstanding and mistakes of the students about fractions two tests are prepared by researchers which are composed with ten open ended questions for 5th grade and twelve open ended questions for 6th grade students. Before preparing tests, related literature has searched, and Secondary School Mathematic Study Program of the Ministry of National Education and the 5th and 6th grade mathematic course books accepted by Turkish Education Board of Ministry of National Education are examined. After that the opinion of two domain experts and ten mathematics teachers who are working at the schools that the tests are applied is gathered and related revisions are made for tests' validity. Content Analysis Method is used on the analysing of the data. The students' solving are examined in three categories as correct, wrong and blank. The analysis of two experts are compared and the reliability level is found as %98. Also when the data are interpreted, directly quotations from the answers of the students are used to gain the reliability and validity. It is seen from the result of the study that students have misunderstanding and mistakes about fractions such as, students are not using the modelling on the operations because of that most of the mistake and misunderstandings are about operations with fractions. Generally, the mistake by the 5th and 6th grade students about addition and subtraction on fractions is that they are making the addition and subtraction by thinking the dividend and denominator separately.

Key Words: Mathematics education, fractions, mistakes, misunderstandings

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Evaluatıon Of Classroom Teachers' Opınıons Relatıng To Teachıng Professıon

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ABSTRACT

All of the professions in a society are important. However, amongst all of these professions, teaching has a distinctive importance. It's because of that teachers are the ones who raise doctors, lawyers, prosecutors, engineers etc. If we want doctors, lawyers, prosecutors or engineers to be competent, then we have to raise the teachers in a better way. Subsequent to that, we have to make teachers love their professions so that they can be successful in their fields. If teachers have negative opinions about their professions, they won't likely to be successful. Aim of this research is to evaluate classroom teachers' opinions relating to teaching profession. This research's study group consists 36 classroom teachers working in Diyarbakır in 2016-2017 educational year. Qualitative research method is used in this research. Research's data was gathered by using semi-structured interview form and all of the gathered data is content-analyzed. The consistency was found to be 94% with the aim of providing coding consistency and coding by two different researchers in the field. During the analysis, codes were created reflecting the opinions expressed by the teachers towards the questions. After encoding, the process of matching the data has been started. In the matching process, the codes are merged to create sub-themes and each sub-theme is placed under the themes. According to the results of the research, the main reasons of teachers' negative opinions relating to teaching profession can be listed as; low-paid salary, discredit in society, working under difficult conditions. Besides, the main reasons of having positive opinions can be listed as; favorable working hours and its being suitable for women. Besides, it is shown that implementations like 'paid (substitute) teachers' affect teachers' opinions about their professions in a negative way. This implementation causes a

misunderstanding amongst society as if teaching is an ordinary profession which can be managed by anyone just because there are no other implementations like ‘paid (substitute) doctor’, ‘paid (substitute) engineer’ or ‘paid (substitute) lawyer’ as an alternative. Thus; this condition causes teachers’ to have a negative perception about their own professions.

Key Words: Teaching profession, Classrooms Teachers, Teacher Opinions

Evaluation Of The Opinions Of Basic Science Department Administrative Students Related To Mathematical Applications Of Worksheets

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ABSTRACT

The work sheet is the materials that help students to organize their knowledge on their own, including the steps that students need to take during the course of any topic, in this respect, work sheets can be used as tools to guide student learning in constructivist learning environments. The worksheets provide a fun learning environment that affects the cognitive aspects of the students positively as well as contributing to the cognitive characteristics of the learners by enabling them to become active and learn permanently and meaningfully. It improves the motivation and aspiration of the lesson by correcting the misconceptions and misunderstandings of the learners. The purpose of the study is to investigate whether elementary education teacher candidates have an effect on the learning of working leaves applied in mathematics lessons and the worksheets received from the book (Ready-to-Game) developed by Frances M., Thompson B. (1994). Qualitative research method was used in the research. The data of the study were collected by using the semi-structured interview form. The content of the obtained data was analyzed. In general, it has been stated that study worksheet are a great contribution to a better understanding of the lesson, but they are concerned that the preparation and application of study worksheets take time and that the curriculum cannot be trained. Preschool teacher candidates; Application is to encourage learners to learn, and teacher candidates of classroom teachers have supported the view that they are more useful to seeing and completing our deficits by repeating the subject again.

Key Words: Worksheets, Basic Education, Teacher Candidates,

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Evaluation of the Results of a Mathematical Literacy Teaching Practice for 5th and 6th Grade

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ABSTRACT

Eliminating the disconnection between school mathematics and everyday life remains one of the fundamental challenges of mathematics education. Many documents highlight the importance of working with actual data in school mathematics courses to eliminate such disconnection[1]. This increases the importance of mathematical literacy, which refers to capacity to use mathematical knowledge acquired in school in everyday life. The improvement of mathematical literacy remains one of up-to-date goals of education. In this case, equipping individuals with mathematical literacy is a main goal.

In this study, the answer to "Is there a significant effect on the mathematical literacy success of 5th and 6th grade students of teaching by teachers who trained[1] about mathematical literacy (30 hours)?" was sought. Two trained volunteer teachers have implemented mathematical literacy practices in the course of "Mathematics Practices" (2h / week). Pre-post test results of students who solved mathematics literacy questions for two lessons per week during 12 weeks were reported in the study. The data collection tool used in this study is the Mathematical Literacy Test (MLT). Although there were 27 fifth grade and 26 sixth grade students, two groups of 25 students' data were used because of the fact that two students in the fifth grade and one student in the sixth grade did not take the post-test. The pre-test and post-test answered by students participating in the study, were read within the framework of the rubrics published by PISA.

[1] Teacher training was conducted through the participation of the Bursa Directorate of National Education and Uludağ University, Project No: KUAP(E)-2015/26 to collect data for the dissertation named "The İnvestigation

The data of the study based on the experimental design, was analyzed by Paired Samples t-test. A paired-samples t-test was conducted to see mathematical literacy teaching effect on student learning comparing pre and post test scores. In 5th grade, There was statistically a significant difference [$t_{(24)}$=-7,587, p<0.01] between pre-test mean ($\bar{x}_{pre-test} = 11{,}24$) and post-test mean ($\bar{x}_{post-test} = 16{,}96$). Effect size (d=1,5) calculated in the test result, indicates that this difference is huge [2]. In 6th grade, There was statistically a significant difference [$t_{(24)}$= -11,423, p<0.01] between pre-test mean ($\bar{x}_{pre-test} = 10{,}60$) and post-test mean ($\bar{x}_{post-test} = 19{,}52$). Effect size (d=2,28) calculated in the test result, indicates that this difference is huge. It has been achieved that teachers, who have studied mathematics literacy, will contribute to the development of mathematical literacy in students. It is also recommended that mathematical literacy courses should be taught from primary school.

Key Words: Mathematical literacy, problem solving, developing mathematical literacy.

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of the Development of Students' Mathematics Literacy Who are Educated by Mathematic Teachers Trained about Mathematics Literacy".

Examination Of Teacher Candidates' Hopelessness Levels For The Future

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ABSTRACT

The word "hope" in English, which is generally one of the most common languages of the world, is preferred to express the word “umut” (Yıldırım, 2015). "Hope" implies the meaning of "expecting or expecting things to happen" or "asking for or expecting" in the dictionary (Mirable, 2004). The concept of hope is a force that motivates the person to act and enriches and motivates the individual's life as a feature that gives a sense of well-being (Ağır, 2007). Despair can be expressed as the inability to feel this power. According to Beck (1963), hopelessness is the expectation of negative consequences if they cannot overcome the failures of individuals, they can never solve their problems, that no one can help them, if they are not objective and realistic reasons, they do not make an effort to reach their aims. The American Psychological Association (APA) (1997) describes the reasons for despair; depending on various factors, the individual's activities are restricted for a long time, and as a result of this, emerging loneliness, deterioration of body health, long-term stress, self-abandonment and to lose abstract values or belief in God. When we look at the work done, it has been determined that there needs to be studies to determine the levels of hope and hopelessness of the younger generation.

The purpose of this study is to determine the hopelessness levels and causes of the students of different departments attending the faculty of education. The study group of the research constitutes first and fourth year students who study in different departments at On Dokuz Mayıs University Faculty of Education. In the study, Beck et al. (1974) developed the "Beck Hopelessness Scale" is used as a data collection tool. The scale is a measure of self-evaluation, consisting of 20 items,

scored between 0 and 1. The reliability and validity study for Turkey was carried out by Seber and Durak (1991). In addition, the Cronbach Alpha reliability coefficient was 0.86. The scale consists of three sub-dimensions: expectations and feelings about the future, loss of motivation and hope.

Findings revealed that the expectations and emotions about the future of the students in different departments are influenced by different variables. It has been determined that the level of motivation in the learning process of the students changes over time depending on different variables. All the conclusions of the expectations, motivation and hope levels for the future of the younger generation will thoroughly be discussed in the congress.

Key Words: Hope, Hopelessness, Beck Hopelessness Scale.

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Examination of Perceptions Related to the Assessment and Evaluation Process of Middle School Mathematics Teachers

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ABSTRACT

Assessment and evaluation is an indispensable element of the learning process. The concept of Assessment and evaluation plays a critical role both in accountability and in the teaching process at the point of increasing the quality of teaching [1]. The aim of the Assessment and evaluation applications should be not only to give grades to the students or to determine the success-failure situation but to orient the students according to their interests and abilities considering the individual differences of the students and to reveal the quality of the teaching service given [2]. This situation raises the issue of determining the perceptions of the teachers who organize the assessment and evaluation activities in the class in relation to the assessment and evaluation process. For this reason, the purpose of the research is to examine the perceptions of the secondary school mathematics teachers about the assessment and evaluation process, taking into account various variables. In addition, it will be possible to determine how mathematics teachers understand the assessment and evaluation process, to develop the proposals for improving the professional skills of the mathematics teachers in the assessment and evaluation to develop suggestions for the in-service training needs of the mathematics teachers.

The survey research was used in this study. A total of 144 middle school mathematics teachers, 59 male and 85 female, participated in the research. In the data collection process, the "Teachers' Understanding of the Assessment and Evaluation Process" scale developed by Brown [3] and adapted to Turkish by Vardar [4] scale was used. There are four sub-dimensions of scale. These are student accountability, school accountability, improvement and irrelevance. The dependent variable of the research is the perception scores of the teachers about the

assessment and evaluation process, while the independent variables are gender, age, years of teaching experience, in-service training situations. In the study, the data obtained by means of measurement were analyzed by descriptive analysis, correlation and one way multivariate analysis of variance (MANOVA).

When we consider the findings of the research, it has been determined that the measurement and evaluation process of middle school mathematics teachers perceives that students should be used to improve their achievement and improve teaching quality. It also suggests that the process of assessment and evaluation can be used to determine whether students are successful at the end of instruction and to what extent and to what extent students are successful. The assessment and evaluation approaches most commonly used by secondary school mathematics teachers are multiple choice tests, true-false tests, short-answer (space filled) tests, projects, and long response written examination.

Key Words: Secondary school, mathematics teacher, assessment and evaluation,

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Examination of Planned and Implemented Lessons

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ABSTRACT

Teachers' attentions, expectations and decisions during planning have impacts on their perspectives and reactions in teaching (Choy, 2015). Thus, planning and teaching are interrelated with each other powerfully (Hughes, 2006) and this relationship influences the quality of instruction and student achievement. However, it is difficult for preservice teachers to prepare plans and conduct lessons (Carrier, 2011). In this direction, the purpose of this study was to determine how closely the implemented lesson reflected the planned lesson and investigate the impact of the planning meetings on the implementation of the lesson. In this research, case study was conducted as a research methodology with the participation of four senior preservice middle school mathematics teachers. They planned a lesson related to fractions together and one of them implemented the lesson in a real class while the others were observing. After implementation, they discussed on what went well or not and shared their ideas on teaching by considering the lesson plan. Data were collected through the video recordings of planning meetings, implementation of the lesson and discussion process. The transcripts of the videos and lesson plan were analyzed through content analysis method to address the similarities and differences between the planned and implemented lessons and present the factors which influence the implementation of the lesson. According to the findings, possible situations included three categories: it was planned and happened, it was planned and did not happen and it was not planned but happened. Besides, it was found that discussions during planning meetings had an impact on the implementation of the lesson. In addition to this, it was observed that collaborative planning was very important for developing and shaping the lesson because of preservice teachers' lack of knowledge and experiences. Finally, they had difficulty in responding to students when they encountered unexpected reactions or events that they had not talked in planning.

Key Words: Planning, teaching, preservice teachers, fractions

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Examination of Preservice Middle School Mathematics Teachers' Levels of Proof about Triangles through Argumentations

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ABSTRACT

Proof has an important place in mathematics. In the literature, there have been research investigating different grade levels of students' proof skills and levels. Balacheff (1988) explores mathematical proof into four groups: pragmatic proof, intellectual proof and demonstration proof. Consecutively, the students at the pragmatic proof level are the lowest level and they try to produce proof by suggesting examples; the ones at the intellectual proof level try to make proof using formulizations. The students at the demonstration proof level are at the highest level and they use theorems and information accepted as true by everybody in their proofs. Moreover, if it is considered that argumentation can be defined as effort and actions of convincing others about a claim, it can be stated that proof and argumentation are related (Pedemonte, 2007). Furthermore, some research have showed that the process of production of proof can be enhanced with the help of argumentation activities (Boero et al., 1996; Garuti et al., 1996; Mariotti, 2001). Therefore, in the current qualitative research, the proofs of preservice middle school mathematics teachers (PMSMT) with and without argumentations were examined. In other words, in order to examine the way that argumentations are related to proof, the proofs produced by the students studying individually and through argumentations were compared. In this respect, the purpose of the study was to answer the research question of "How do argumentations affect PMSMT's proof levels?" With this aim, initially the participants were asked some problems about proof related to triangles and to work individually on them. Then, they were participated in whole class discussions and produced argumentations about these problems. Afterwards, the proof levels of PMSMT were compared focusing on the

proofs produced in the process of working individually and formed through argumentations. The qualitative data analysis process was performed by using Toulmin's argumentation model and Balacheff's proof levels. The process of data analysis is continued. The findings will be reported.

Key Words: Argumentation, proof level, trieangles.

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Examination of Realistic Problem Solving Ability of Preservice Teacher

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ABSTRACT

Mathematics education should encourage students to acquire a critical thinking approach that provides the connection between mathematical concepts with problems and situations that may appear in daily life. The studies of problem solving in a mathematical teaching process are more important in terms of changing needs and the age we live in. The focus of mathematics education should be the problem solving according to the previous study that was related to the problem solving [1]. Also, the problem solving ability in the elementary mathematics curriculum which is being conducted in our country, is expressed as a basic skill that students need to improve. Furthermore, it is said that the problem solving activities should be good agreement with real life. The solving of problems requiring realistic answer had an importance due to Realistic Mathematics Education reforms in mathematics education [2]. In Turkey, these problems have been used in mathematics curriculum since 2005. The problems requiring a realistic answer does not indicate an absolute truth, and there was no known method or equation. More than one strategies were necessary for the solving of these problems. The solving way of problems requiring a realistic answer strongly depends on the experience of the person who is confronted with this problem and whether or not it faces a similar problem. The problem can be a realistic problem solving item for a student today, while the same problem will be part of a regular set of items tomorrow.

In this research, the problem solving abilities of preservice teachers were examined through their answers to problems requiring a realistic answer in mathematics and other lessons. This study is a descriptive survey model. This group

consists of 89 preservice teachers. 43 of them were told that the problems requiring a realistic answer should be solved for the research in an elective lesson of child singings, whereas 46 of them were told that there was a mathematical problem that should be solved in an elective lesson of mathematics applications. The data collection tools used in this research consist of a problem solving attitude scale and 5 word problems requiring a realistic answer. Until now, the findings obtained from the data analysis have revealed that preservice teachers could solve the unrealistic problem. The analysis of dataset has been carried out.

Key Words: Preservice Teachers, Problem Solving, Realistic Answer.

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Examination of the Relation between Computer Literacy and Mathematics Literacy among the Sixth Grade Students

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ABSTRACT

With respect to literacy, new definitions are being made every day in the literature. Every new interpretation leads to the idea that it can change depending on the current environment, tools used and/or purpose intended and there may be different types of literacy including computer literacy, media literacy and visual literacy (Reinking, McKenna, Labbo and Kieffer, 1997; Tuman, 1994). This is also valid for computer literacy and mathematics literacy. Therefore, studies on computer literacy and mathematics literacy are always needed. In the present study, the objective was to determine the computer and mathematics literacy levels of secondary school 6th grade students and to reveal the relation between these levels. In this scope, sample consisted of province, district and village schools as well as private schools. The relation among the computer and mathematics literacy levels of the students selected from various regions was determined and the difference among the levels of those schools was detected as well. Since the relation between computer literacy and mathematics literacy among 6th grade students was determined, the method of the study was relational screening model. Relational (correlative) studies aim at determining whether there are relations among two or more variables (Karasar, 2009). Population of the research was composed of the 6th grade students of a secondary school found in Elazığ while the sample consisted of 139 6th grade students. In the study, “Mathematics Literacy Scale” and “Computer Literacy Scale” which were developed by the researcher and tested for validity and reliability were used. The data obtained were analyzed by means of specific statistical techniques. After the findings were analyzed, results were discussed in line with the literature and necessary recommendations were made.

Key Words: Computer Literacy, Mathematics Literacy, Literacy.

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Examination of The Views of Sophomore Pre-Service Mathematics Teachers on The Qualifications of A Good Mathematics Teacher

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ABSTRACT

The qualifications of the students that constitute the target group of education activities are related to the qualifications of the teachers. The quality of the teacher plays a very significant role in the achievements of educational systems (Köseoğlu, 1994). The present study aims to investigate the qualifications of a good mathematics teacher according to sophomore pre-service mathematics teachers. In the study, one of the qualitative research methods, the case study technique was selected because the study aimed to investigate the knowledge of pre-service teachers in depth. The study group included 66 pre-service math teachers, 48 females and 18 males, who were attending their second year in Fırat University, Faculty of Education, Department of Elementary Education Mathematics Teaching in 2016 – 2017 academic year. The data obtained with semi-structured interviews conducted with pre-service teachers were analyzed with content analysis. After the analysis, the data were grouped under specific themes to determine the sub-dimensions of the data. Frequency and percentage figures for the determined sub-dimensions were calculated and evaluated.

According to the sophomore pre-service mathematics teachers, ideal mathematics teachers should demonstrate the students that mathematics is intertwined with daily life, have extensive knowledge on the field, be able to instruct the class by materializing the content since it is an abstract course, and know how to transfer the knowledge in addition to being knowledgeable in the field. Furthermore, they stated that teachers should be both role models and act as guides for students both in the school and out of the school. Furthermore, they should make the students

love mathematics, overcome the negative prejudice against mathematics, open up the horizon of the students, instruct the course with appropriate methods and techniques, love the students, love their profession, be patient, aim to teach all students, use a fluent and comprehensible language when instructing the class, instruct the class by recognizing the individual differences, understand the student psychology, establish good communications with students, constantly improve themselves, prepare for the classes, and ensure that the students participate in the class. It was stated that teachers with these qualities would instruct their classes better and prepare students for the future better.

Key Words: Pre-service Mathematics Teachers, Teacher Qualifications.

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Examination of Visual Mathematics Literacy Self-Efficacy Perceptions of Middle School Students in Terms of Variable Variations and Student Opinions

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ABSTRACT

The aim of this research is to examine and interpret the visual math literacy (VML) self-efficacy perceptions of the secondary school students in terms of variables such as gender, success in mathematics, academic achievement, ability to relate mathematical concepts to everyday life and integrate everyday objects with mathematics. Mathematical literacy as the capacity of the Economic Development and Cooperation Organization (OECD, 2003) to understand and recognize the role that mathematics plays in the world, using mathematical thinking and decision-making processes to solve possible problems today and tomorrow as a human thinking, producing and critical individual The problems encountered in daily life by VML, Bekdemir and Duran (2012), which are formed by the combination of visual literacy defined by Debes (1969) as having the ability to distinguish and interpret visual movements, objects, symbols and other stimuli around one's visual or spatial, To be able to perceive, express, interpret, evaluate and use spatial information mathematically. The target group of the study, which is carried out by the mixed method, constitutes 74 students who read in the 8th grade of middle school in the spring term of 2016-2017 academic year and are selected by simple random sampling method. VML Self-Efficacy Perception Scale, which determines students ' VML self-efficacy perceptions, Personal Information Form, which explains some socio-demographic characteristics, and Student Opinion Form, which determines the students' thoughts about VML, were used as data collection tools in the study. In the analysis of the data, descriptive statistics, t test, one way analysis of variance and Tukey method were used in multiple comparison methods. As a result of the research, there was no significant difference between students' genders and VML self-efficacy perceptions but there was a significant difference in terms of academic achievement, ability to relate mathematical concepts to everyday life, and integration of everyday objects into mathematics. The findings were discussed in the light of the literature and some suggestions were made

Keywords: Visual Literacy, Mathematical Literacy, Visual Math Literacy, Perceived Self-efficacy

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Examining Preservice Teachers' Noticing Skills about Triangles

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ABSTRACT

Noticing is a part of everyday life and a concept that includes focusing on instructional issues and making sense of the events (van Es, 2011). Noticing skills are important in terms of increase in both quality of instruction and achievement of students (van Es, 2011). It is one of the essential skills that both teachers and preservice teachers need to gain (Mason 2002). On the other hand, triangles are one of the most basic and common geometric figures which are used in constitution of other geometric shapes and structures, calculation of their areas and examination of their properties (Fey, 1982). At this point, the aim of this study was to determine preservice middle school mathematics teachers' noticing skills in terms of their content knowledge on the definition of triangles and their properties. Case study was used to gain an in-depth understanding of the situation. The participants were composed of 22 junior preservice middle school mathematics teachers enrolling the department of elementary mathematics education at a public university. The data collection process included two stages. At first stage, peer group discussions were conducted and preservice teachers were asked to define triangle and triangle types and also indicate the relationship between them through a diagram on a paper. At second stage, whole group discussions occurred and preservice teachers shared their triangle definitions and discussed on the correctness, necessity and sufficiency about these definitions. The data were obtained from written documents and transcripts of whole class discussions. The data were analyzed through content analysis technique. According to the findings, the triangle definitions and comments on the properties of triangles made by preservice teachers showed that their noticing levels regarding this subject were low. However, it can be said that whole class

discussion process helped them to define the concept of triangle more correctly since sharing different ideas, indicating incorrect approaches and using new knowledge enabled to understand the missing points.

Key Words: Noticing, triangle, preservice teachers

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Examining the Patterns Generalization Strategies of 6th Grade Students

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ABSTRACT

The view that mathematics is the generalization of relations shows the importance of generalization in mathematics teaching (Lee, 1996). Therefore, it is highlighted that exploring and generalizing the relations in patterns may improve the algebraic thinking skills of children who are at their early ages (Armstrong, 1995). Investigating students' generalization strategies regarding patterns, which were included in every grade from primary education first grade to eight grade in the mathematics curricula in Turkey reconstituted from 2005 to 2013, is considered significant for future algebra learning.

The limitedness of the number of studies dealing with patterns generalization in Turkey indicates the need for research in this area. However, the literature research suggests that patterns and patterns generalization strategies have been mostly carried out with teachers or prospective teachers.

Recognition of the generalization strategies students employ in patterns is deemed important to contribute to their algebraic thinking and construction of the concept of function. It is thought that conducting research on the generalization of patterns at the sixth grade level, in which students express the general rules of patterns symbolically, go through the process of transition to algebra from arithmetic, and construct the basic concepts of algebra, will contribute to the literature. Furthermore, the analysis of the patterns generalization strategies of 6th grade students would enable us to understand the cognitive needs of students in a comprehensive way.

In this context, the purpose of the study was to examine the patterns generalization strategies of 6th grade students. In this study, a case study was use as research methodology. The research sample were composed of 232 students at 6th grade, determined by randomly from in a middle school in Elazığ in the spring term of 2015-2016 academic year. The data gathering tool was a pattern test composing of three linear pattern problems. Answers of the participants were examined by using qualitative and quantitative analyses. In data analysis, at first the answers are examined and it is decided that the answers are right or wrong. In the second part of data analysis, the collected data is classified through the generalization strategies in the related literature.

The findings illustrated that 6th grade students mainly used “recursive or additive strategy” to find near terms and obtained the highest number of correct answers through this strategy. The students generally used the “recursive or additive, multipliying with difference, whole-object or proportion” strategies to find far terms and obtained incorrect answers. So these students were more successful on finding the near terms instead of finding the far terms.

They did not consider the given figures or the structures of those figures. They focused on “numerical strategy” in order to find the rules of the shape pattern problems. The students rarely used “explicit strategy”. Also there was a lack in the exploring the pattern rule and generalization. It is suggested that teachers should mainly use activities in which patterns generalization strategies can be used, and teach the relationship between term and order of term.

Key Words: Mathematics education, patterns, generalization strategies

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Experiencing the functions of proof in the dynamic mathematics software

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ABSTRACT

It is considered that individuals who understand mathematics and can implement their mathematical ability and knowledge in real life to shape the future in this changing world. As they will be engaged in more active roles, it is important that the process of teaching and learning of mathematics should be evaluated regarding the demands of the 21st century [1] and the difficulties encountered should be eliminated. In this process, proof is one of the most difficult topics for students [2] and a significant part of mathematics [3]. Proof provides students with opportunities to enhance their mathematical understanding. However, using proof for this purpose is very difficult in the learning and teaching process. The fundamental functions of proof and proving in this process are verification and explanation [3]. In this context, the purpose of this study is to provide students with opportunities to experience these fundamental functions of proof and proving in the dynamic learning environment. For this purpose, the details of how to use the tools of the dynamic mathematics software GeoGebra are discussed in terms of using these fundamental functions of proof in the learning and teaching process. Fifteen dynamic visual proofs that are constructed with GeoGebra software are investigated by using document analysis. Consequently, it is considered that GeoGebra tools may provide students with opportunities to experience functions of proof in the learning environment.

Key Words: Proof, The functions of proof, Dynamic mathematics software, GeoGebra

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MATEMATİKÇİLER DERNEĞİ

How does “Mathematics Education Literature” approach the “Mathematical Modelling”? : A Thematic Review

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ABSTRACT

Mathematical modelling is a shining star of mathematics education literature for a while now. And it becomes a catchy issue in these days in Turkey, because PISA results show that the questions Turkish students failed most, requires mathematical modelling. It seems that mathematical modelling will be a trending topic for the researchers of mathematical literacy, so we think that examining past studies about the issue would be a loadstar to the future studies.

In this study, we use thematic review method in order to analyse the studies on mathematical modelling. In thematic reviews, one can identify each study according to thematic matrix, underline the general tendencies, explain differences and similarities and focus on the unique qualifications of each study [2];[4]. In this regard, there are some differences between thematic review and literature review and meta-analysis. Literature review is an examination, synthesis and abstract of the researches about a specific research problem [1]. Meta- analysis is a grouping of similar researches about an issue, a study area or a theme, under some criterion and an interpretation of quantitative results of them [3]. Taken all together, thematic review matrix by Ormancı, Çepni, Deveci and Aydın [5] is utilised after some editing in accordance with the aim of the study.

In this study, we divide the studies into two groups: their general features and content features. We ask and respond several questions that help to understand the nature of the studies: (1) How is the general features of the studies on mathematical modelling (will be mentioned “studies” here in after)? (2) What are the reasons of the studies? (3) What are the aims of the studies? (4) What are the methods, sample/participants and data collection tools of the studies? (5) What are the

subjects of the studies? (6) What are the results of the studies? (7) What are the implications of the studies?

In conclusion, we try to examine and explain mathematical modelling studies in mathematics education literature.

Key Words: Mathematical literacy, mathematical modelling, thematic review.

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Investigation of Field-Dependent and Field-Independent Cognitive Styles of Seventh Grade Students in Geometry Problem Solving and Mathematical Process Development

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ABSTRACT

In the ever-changing and developing world, those who understand mathematics and have mathematical competency will have significant opportunities and options to shape their future. Mathematical competence opens the doors to a productive future. The lack of mathematical competence closes these gates. Everyone needs to understand mathematics. Everyone needs to understand mathematics (NTCM, 2000, s. 1). For this reason, the mass of mathematical formulas must be moved beyond being a lesson, and environments should be created where students can experience mathematics by thinking and doing. In the new mathematics curricula ,students will experience mathematics, develop problem-solving skills with activities that will help them develop their reasoning skills, use the learned knowledge not only in mathematics but also in other disciplines outside mathematics; emphasizing the importance of the environment in which they can transfer their written and verbal information. As a matter of fact, Ministry of National Education (Ministry of National Education [MEB], 2005) teaches that each student has different intelligence areas and learning styles, and that students can learn better with this kind of education.

Compared to European countries it has been pointed out that in our country, students need to be updated in their educational curricula due to their personal development and the need for individualized learning approaches. When we examine the mathematical learning areas prepared in this regard, we can see that geometry takes precedence. Geometry aims to give students various skills such as realism perception, providing visual intuition and general interior visuals. For this reason, geometry teaching should be in accordance with the learning and development levels

of the students. Students need to explore and experience in order to learn geometry effectively. For this reason, geometry education should be given to the students via rich experiences especially starting from the primary education stage (Kılıç, 2003). Within these rich experiences, in the courses prepared according to the cognitive style of the students, the problem solving and mathematical process skills in the mathematics learning areas should interact with each other. These skills, which people can use during and after their school life, are located in mathematics learning areas. For this reason, the purpose of the research is to examine mathematical process developments in solving geometry problems of seventh grade students in field-dependent and field-independent cognitive styles.

In this research, geometric problem solving and mathematical process skills of the seventh grade students in field dependent and field independent cognitive style are examined. For this purpose, data was collected from 125 students in a state secondary school affiliated to the Ministry of National Education located in the province of Tarsus, Mersin in the academic year of 2015-2016. The data was gathered with two different tools: 'Group Embedded Fıgures Test' and 'Geometry Reasoning Assessment Scale' consisting of open ended problems. The Group Embedded Fıgures Test' developed by Witkin et al. has determined the level of field dependent and field independence of students. With the Geometry Reasoning Assessment Scale, students' problem-solving skills and reasoning skills are measured. Relational search model was used in the study. Descriptive and inferential (multiple regression) analyzes were used in the study. As a result of the findings, it was determined that students were slightly below the average of problem solving skills and below the average for mathematical process skills. Problem-solving and mathematical process skills are also found to be higher in students with field-independent cognitive style. In addition, when the mathematical process skill increases, the problem-solving skill increases, and as the cognitive style score increases, the problem-solving skill and the mathematical process skill increase, but the mathematical process skills do not differ depending on the problem-taking dependency on problem solving. Despite the fact that learners have a connection between field-dependent and field-independent mathematical process skills and problem-solving skills, it has also been found that these cognitive styles have no effect on mathematical process skills and problem solving.

Key Words: Problem Solving, Mathematical Process Skills, Field Dependent, Field Independent, Geometry.

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Investigating Seventh Grade Students' Performance in Reading and Interpreting Data in Mathematics Problems

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ABSTRACT

In the developing world, graphs, charts, tables and data have become a part of today's information society (Mooney, 2002). Accordingly, the research point out that as early as middle school, students should have experience collecting, organizing, representing and interpreting data given in graphs, tables or charts (e.g., Mooney, 2002; NCTM, 2000).

This study examined six seventh-grade students' performance to read and interpret the data presented in graphs and tables while they were working on several real world problems. Three problems, called as '' Memory Stick'', ''Holiday Apartment'' and ''DVD Rental'', were used in the study. These problems were selected among the mathematics items developed for PISA 2012 survey (see https://www.oecd.org/pisa/pisaproducts/pisa2012-2006-rel-items-maths-ENG.pdf).
The mathematical content of these problems involved students' dealing with the graphs or tables of data. The participants of this study were six seventh grade students. The students worked individually during almost 40 minutes (one class hour) for each problem. The data sources for this study were students' all written papers including their solutions to the problem. For the data analysis, the data were qualitatively examined with respect to students' understanding and interpretation of the graphs and tables. The findings of the study displayed that except one student, five students' performance in reading and interpreting data presented in graphs sand tables was inadequate. Although students understood the real world problem situations correctly, they had difficulty in interpreting the data given in graphs or tables. Particularly, students could not decide which data given in graphs or tables

are useful for them, and they could not select necessary data/information for solving problems. Moreover, students generally interpreted the real life situations based on only their own daily life experiences; therefore, they made some logical error. For this reason and they could not reach the expected answers of problems.

In order to evaluate the given information/data and to reach the valid results for the real life problems, reading and interpreting the data presented in graphs and tables is a required competence for students. This study initially suggests that in order to develop their students' ability to read and interpret data, teachers should be aware of their students' both competencies and difficulties. Moreover, in their lessons, teachers should provide more opportunities for students to work on real-life problems in which students are needed to interpret the data given in graphs and tables and to discuss the data sets and their graphical representations.

Key Words: reading and understanding graphs and tables, real life problems, middle school students.

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Kimax: A Maxima Package for Analysis of Reaction Kinetics

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ABSTRACT

In this study, we develop a chemical reaction analysis package called “Kimax” using Maxima [1,2], a GNU free symbolic algebra system.

Kimax derives and analyzes differential equations, also called rate equations, corresponding to a large class of chemical reactions of both reversible and irreversible types [3]. At first, it enables user to observe differential equation or system of equations corresponding to a given reaction, and then obtains and displays desired solution components as time evolves.

Some reactions lead to differential equations that can be solved analytically, while others lead to equations which can only be handled numerically [4,5].

A comprehensive analysis of a given reaction requires that one observe the reaction rates and equilibrium states based on the parameters such as stoichiometric coefficients, initial concentrations and rate constants [6]. An interactive tool providing such an analysis leads to better understanding of both the given chemical reactions and the corresponding system of differential equations.

Kimax is user friendly tool that obtains stoichiometric coefficients, initial concentration and rate constant from user and obtain corresponding differential equation or system of equations and then provide solutions (analytical or numerical) so that one can observe to concentration of reactants and products simultaneously. Kimax can be easily added to the existing packages of Maxima and used freely for educational purposes.

*This work is part of the master thesis of the first author at Karadeniz Technical University.

Key Words: Reaction Kinetics, Maxima, Symbolic Algebra

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Learning Proof as Collective Mathematical Activity

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ABSTRACT

Proof is a crucial activity in which students of all ages should be involved if they are actively engaging with and learning mathematical content as suggested by both policymakers (CCSSO, 2010) and mathematics educators (Knuth, 2002). Yet, as traditionally taught, proof often appears to be a formal, rote activity with little meaning to students. Research indicates that students at all levels struggle to understand and construct proofs (Harel&Sowder, 1998), and that teachers have difficulty fostering students' ability to justify and prove (Bieda, 2010). Additionally, many teachers find teaching proof difficult, often due to their beliefs about what constitutes proof and their perception that proof is not appropriate for all grade levels. Teachers thus need to deepen their knowledge to foster students' understanding of proof and justification.

We take proof to be a learning activity with a social character (Cobb&Yackel, 1996) in mathematics classroom communities where learners communicate their reasoning and justify their arguments. Thus, proof is an argument that consists of 1) a connected sequence of assertions about a claim and 2) the norms of acceptability for making and communicating those arguments established by a classroom community (Stylianides, 2007). Proof here is further framed as an activity that helps students develop reasoning and justification skills, even if the proving activity does not include development of a formal mathematical proof.

A proving activity often begins with participants developing conjectures, and then justifying their arguments and explaining their reasoning. This process does not necessarily lead participants to develop a formal proof, but allows them to understand ideas and communicate with others mathematically. Proving activities, as a tool for

learning, allow students to create conjectures, explore whether the conjectures are true, and develop and evaluate justifications and proofs.

The context for this study is a university Professional Development course called “Mathematical Knowledge for Teaching: Reasoning, Justification and Proof.” Twelve teachers enrolled in the course with an average teaching experience of nine years.

Data analysis involved looking for patterns in the classroom observations, field notes, and artifacts. We adapted a methodology proposed by Cobb and colleagues to analyze the classroom practices in terms of how the community determined what constitutes valid proof, and employed a constant comparison method to analyze the data.

Initial results indicate that at several points throughout the class, participants were communally involved in developing a conjecture or justification through discussion and that the nature of such discussions evolved as participants were pressed to think more about the nature of proof. In general, what was taken as proof shifted over the duration of the course, moving from example-based towards more deductive justification. Though the instructor provided definitions for deductive proof, what the class collectively accepted as deductive proof was largely constructed by the participants based on what was both mathematically correct and communally understandable. What became apparent from the preliminary analysis of classroom data is that the whole classroom discussions as a collective mathematical entity were extremely productive in terms of developing practices related to proof.

Key Words: Reasoning and Proof, Teacher Education, Professional Development.

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Mathematics Literacy In Mathematics Education Process: A Thematic Review

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ABSTRACT

The value of mathematical activities that focus on the majority of people in society (and therefore students) can only be determined by the development of mathematics literacy skills of students which can provide contribution [1]. In the last 10 years, the concept of "mathematical literacy" has found itself in almost every phase of mathematics education reform. With the participation of OECD countries and many other countries, the effect of international scale evaluation studies such as PISA and TIMSS has increased the importance of mathematical literacy and as a result many studies and projects related to mathematical literacy have been carried out. As an individual's capacity, mathematical literacy was defined to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen [2].

It can contribute mathematics education to find answers questions like; What should be done to educate students as mathematics literate individuals? What kind of mathematics education should be instructed? How to organize learning environments and teaching programs? In this context, the analysis of researches and studies related to mathematical literacy can provide important contributions for educators, researchers and teachers. In this study, it is aimed to analyze and examine the studies carried out on mathematics literacy in mathematics education process. For this purpose, the method of this study, which is planned qualitatively, was determined as thematic review which based on document analysis. In the research, mathematics literacy articles in the mathematics education process were analyzed using a matrix

containing general characteristics (type, year and demographic characteristics of the articles) and content characteristics (justifications, purpose, research method, sampling, data collection, results and suggestions).

A common theme running through the research on access to mathematical literacy is that communicating on all levels is dependent on the nature of classroom relationships and students' positioning within them: they need to be motivated and enabled to reflect and argue, to make suggestions and challenges, and to draw on their own experiences. In enabling access to mathematical literacy, they are all dependent on a re-figuring of classroom relationships to include an explicit focus on investigation, explanation and justification in mathematics. Although school mathematics was undergoing fundamental changes, including the unification of traditional school mathematics subjects (arithmetic, algebra, geometry) into the inclusive subject of mathematics at the time, the concept of mathematical literacy was still not elaborated in formal education.

Key Words: mathematics literacy, mathematics education process, thematic review.

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Mathematics Subjects Prospective Mathematics Teachers Have Difficulty to Teach

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ABSTRACT

Mathematics curriculum aims to train individuals who value mathematics, think mathematically, use mathematics, and solve problems One of the most effective factors on student achievement is the curriculum used according to studies comparing mathematics success. In this context, the mastery of the subjects of the curriculum of the teachers who are the practitioners of the program is important. The purpose of this study is determine the mathematics subjects that prospective teachers have difficulty to teach and to develop a solution proposal for these subjects in teacher education. Descriptive survey model was used in the study. The study group of the study is composed of 100 high school mathematics teacher candidates who are taking Teaching Mathematics course. Data were collected by a form containing all mathematical topics in the 9-12 grades mathematics curriculum which has been developed by the researchers. Teacher candidates were asked to select the first 3 subjects they would not prefer to teach. They also explained the reasons for their preferences. In the analysis of the collected data, the percentage and frequency values were utilized and the analyzes are continuing. As a result of the research, the subjects that high school mathematics teacher candidates have difficulty to teach will be revealed and the necessary precautions to be taken in teacher education and the solution recommendation will be discussed with the support of the literature.

Key Words: Prospective mathematics teachers, teacher education, mathematics curriculum.

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Mathematics Through STEAM Perspective

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ABSTRACT

In the first quarter of 21st century, there have been significant improvements in science, education, technology and economy. In general, these changes which are called information era paradigm have deeply affected many fields such as mathematics [1]. STEAM is a new approach consisting of first letter of Science, Technology, Economy, Art and Mathematic [2, 3]. In other words, STEAM reflects the mathematics perception of information era. Because today, pragmatic point of view effects every part of our life and many fields including mathematics are considered as potentially value-added concept and as an input for technology [4]. This context caused mathematics to be thought as an engineering project with science, innovation, technology, economy and art. Therefore, mathematics which is a stakeholder of STEAM is called 'new mathematics'[5, 6]. As a shareholder of STEAM, mathematics has become one of the base of technology and economy. The role of new mathematics in STEAM is rather *a bridge*. Such a point of view emphasizing the role of mathematics in wholeness of STEAM is very important for mathematics teaching[7, 8]. In near future, it is easy to estimate that STEAM approach will be effective on development of mathematics curriculum, mathematics teaching and mathematics projects. Hence, for appropriate mathematics teaching based on information era paradigm, knowing the importance and role of mathematics in STEAM wholeness is crucial. To succeed this, determining the opinion of mathematics teacher about the place and role of mathematics in STEAM is important.

The aim of the study which will be carried out through descriptive method is to determine the place and role of mathematics in STEAM based on mathematics teachers' and scholars 'opinion and to analyze these opinion according to some variations. The research will be carried out with mathematics teachers in Şanlıurfa and instructors at Harran University who work in 2016-2017. Data will be collected through a questionnaire developed by researcher and will be analyzed by descriptive statistic techniques. According to the results, with the sample of Şanlıurfa, the role and place of mathematics in STEAM will be determined and some suggestions will be put forward.

Key Words: STEAM, Mathematics teaching, New mathematics.

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Mistakes of High School Students in Use of the Babylonian Number System: The Base Arithmetic and Place Value Concept

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ABSTRACT

Aim of Mathematics History which is the elective mathematics course in the secondary education curriculum is to contribute to the mental development of the students by providing them with different areas of mathematics. The activities that conducted for the history of mathematics within the scope of this course provide students to understand today's mathematics better and to associate with today's mathematics [1]. One of the topics of mathematical history that can provide the students to relate to the present number system is the Babylon Number System. The Babylon Number System requires students to know basic and important skills such as base arithmetic and place value concept. It also allows student to associate this information with current mathematical topics. In this direction, the study examines the mistakes that senior high school students made based on the concept of base arithmetic and step in using the Babylon Number System. In the study, Phenomenological method used as a qualitative research design. Phenomenological method is used to investigate the real causes behind a concept. Participants of the study are 14 senior high school students in city of Bayburt. In the study, activity cards and observations are used as data collection tools. In the activity cards, students are given tasks that require the use of the base arithmetic and stepping concept. The observations are used as supporting elements for the activity cards. The data of the study is analyzed by the content analysis method. As a result of the study, it is determined that students did not fully understand the concept of base arithmetic and place value concept. In addition, it is determined that there is a difficulty in

associating the present number system with the Babylon Number System because the concept of base arithmetic and step cannot be conceptually learned.

Key Words: Babylonian Number System, Base Arithmetic, Place Value Concept

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Out-of-school learning environments in Mathematics Teaching: Science Museums and Science Centers

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ABSTRACT

Falk and Dierking (2002) define informal learning as learning which occurs as a result of the individual's own will according to their interests and needs, and their interaction with the environment. This kind of learning, which takes place spontaneously in accordance with the individual's needs, is also identified as out-of-school learning (Rennie, Feher, Dierking & Falk, 2003). Science museums and science centers which are examples for these learning environments can provide students with fun and permanent learning opportunities and allow each student to follow their individual paces (Öztürk, 2014). The science museums and science centers whose numbers have increased with the TÜBİTAK 4004 – Science Center Establishment Support Program in the recent years and examples of it can be seen in other countries, contribute to learning positively (Anderson vd., 2000). Cooper (2011) claims that informal learning has an important role in supporting formal mathematics learning. In this study, the aim is to present the condition of the mathematics oriented exhibition settings of science museums and science centers in Turkey by observing them, and also to reveal the circumstances of the out-of-school learning environments which are required for mathematics education. In this study, which is based on qualitative approach, 1 science museum (mathematics museum), 1 industry and industrial exhibition, and 9 science centers from 8 different cities were observed specific to mathematics by case study technique, and semi-structured interviews were done with 13 explainers. As a result of these observations, it was deducted that mathmetics exhibition settings were fewer compared with other fields (like physics, chemistry, biology, astronomy) in 7 science centers, and there was

none in 2 science centers. One of the two observed museums holds exhibition settings which mostly consist of mathematical content under the name of "Mathematics Museum". In the other museum, mathematics exhibition settings are included in the curriculum in the museum. The exhibition settings in science centers are mostly gathered around the same themes (Tangram, Probability, Pythagorous Theorem) and usually appeal to every age group after primary school. However, science museums hold more settings which appeal to secondary school and especially highschool age groups, in addition to the themes in science centers. Explainers that work in science centers similarly pointed out that the mathematics exhibition settings in science centers are insufficient in interviews that were done with them. The fact that there is no specialist explainer staff in mathematics, and the shortage of technical support were pointed out as the reason to this insufficiency. Explainers believe that the number of mathematics exhibition settings in science centers should be increased because of reasons like; inseparable from other disciplines and that it should be concentrated on as much as the other fields. Explainers in science museums on the contrary, said that there are exhibition settings suited for each age groups present, therefore the settings are sufficient. Moreover, all of the explainers stated it is important that an explainer in mathematics field be responsible for the existing exhibition settings for the setting to draw attention.

Key Words: Informal Learning, Out-School Learning Environments, Science Centers, Science Museums, Explainer, Mathematics Museum.

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Pre-Service Elementary Mathematics Teachers' Strategies for Multiplication and Division with Fractions

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ABSTRACT

Fractions and computations with fractions are considered one of the most complex subjects for the students in early years of their school experience, even into their higher grades [1], since they are complex structures being composed of many relations [4]. Those relations can be constructed with conceptual understanding rather than procedural skills. However, having lack of conceptual understanding usually makes students, and even for teachers, use standard algorithms like inverting the divisor and multiplying fraction or other rules of computation which are generally expected to be forgotten.

Computation with fractions requires understanding of operations and fraction sense. In order to develop fraction sense, students should be given sufficient opportunities to produce their own strategies rather than rules or procedures taught by teachers. Those personal or flexible strategies are called as invented strategies referring to "any strategy other than the traditional algorithm" (p.218) [5] Using such strategies would then contribute to develop standard algorithms [2].

In the related literature, it is argued that pre-service teachers' performances in computation with fractions has a reflecting role on their future students' learning in the same subject [3]. In other words, when pre-service teachers acquire conceptual knowledge of any mathematical subject, they most probably reflect their knowledge to their students on the same manner. From this perspective, pre-service teachers' abilities in using strategies while solving multiplication and division with fractions are

considered important in terms of constructing a conceptual understanding for students. Thus, the aim of the study was to examine whether pre-service teacher's flexible strategies for multiplication and division with fractions according to their grade level. The participants were composed of 173 pre-service elementary mathematics teachers including 45 freshmen, 64 sophomore, and 64 junior students studying in a public university in Ankara. In this study, pre-service teachers were asked to solve two computations of fractions questions, $\frac{2}{5} \text{ x } 2\frac{1}{2}$ and $\frac{1}{2}:\frac{1}{8}$, without using the rule-based strategy (e.g. inverting the divisor and multiply). The overall findings revealed that although pre-service teachers were able to arrive at correct response, they rarely used various flexible strategies while doing computations with fractions. The common strategy for each computation with fractions drew upon using *region model* that is more concrete than other strategies. The other strategies used for multiplication with fraction were defined as *using unit fraction*, *repeated addition*, and *distributive property.* For division with fractions, using *unit fractions*, *repeated subtraction* were defined as flexible strategies. It was also found that junior students were better in using flexible strategies while comparing with the others.

Key Words: Pre-service elementary mathematics teachers, multiplication and division with fractions, flexible strategy.

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Preservice Primary School Teachers' Views on the STEM Method

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ABSTRACT

STEM method is the abbreviation of the initial letters of Science, Technology, Engineering, and Maths and refers to an integrated approach of instruction covering science, technology, engineering, and maths education. It is a learning-teaching system with interdisciplinary practices [1, 2]. The studies focusing on making STEM Turkish suggest using the abbreviation FeTeMM as its Turkish counterpart [3]. FeTeMM [Eng. STEM (Science, Technology, Engineering, Mathematics)] instruction aims to prepare students for the 21st century by equipping them with problem solving, inquiry, critical and analytical thinking skills.

The purpose of this study is to reveal preservice primary school teachers' views on STEM Instruction and analyze the association between their views and the variables of gender, grade, and the type of high school that the preservice teachers graduated from. Survey model, which is a descriptive research method, was employed in the study. The sample of the study consists of preservice primary school teachers studying in their 3rd and 4th years in a faculty of education department of primary school teaching during the 2016-2017 academic year. In this study, Integrative STEM Teaching Intention Questionnaire, developed by Lin and Williams [4], and translated into Turkish by Hacıömeroğlu and Bulut [5], was used as data collection tool. The data were analyzed via t-test and ANOVA.

According to the results, the preservice teachers have positive attitudes towards STEM instruction. It was also found out that the preservice teachers' attitudes towards STEM instruction do not change depending on grade, gender, and high school type.

Revealing preservice teachers' views on STEM instruction method is important to determine and decide on its practicability in the future curricula.

Both in-service teachers and preservice teachers need to develop their students' engineering, design, problem solving, and high-level thinking skills by engaging them in such skills. Hence, STEM practices should be integrated to both in-service and preservice trainings for in-service and preservice teachers. In this sense, future researchers and curricula practitioners are under much responsibility.

Key Words: Science, technology, engineering, maths, preservice primary school teachers.

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Pre-service Primary Teachers' Views about Origami Usage

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ABSTRACT

Geometry teaching and learning is a familiar concept in literature. Geometry instruction helps learners develop an understanding of "geometric shapes and structures and how to analyze their characteristics and relationships" [2]. Origami, is an ancient Japanese art of paper folding activity. Origami develops spatial thinking while interpreting and carrying out its instructions to do. Spatial thinking ability develops in verbal and graphical ways, including reversing, rotating, and inverting a model, or translating steps to the mirror side. The purpose of this study is to investigate beliefs and views of pre-service primary teachers about usage of origami in their mathematics courses.

Qualitative methodology was used to collect data. Pre-service primary teachers self-reported views related to origami related mathematics instruction was examined through their reflection papers. The purposive sampling method was used which enables to select the sample in accordance with the specific purpose of the study [1]. The selection criterion in this process was preservice teachers' lesson experience which was based on activity based on usage of origami in mathematics education. 72 of 3rd year pre-service primary teachers who are students of state university at the Central Anatolia were participated to study. They wrote their views about usage of origami in mathematics instruction based on their experience with origami activity.

The use of origami based mathematics instruction is gained attention and origami related objectives and activities was included in the curriculums [3] of different nations but detailed research should be conducted with the pre-service and in-service teachers.

Pre-service primary teachers stated that origami can be used as a learning-teaching tool in elementary-level classes (1-4) in mathematics. Moreover, they mentioned that origami could have positive effects on spatial ability, cognitive development, instructional processes, and students' motivation to learn. Origami also can be used making learning fun, developing creative thinking skills and imagination, supporting audio-visual learning, providing concrete examples, promoting spatial skills, supporting learning by doing, connecting to daily life. At the end, these opinions of pre-service primary teachers indicated the potential of origami as an effective teaching and learning tool.

Key Words: pre-service primary teacher, origami mathematics, teaching, view, qualitative study

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Prospective Mathematics Teachers' Perceptions of Linear Dependent - Linear Independent Vectors

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ABSTRACT

In the 2015-2016 academic year, the linear algebra course was conducted by the researcher. During the lessons, it was determined some troubles related to learning of freshman students. It was seen that prospective mathematics teachers could solve questions of linear algebra course algebraically but they could not understand and /or could not solve some questions requiring simple geometrical comments. Determining this problem that mathematics teacher candidates' difficulty in solving those kind of questions encouraged the researcher to start this research. In the study, it was aimed to examine the prospective mathematics teachers' perceptions on linear dependence and linear independence of vectors, one of the main topics in linear algebra course. The study was carried out with a qualitative approach and focused on the prospective mathematics teachers' perceptions of the geometric positions of linear dependent and linear independent vectors in two dimensional Euclidean space. A test developed by the researcher consisting of 9 open-ended questions was used as a data collection tool. The test was administered to the first year students in the mathematics teacher program in the spring term of the 2015-2016 and spring term of the 2016-2017 academic years in Dicle University, Ziya Gökalp Faculty of Education. In the spring term of the 2015-2016 academic year, 21 students and in the spring term of the 2016-2017 academic year, 13 students participated in the study. After administering of data collection tool, geometric comments were made on the positions of linear dependent and linear independent vectors in two and three dimensional space and presentations were made in computer environment using Geogebra 3D program. At the end of the study, it was determined that the students realized better understanding of the algebraic

expressions of vectors and realized a more conceptual learning of vectors through the computerized visualizations.

Key Words: Linear dependency, linear independency, vectors, prospective mathematics teachers.

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Prospective Middle School Mathematics Teachers' Learning through Investigating Student Work: A Case of Algebra

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ABSTRACT

"Effective mathematics teaching requires teachers' understanding what students know and need to learn…" (NCTM, 2000, p.16). Teachers need to know students' challenge in a particular topic which students are likely encounter in learning this topic (NCTM, 2000). The studies are needed focusing on the contexts where prospective mathematics teachers may learn about what students know about a particular topic. This study specifically examined prospective middle grade mathematics teachers' learning about students' use of letters in algebra while they were working on students' actual works. The participants of this study were 44 prospective middle grade mathematics teachers, who were in their third years of their mathematics education program. The data was collected during the fall semester of 2016-2017 academic years. Before the research was conducted, in accordance with the aim of the study, by using Küchemann's (1978) test items, an algebra test was designed by the authors, and it was implemented to almost sixty seventh-grade students. In this way, the student data for presenting prospective teachers were obtained. The actual data for this study was collected in a method course of a public university. During the data collection process, prospective mathematics teachers worked in a group of 4-5 for all activities. In this process, prospective teachers were initially asked to predict about students' possible solutions, and then they were asked to examine students' solutions manifested in their written works and to reflect their learning through analyzing students' solutions. They were also asked to write down their all reflections. Moreover, at the end of the study, prospective teachers were

implemented self-report questionnaire where they evaluated the contributions of the students' works to their learning. The data were collected through prospective teachers all written documents. The data were qualitatively analyzed. The findings of this study displayed that prospective mathematics learnt about students' various common difficulties and misconceptions regarding use of letters. For example, prospective teacher groups commonly indicated their learning regarding students' tendency to give a numerical value to variable or students' ignorance the letter while adding a number and an unknown. Moreover, they also expressed that students ignore the negative values (integers) for variables. On the other hand, prospective teachers also indicated their observations regarding students' lack of prior knowledge regarding equation concept and area of rectangle concept. Many of the prospective teachers explained they have never predicted that students could make such mistakes.

This study suggests that investigating students' written work as a practice-based material has potential to improve prospective teachers' knowledge concerning what students know and how students learn (e.g., Ball, 1997; Smith, 2001). This study contributes to the growing body of research concerning prospective mathematics teachers' knowledge of student algebraic thinking.

Key Words: Algebraic ways of thinking, prospective middle grade mathematics teachers, pedagogical content knowledge

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Secondary School Mathematics Teacher Candidates Associate the Egyptian Number System and Arithmetic with the Number System and Arithmetic Used Currently

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ABSTRACT

Knowing the mathematics history by secondary school students helps them understand the reasons behind mathematical rules and improve their thinking [1]. Knowing and understanding of the mathematics history by the students is directly related to the mathematical history knowledge of mathematics teachers [2]. In this sense, it is very important for the teachers to know the history of mathematics and to make connection between it and today's mathematics subjects. In the direction of the aforementioned reasons, the study is done to examine making connection between Egyptian Number System and arithmetic used currently by secondary school mathematics teacher candidates. In this study, case study method is used from qualitative research designs. 23 first grade middle school mathematics teacher candidates participated in the study. Participants took the history of mathematics and volunteered to participate in the study. Participants were given an event card prepared according to the Egyptian Number System, containing arithmetic operations and asked them to make a solution. Thereafter, some questions like "What is the relationship between the Egyptian Number System and the present number system?", "What is the relationship between the arithmetic operations done by the Egyptians and the arithmetic operations today?" are asked to the participants by carrying out semi-structured interviews with them. Content analysis is applied to the obtained data. As a result of the work, though the symbols of the Egyptian number system and the numbers used today are different, it is obtained that main idea of these number systems is similar. Some of the similarities; both numerical systems are based on the 10-base system, the order of arithmetic operations is the same, and

the logic behind arithmetic operations (such as the multiplication is the abbreviated collection) is the same.

Key Words: Egyptian Number System, Egyptian Arithmetic, Present Number System.

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Teaching With Ratio-Proportional Problem Based Learning Approach: Scenario Example

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ABSTRACT

Problem based teaching is a learning approach that students perform in the context of problem solving exercises around a complex problem that has not a single solution to learning (Hmelo-Silver &Barrows, 2006:24). Therefore, it is one of the active learning approaches that should be used most in classrooms as it brings daily life problems to learning environments.

All students focus on the same scenario and the scenarios reflect the real situation. By means of the scenarios, the students are exposed to various problems, producing multiple paths for solving and constantly seeking to learn (Çayan ve Karslı, 2014:1438). Scenarios are educational devices that can awaken student curiosity, attract attention and attention, offer new clues to the student as he or she goes to the desired destination, and keep the learning impulse constantly alive.

The aim of this study is to introduce the probabilistic approach to the rate-proportion in the 7th grade mathematics curriculum of secondary school as a teaching material developed as teaching material. The improved scenario comprises of the learning outcomes which are 'Examines real life situations and tables and decides whether two multiplicities are inversely proportional.' and ‘Solves problems related to true and inverse proportion'. The prepared scenario consists of two sessions. The first session consists of three parts and the second session consists of two parts. It has been noted that the prepared script is open to debate in the way that it can be applied to achievements and group work appropriateness, to include problem situations that are constructed from everyday life, and to propose many solutions to problem situations. In addition, the screenplay has been supported by

visual materials to capture the attention and attention of the students. The prepared scenario has been presented to the opinions of 4 faculty members and 3 mathematics teachers, each expert in the field. The necessary arrangements have been made after the obtained feedbacks.

A pilot application has been carried out in order to carry out the validity and reliability studies of the scenario. As a result of the implementation, the scenario was rearranged and finalized. As a result of the studies made, the validity and reliability of the ratio-proportion issue has been provided and a scenario consisting of two sessions has been prepared so that each session would be a learning target of a session.

Key Words: Scenario, problem based learning, mathematics education.

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The Attitude Of The Teacher Candidates Toward Teaching Profession

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ABSTRACT

The profession is indicated as to produce beneficial product and service and of activity, which made to earn money in return and depends on the knowledge and skills acquired from education, with rules set by society (Kuzgun, 2000). Teaching profession is an occupation area ,which has social, cultural, economic, scientific and technological scopes, with a professional statues that requires professional formation and academical study which takes special expertise knowledge and skill as a basis (Erden, 1998: 27). In the 43rd article of Basis Law of National Education number 1739, the teaching profession is described as a special expertise profession, which takes government's education, training, and management duties related to this (MEGSB, 1987). No matter which branch it is, teacher's attitudes toward their professions has importance on them doing teaching profession more fondly and being more successful. Positive attitude towards the teaching profession will positively affect all elements related to the teacher if they are consistent after they pilot their career (Pehlivan, 2008). In this context, characteristics of the individuals and their attitudes towards the career they choose have an important role in career choice.

It will be directive what type of training they, the teacher candidates that will work in educational organizations, must be given in pre-vocational training in case it's known their attitude towards the profession (Üstüner,2006). For this reason, athe aim of the study is to determine the attitudes of the students studying different branches in Education Faculty. The study group of the research is consists of 1th and 4th graders studying in different branches in Education Faculty of Ondokuz Mayıs

University. In this research, “Attitude Scale Toward Teaching Profession” was used as data collection tool, which was developed by Üstüner (2006). This scale is a Likert-type unidimensional scaling method, consisting of 34 items. The study of validity and reliability of this scale was made by Üstüner (2006). This scale’s reliability coefficient was found as 0.72. Besides, inner consistency (Cronbach Alpha) of this scale is 0.93. Findings elucidated that attitudes towards the teaching proficiency of students from different branches have been affected by different varieties such as sex, branch, active degree.

Key Words: Attitude, attitude scale, attitude scale toward teaching profession.

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The Creative Application on Teaching Math Concepts: Photography

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ABSTRACT

Many research show that making school more interesting and productive for students and teachers is getting harder, especially in mathematics. For the solution, if we look over the leaders of education systems which are five countries of East Asia (namely China, Hong Kong, Japan, Korea and Singapore) and America, the educational opportunities are based on technology. The reality of world is that using technological tools become the main necessity of today's world. To share your knowledge accumulation, one of the most popular technological way is photography. Photography is the result of combining several different technical discoveries. A long time ago the first photographs were made Greek mathematicians Aristotle and Euclid described a pinhole camera in the 5th and 4th centuries BC. Especially, geometry which is one of the main branches of mathematics relates to visual aspects of our surroundings. The general purpose of education is skilfully utilization of student-centred research, exploration and problem-solving activities. In geometry, students can theoretically understand of definitions, equations of structures. On the other hand, the application part is also important to complete the theory. In this study, we show that by exploring the relation between geometric shapes and photography in daily life, students not only recognize the shapes but also understand their purpose and their creation. Furthermore, some mathematical soft wares which are useful to create figures in two and three dimensions are introduced.

Key Words: Teaching mathematics, photography, mathematical programming.

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The Effect of Cooperative Learning Method on Reflective Thinking Skills towards Problem Solving

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ABSTRACT

Reflective thinking can be described as an active and intentional process involving the criticism of premises about the problem-solving process or content (Dewey, 1933; Mezirow, 1991).

The mathematics curriculum renewed based on the constructivist approach considers students as active participants. It highlights the formation of classroom environments where students can do querying, establish communication, think critically, and share their opinions easily (Ministry of Education's, 2013). Reflective thinking plays an important role in activating the critical thinking skill (Choy and Oo, 2012), which is among the objectives of the curriculum.

Though the international literature contains many studies investigating reflective thinking skills, there are a limited number of studies conducted on this matter in Turkey. These studies have mostly focused on teachers and prospective teachers. There are a very limited number of studies dealing with students' reflective thinking skills. Moreover, there is no study on the activities or methods that play a role in the development of students' reflective thinking skills in the mathematics course. In this context, it is thought that research focusing on students' reflective thinking skills in the mathematics course will contribute to the literature.

Considering that reflection is realized within an active process (Gelter, 2003) and shaped in the context of social environment (Choy and Oo, 2012), it is clear that the role of cooperative learning in the development of reflective thinking skills should be investigated. Therefore, it is thought that this study will present a different perspective on the effectiveness of cooperative learning. In this context, the purpose of the study was to investigate the effect of cooperative learning method on reflective thinking skills towards problem solving of seventh grade students.

The research participants were composed of 71 seventh grade students attending a middle school in Elazığ in the autumn term of 2016-2017 academic year. The research has been designed as the pre-test post-test control group quasi-experimental design. Cooperative learning method was used experimental group (n=36). Student Teams-Achievement Divisions was used in the experimental group and there was no intervention for the control group (n=35). Data were collected using the "Reflective Thinking Skills towards Problem Solving Scale" developed by Kızılkaya and Aşkar (2009). Data was analyzed by performing a t-test for dependent and independent samples. According to the results of the data analysis, at the end of the interventions, it was seen that the levels of reflective thinking skills towards problem solving of the experimental group were significantly higher than of the control group. According to the data obtained from the study it can be stated that cooperative learning method has positive effects on students' reflective thinking skills towards problem solving. It is suggested that cooperative learning method should especially be used for developing students' reflective thinking skills towards problem solving in mathematics course.

Key Words: Cooperative learning, reflective thinking, mathematics education, problem solving.

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The Effect of Math Worksheets based on Multiple Intelligences Theory on the Academic Achievment of the Students in the 4th Grade Primary School

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ABSTRACT

The worksheets are important materials including the steps that students need to take, help students organize their knowledge, as well as whole class' attendance to the activity at the same time. (Aydoğdu and Kesercioğlu, 2005). The aim of this research is to examine the effect of Math worksheets based on the Multiple Intelligences Theory on the academic achievement of students in the 4th grade primary school. The sample of the research consists of 64 (32 experimental and 32 control) students who are studying in the 4th grade in a primary school affiliated to the Ministry of National Education in the province of Bağlar, Diyarbakır in the academic year of 2016-2017. Experimental design with pre-test and post-test control group was used in the study. The obtained pre-test and post-test data were analyzed with SPSS 22.00 package program. Content analysis of interview data was conducted. According to the results of the research, it is shown that Math worksheets prepared on the basis of the Multiple Intelligences Theory has increased the academic achievements of the students in general. According to these results, it can be said that the preparation of Math worksheets according to students' different intelligence fields can positively influence the academic achievement of the students.

Key Words: Multiple Intelligence Theory, Worksheets, Academic Achievment

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The Effects of The Mathematics Literacy Education of The 6th Grade Students to Mathematics Literacy Achievement

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ABSTRACT

The Program for International Student Assessment (PISA) of the OECD describes mathematical literacy as: “an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen” [5].

According to de Lange, mathematical literacy is the overarching literacy that includes numeracy, quantitative literacy and spatial literacy. Each of these type of literacy empowers the individual in making sense of and understanding aspects of the world and his/her experiences.[4].

The purpose of this study is to investigate the effects of mathematics literacy education on secondary school sixth graders on the success of mathematics literacy and to examine the changes in attitudes and motivations towards mathematics.

The study was on the 56, 6. grade students who are studying in the secondary school of Karacabey district of Bursa. All students attending randomly selected 6/A and 6/B branches of the school participated in the experimental study.

In the study pre-test-post-test experimental design which has control group was used. In the study, mathematical literacy training was given to the experimental group for 12 weeks and the traditional teaching was continued in the control group. Mathematics Literacy Test was applied to the students as pre-test and post-test. Also, the changes in attitudes towards mathematics were examined by applying the Mathematical Attitude Test and whether there is an increase in motivation for mathematics using the Mathematical Motivation Scale. The data obtained in the

study were evaluated using the SPSS package program. It was examined that whether there is a meaningful difference between pre-test and post-test average points using t-test for data analysis and control test was applied. Analysis results show that the student bred by teaching activities of 5E learning.

The lessons were performed using teaching activities of traditional methods for control group and lesson activities planned by 5E teaching model for experiment group throughout twelve weeks. Mathematics Literacy Test was applied to the students as pre-test, post-test and permanence test. In addition, the changes in attitudes towards mathematics were examined by applying the Mathematical Attitude Test and were measured whether there is an increase in motivation for mathematics using the Mathematical Motivation Scale. The data obtained in the study were evaluated using the SPSS package program. In the analysis of the data, t-test was used to determine whether there was a significant difference between groups in the mean of pre-test and post-test and permanence test scores, changes in mathematics attitudes and motivations are examined.

As a result of the findings, it has been observed that mathematics literacy training significantly increased the mathematical literacy achievements of sixth grade students. Moreover, as a result of the application, it has been detected that the attitudes of the sixth grade students towards mathematics change positively and increased motivation.

Key Words: Mathematical Literacy, Mathematical Education, PISA

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The Impact of Intelligence Games on Gifted/Talented Students' Analytical Thinking and Decision Making Skills

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ABSTRACT

Education of gifted/talented students is an increasingly developing field and studies in this field are raising awareness to teachers and families in many ways. Schools and families are academically putting pressure on gifted/talented students, because they thinks students should achieve higher success. Moreover, students with different interests are bored, because they are deprived of the subjects they want to study in depth. Therefore, a lot of unwanted behaviour occurs. Students have various needs to meet such as cognitive abilities and different thinking skills. One of the educational tools needed to meet these needs is undoubtedly the game. A well-planned and purpose-driven game is an endless adventure for students. Moreover, intelligence games can be used as an effective tool in the development of mental capacities and skill of students, in becoming more entertaining by gamification of real problems (Meb, 2013). In the Turkish education system, intelligence games are being taught as an elective course and are used stepped curriculum. Stepped curriculum is a hierarchical structure and it contains three basic stages: Beginner (D1), Intermediate (D2), and Advanced (D3). Furthermore, the types of games that represent the learning fields of the program is reasoning and transaction games, verbal games, geometric-mechanical games, strategy games, memory games and intelligence questions (Meb, 2013).

In this study, various games were played to gifted/talented students based on Karnes and Bean (2009)'s map of thinking description cluster map. In addition, the impact of these games to improving analytical thinking and decision-making skills on students were examined. In this study, the types of games such as reasoning and transaction games, memory games and intelligence questions were used by applying single group pretest-posttest experimental research design. Various tests such as kakuro, memory games and "Analytical Thinking and Decision Making Skills Assessment Test" which are composed of these game types were prepared according to D1, D2 and D3 difficulty levels. These tests which are valid and reliable were applied to 22 gifted/talented students in a private school as a pre-tests in the 2016-2017 academic year. The appropriate/accidental sampling method was used to choose a private school. Furthermore, the purposive sampling was used to choose gifted/talented students in this school. Following the pre-test, "Rat A Tat Cat" and "Sleeping Queens" games were played to students 16 lesson hours in total for 4 weeks. After the 4-week event period, the tests were re-administered to the students and the data were recorded. As a results of the analysis, a significant difference was found between the pre-test and the post-test, and it was emerged that the games positively affect to gifted/talented students for analytical thinking and decision making skills.

Key Words: analytical thinking, decision making, intelligence games, gifted/talented students

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The Investigation of the Students' Cognitive and Metacognitive Competencies According to the Different Variables

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ABSTRACT

It is important how individuals learn information and how construct them. In this process, metacognitive competencies of individuals can play a role. Despite metacognition was suggested by Flavell (1979), it is theoretically an older concept. Individuals with metacognitive competence can plan the learning process, control, evaluate and organize themselves according to the learning environment (Schraw, 2009). Therefore, metacognition is important in the education of individuals (Hacker, Dunlosky & Graesser, 2009). As ages of the individuals progress, the levels of metacognition increase. However, individuals may not have fully knowledgeable about metacognitive skills and competencies (Baker, 1989). Then, it can be said that to determine the motivational, cognitive and metacognitive levels of students is important for experimental or review studies to change and improve the academic achievements and attitudes of students. Moreover, this study will provide the significant contributions to the literature by using a scale has such sub-dimensions: self-sufficiency, metacognitive strategies, using the learning strategies, organizing the learning process and evaluating the learning process. The purpose of this study was to examine the above mentioned sub-dimensions towards motivational, cognitive and metacognitive competencies of middle school students according to the gender and class level variables.

This study was designed as descriptive study and conducted by using screening model. The participants included 366 middle school students. As the data collection tool, “Motivational, Cognitive and Metacognitive Competencies Scale” adapted from English to Turkish by Aktamış and Uca (2010) was used. The scale of 26 items consisting of five sub-dimensions. The collected data were analyzed using SPSS (Statistical Package for the Social Sciences) program. First, it was examined whether the data are appropriate for normal distribution, or not. As the results of the data analysis, statistically significant differentiation was observed for men in the motivational, cognitive and metacognitive competencies levels of middle school students. When it is examined according to the dimensions of the scale, there was no statistically significant difference for such dimensions: “self-sufficiency”, “metacognitive strategies” and “using the learning strategies” according to the genders. However, statistically significant differentiation was observed for men in such dimensions: “organizing the learning process” and “evaluating the learning process”. On the other hand, there was significant difference for the class level in all sub-dimensions.

Key Words: Metacognitive competence, self-sufficiency, using the learning strategies, organizing and evaluating the learning process**.**

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The Reflectıon Of Mathematıcal Modellıng To Teachıng Tools: A Textbook Analysıs

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ABSTRACT

Mathematical modeling (MM) is a versatile, cyclic, and complex process in which a real life problem situation is solved by a mathematically developed model where the cognitive activities are heavily used during the solution (Berry&Houston, 1995; Lesh&Doerr, 2003). MM allows students to see role of mathematics in the real life by treating it in the same context as learners' real life. This study focus on how mathematical modelling and the concept of model and modelling are used in the middle school mathematics textbooks. The study was a document review and content analysis method was used for analyzing textbooks. In the analysis, first, the concepts of model and modeling in the middle school mathematics textbooks were examined. Then, the use of model and modelling words were separately coded for each problem and activity based on learning field, class level, section where the model was used (instruction-evaluation sections), and the types of models used. Finally, all problems and activities were recoded into six subcategories: counting scales, fraction cards, algebra tiles, real life context models, counting blocks, and visualization models. The results revealed that the concept of modeling in the textbooks was used to transform mathematics into a visual form or to embody it, and mathematical models were used as only concrete models or for visualization purpose. In the textbooks, the word model was used 44 times in the fifth grade, 69 times in the sixth grade, 31 times in the seventh grade, but none of model or modelling words used in the eighth grade. While the concept of the model was not included in the subjects in the probability and data processing learning areas at all class levels, it is used mostly in numbers and

operations (79.2%), in geometry and measurement (21.5%), and in algebra (6.3%) learning areas. The use of models based on subject was used extensively in the areas of accumulation in some subjects, especially fractions (30.5%) and integers (27.8%). The use of models were often in the form of counting scales and blocks, fraction cards, and algebra tiles, or in the form of transforming mathematics to a visual form as shape or number lines in order to make the mathematical situation more understandable.

This study showed that, the concept of modelling in the textbooks is treated as a "modelling mathematics" rather than MM and those models are used intensively in some topics, but not used for others such as probability. Modeling mathematics is defined as the use of mathematical representations to describe concepts and ideas (Cirillo, Pelesko, Felton-Kosetler&Rubel, 2016). In modelling mathematics, there is a tendency from mathematics towards real-world, while in MM; this direction is reversed (from real-word towards mathematics). In existing textbooks, modeling is used in a different meaning from MM, while models are limited to concrete and visual constructions. To conclude, it is suggested MM should be included in the curriculum and textbook at all grade levels so that it can provide a rich learning environments for students by engaging in mathematical modelling activities.

Key Words: Mathematical Modelling, Modelling Mathematics, Textbook.

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The Review on The Primary Education Math Teaching Candidates' Views about The Course of Special Teaching Methods

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ABSTRACT

The goal of this study is to provide the views of teaching candidates who receive the training in the program of Primary Education Math Teaching about the course of Special Teaching Methods. As it is aimed to examine thoroughly the data from the study, the case study from the qualitative research methods was preferred (Yıldırım& Şimşek, 2005). For this purpose, this study was done on the teaching candidates who received the training at 3. and 4. grades in the program of Primary Education Math Teaching in The Department of Science and Math Education, The Faculty of Education in Fırat University in 2016-2017 Education-Training year. The semi-structured interviews were done with the 85 primary education math teaching candidates in total as 20 males and 65 females in the research. The data from the result of interviews was categorized under the certain themes as it was analyzed by the content analysis (Miles & Huberman, 1994; Yıldırım & Şimşek, 2005).

According to the data at the end of the research; the majority opinion of the teaching candidates related to the course of Special Teaching Methods is that this course is necessary for the teaching candidates and it will be useful to use the proper method and techniques for the target and objectives in this course. The teaching candidates stated the necessity of course in theory and that the theoretical section is a prior condition for the practice. According to some of the teaching candidates, it was stated that this course's theoretical section should be reduced and this section went on boringly. The opinions of teaching candidates related to the practice part of the course were generally seen as it makes whatever is learnt in the course

permanent and it makes that the course is conducted in more effective and efficient way. When the views of teaching candidates related to the use of materials in the course are reviewed, it is as the use of material in the courses materializes the teaching, and as it makes the learning permanent. Moreover, the teaching candidates recommended that the use of material is increased.

The teaching candidates stated that it is important that the instructors who give the course of Special Teaching Methods should be the expert persons at their field and they should develop themselves much more. They stated that it is necessary to consider the practice's grade point average in the field of assessment and evaluation for the course of Special Teaching Methods, moreover it will be good that the outcome-and-process based evaluation is done. A great part of teaching candidates stated that they believe that this course has got a great contribution to the teaching profession, and they said that this course provides an occupational experience for the teaching candidates. Moreover, they focused that the hours of this course's practice should be increased and the teaching candidates should have much more active role in the course as the recommendations to increase the course's effective and efficiency.

Key Words: Math Teaching Candidates, The Course of Special Teaching Methods.

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The Role Of Problem Solving On Students' Mathematical Thinking Skills In Technology-Supported Learning Environments

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ABSTRACT

In the present and last century, science and technology have been developing in a dizzying rapidly. In the report that is published by NCTM (2000) (the National Council of Teachers of Mathematics), it is noted that the need for ability to using and interpretation of mathematics in everyday life is gradually increasing and mathematical thinking and problem solving are needed more in many areas. Mathematical thinking is a tool that people use to understand everyday life better and to act on them. In order to be able to conceive mathematical thinking, researchers have studied to determine mathematical thinking's features, functioning, components, and the difference between mathematical thinking and other thinking. In this study, the process of Burton and College (1984) is taken into account in terms of the ease and usefulness of examining mathematical thinking processes. Burton and College (1984) examined mathematical thinking processes in four stages: specializing, generalizing, conjecturing and proving.

Problem solving is the basis for mathematical thinking. Mathematical thinking and problem solving are discussed together by many researchers [2], [3], [4]. When faced with a problem, mathematical thinking processes will be started to use in the process of examining in various viewpoints rather than the answer of problem [5]. There are many factors that influence mathematical thinking skills. Problem solving is not enough to develop a person's mathematical thinking [6]. Education is one of the area which we used technologies. There are contributions such as the fact that education technologies are suitable for students with different skills, the availability of

a large number of materials, the interest and motivation of the students, and the saving of time [7].

In the developing world, there is need to individuals who have mathematical thinking skills, who are able to benefit from technology and overcome the everyday problems they are facing. For this, we have launched a project to explore opportunities and improve the technology in our country (Fatih). The purpose of this research is to determine the conclusions of the technological class environments and tablets used by Fatih Project and the students' contribution to mathematical thinking skills in problem solving. To do this, the students were given activities (work papers), which were prepared by taking expert opinions, in every week. In this study, we used case study method of qualitative research techniques.

Participants of the study were twelve 11th grade students who were able to use geogebra as a tablet and dynamic mathematics program in a determined Anatolian high school. In the data collection process we used the tools such as individual interview records, work-sheets, video recordings, audio thinking records, screen displays on tablets. By using these data, we investigate the aspects, the frequency and the use of purpose of educational technologies in mathematical thinking processes stages: specializing, conjecturing, generalizing and convincing. Finally, we make some suggestions about how students can use technology in mathematics education so that the use of technology in learning environments can reach the goals set in the secondary school mathematics curriculum.

Key Words: mathematical thinking, problem solving, education technology.

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Undergraduate Mathematics Majors' Proof Schemes in Analysis: An Exploratory Research

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ABSTRACT

Studies conducted on analysis concepts showed that continuity, differentiability and integral subjects are generally studied in their internal boundaries and special emphasis is put on 'proof' process.[1] It is also important to understanding that how proved statements are comprehended by students. In this respect, the arguments used in proof process and students' proof schemes are crucial components which might be considered at university mathematics. In this study, binary relations among the continuity, differentiability and integrability concepts, which are three key concepts of analysis course, are evaluated within the frame of proof schemes. Two specific research questions are posed for this purpose: (*i*) How undergraduate mathematics majors' construct proofs of some basic theorems in analysis? and (*ii*) What kinds of proof schemes prefer by undergraduate mathematics majors in proving process?

Survey research design was considered to be suitable for the study as this research contains the descriptive aims for evaluating knowledge of proof process. The participants of the research are 172 freshman in the mathematics departments of three state universities. The participants consist of students attending Analysis courses and the applications have been performed at the end of these courses. A set of propositions has been presented for the relations between the concept of continuity, differentiability and integrability.

The findings show that invalid arguments were used in 37% of the answers given to the all propositions and no justifications were done in 29% of them. The two thirds of the students having answered the propositions as 'true' or 'false' could not explain their answers or present valid arguments shows that the relations between

related concepts are poorly understood. As Juter [2] says, justifications used by the students were often not sufficient to be called proofs, mostly due to a lack of arguments. However, it has been determined that external proof schemes for true propositions and empirical schemes for false propositions are two mostly used arguments.

Key Words: Propositional knowledge, analysis concepts, proof scheme.

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Use of Technology and Materials in the Research of the Spatial Thinking of Gifted Students

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ABSTRACT

Kurtdaş (2012) implies that the standards of the education which is provided to individuals, the determination of the education which the individuals require and the administration of the specified education are important issues. In this regard, he also states that the education of the gifted students has a significant importance. In Turkey, Science-Art centers (Bilim-Sanat Merkezleri) are established to select and station gifted students. According to these centers; gifted children/students are determined by experts as the ones that display higher performance then their peers in intelligence, creativity, art, leadership capacity or special academic areas. (Meb Bilim ve Sanat Yönergesi, 2007). As for spatial talent, it is defined as the visualization, manipulation, as a whole or seperately, and introduction of the objects in space within mind from a different point of view. (Yıldız & Tüzün, 2011). Özyaprak (2012) concluded that there is a significant difference between gifted students and average students in terms of visual spatial talents and that this difference is in favor of the gifted students in her research. Computers are often used as a tool in the improvement of spatial talents. Şimşek and Yücelkaya (2014) observed that technology supported administrations increase spatial talent scores compared to baseline. In this study, the use of technology and tangible materials in the observation of the spatial thinking of gifted students with the help of the education experiment technique is studied. The research is conducted with 2 gifted students who study at Study Science and Art Center (Çalışma Bilim ve Sanat Merkezi). The subject of "three-dimensional shapes which are formed from sixth(6th) class unit cubes" is studied to observe the students' spatial progress. At the end of the

research, it is deduced that computers are not sufficient in the imagination of the concomitance of shapes which are produced by unit cubes with the help of computers, and that the students experience difficulties in creating the shapes where they feel the need to use materials. It is observed that they have difficulty in spatial thinking during the constitution of the shape, they err because of the perspective not being constituted correctly (that they have difficulty in grasping the point of view from the aspect that the issue approaches), and that this causes problems with their spatial thinking.

Keywords: Gifted Students, Science-Art centers, Spatial Thinking, Cubes, Use of Technology

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Visual Math Literacy in the Lower Dimensions What is the Strength to the Procedure of Its Success of Geometry?

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ABSTRACT

Visual literacy, reading and interpreting Visual elements with a capacity of thinking and learning skills with Visual elements together, I mean thinking visually (Hortin, 1980). Visual math literacy is the Visual or spatial problems encountered in daily life, on the contrary, it can perceive Visual or spatial information mathematically, interpret, evaluate, and express, is the ability to use efficacy (Duran ve Bekdemir, 2012). Information about the learning space geometry math learning field as a result of Visual literacy and Visual relationship with mathematics literacy concepts. From this point move the Visual math literacy "Visual or spatial problems encountered in daily life, on the contrary, it can perceive Visual or spatial information mathematically, interpret, evaluate, and to express the ability to use the competence" is defined in the form of (Duran ve Bekdemir, 2012).

In this study, prospective teachers with Visual Math Literacies examination of the relationship between the achievements of geometry. The geometry of the Visual Mathematics literacy achievement is what level is also another topic explored procedure. Working methodically, scan designed model with quantitative. Research was conducted on the teacher candidate 232 As data collection tool "Visual Mathematics Literacy Achievement Test" scale "and" geometry is applied. The analysis of the data correlation and multiple regression analysis methods. Prospective teachers as a result of Visual data analysis math literacy among the lower perception of geometric field size and geometry achievement positively identified a meaningful relationship. In addition, the success of the perception of Visual geometry math literacy has been found to be a meaningful procedure.

Key Words: Math Teacher Candidates, The Success Of Visual Geometry Math Literacy, Mathematics Education.

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Visualizing Daily Life Trigonometric Half Angles Formulas in Secondary Education

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ABSTRACT

This aim of this study is to prove “half angles formulas” which is a sub domain of trigonometrical learning areas, numerically, algebraically and visually and also to introduce them to other researchers.

Mathematic, which consists of relations between abstract concepts tries to model the objects, situations and events in nature. It is an important field. Proof, as a theme, has an important place in mathematical educational program. It played an important role for current mathematical knowledge accumulation.

Visualization and representation are core structures of understanding the abstract mathematical concepts. Visualization provides us to perceive the mathematical concepts via our sense organs (Bagni, 1998). Because, human vision is not enough to see most things. Some things are not seen, because of their being so small or far. Such tools as microscope and telescope are developed to be able to see them. There are something that we can not see, even if we use a special tool. Abstract mathematical concepts are one of them (Arcavi, 2003). Abstract concepts should be made visible and imaginable as possible, in order to understand invisible mathematical concepts. Visualization makes an object, event or a process visible. Visualization is a method that, makes invisible things visible (McCormick vd., 1987, Bagni, 1998).

Trigonometry is an important theme of mathematical educational program in secondary education. Half angles formulas which is one sub domain of trigonometry ,comes across in daily life as many physics problems. Half angles formulas are especially used in mathematics and physics lessons and also widely in engineering. Students usually memorize most equations and use them to solve routine exercises, when they study on trigonometry. Students just put the values in equations, when

they are asked to prove in trigonometry.It may not usually be suitable for "the nature of proof". Students' acquiring how and where to get the half angles formulas is more suitable than memorizing them. Multiple representations and visual proofs may be useful for students to understand the proofs of formulas better. Multiple representations and visual proofs are important for mathematical education, too.

Karadağ& McDougall (2009) state that, creating mental representations at the end of learning processes is so important. Children can easily create mental representations related to mathematical concepts or events they observed in their mind, at the end of an instruction via visualization. Visual representations support the learning as they are tools for understanding the concepts.

This study aims to prove "half angles formulas" which is a sub domain of trigonometrical learning areas, numerically, algebraically and visually and also to introduce them to other researchers.

Key Words: Visual proof, algebraic proof, trigonometric half angles formulas

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What do Mathematics Teacher Candidates Think about the Connection between Real World and Mathematics?

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ABSTRACT

Connecting classroom mathematics to the real world is an important issue in teaching and learning mathematics because real world–mathematics connection provides many benefits for students' learning mathematics (Gainsburg, 2008). However, the emphasis in school mathematics curricula is mainly on abstract mathematical concepts and in mathematics classrooms, the effort made by teachers to connect mathematics to the real life is also reported as inadequate (Karakoç & Alacacı, 2015). Many different factors such as curriculum, resources, time, teacher knowledge, belief or recognition may influence teachers' use of real world connection in mathematics classrooms. This study aimed to investigate the mathematics teacher candidates' views regarding real-world and mathematics connection. For this aim, the data were collected through questionnaire. In accordance with the aim of the study, an open-ended questionnaire was designed by the author, and it was implemented to students (teacher candidates) who were in their 2nd and 4th years of their mathematics education programs in a public university and who would become middle-grade teachers (grades 5-8). 101 students (53-second grade and 48-fourth grade) voluntarily participated in the study. In the questionnaire students were asked, for example, (i) What is mathematics? (ii) Is there mathematics in your daily life? Where do you use mathematics in your daily life? (iii) Do you use the mathematics you learnt in mathematics lessons in your daily life? (iv) How your knowledge of mathematics help you to solve the problems that arise in your everyday life? (v) What might be the benefits and difficulties of using real world problems in mathematics lessons? The data are being analyzed qualitatively. The data analysis has started

with reading and coding of all answers given by teacher candidates. In-depth analysis of the data is still in progress.

The initial findings show that many of the teacher candidates are aware of the use of mathematics in a variety of daily life situations. Many of them indicated that they need to apply the basic mathematics (e.g., numbers, rate-ratio, profit and loss, average, measurement etc.) in their everyday life. On the other hand, some of them emphasized that until this time, they did not need to use several mathematical concepts/topics (e.g., integral, derivative etc.), which they learnt in their high schools or university education, in their everyday life because these topics are abstract. Moreover, almost all teacher candidates positively expressed their views regarding the necessity of using real world problems in mathematics lessons because they believe that such problems provide many benefits in students' understanding of mathematics. In depth analysis of data, the relationship between teacher candidates' perceptions of mathematics and views regarding the use of mathematics in the real world will also be examined and the implications of this study will be discussed.

Key Words: Mathematics, mathematics in real life, mathematics teacher candidates

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POSTER

A Class Of First Kind Volterra Integral Equation

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ABSTRACT

In this study, a class of first kind of Volterra integral equation is investigated, after their reduction to some studied types. Existence of solutions to our approximating equation is proved.

We consider the equation

$$u(x,t) = f(t,x) + \int_0^t K(t,\tau,x,\nabla_x u(\tau,x))d\tau, \tag{1}$$

in which f and K are known functions of their arguments,

$$t \in R^+, x \in \Omega \subset R^3.$$

with Ω domain. We shall investigate (1.1), in regard to the existence of approximate solutions on some set Ω . Such equations appear in Energetics (Corduneanu, 2010, [3]).

In case $\Omega \subset R^3$, which means we limit our considerations to the case of one space variable, the Eq. (1) takes the simplified form [1],

$$u(t,x) = f(t,x) + \int_0^t K(t,\tau,x,u_x(\tau,x))d\tau, \tag{2}$$

and constitutes a partial integrodifferential equation for $u(t,x)$, which appears together with its first derivative. It is adequate to associate with (2) an "initial type" condition of the form

$$u(t,0) = u_0(t), \qquad t \in [0,T] \tag{3}$$

assuming that $\Omega = [0,X], X > 0$. Let us denote, $u_x(\tau,x) = v(\tau,x)$, and taking (1.3) into account one obtains

$$u(t,x) = f(t,x) + \int_0^t K(t,\tau,x,v(\tau,x))d\tau, \tag{4}$$

which together with

$$u(t,x) = \int_0^x v(t,\xi)d\xi + u_0(t), \tag{5}$$

lead to the equation

$$\int_0^t K(t,\tau,x,v(\tau,x))d\tau - \int_0^x v(t,\xi)d\xi = u_0(t) - f(t,x), \tag{6}$$

It is known that first kind Volterra equations, like (6) are, in general, deprived of solutions. Becuse of this we will consider a linear case (in *v*). We shall take their approach/result [4], as the initial point of the procedure to investigate a class of equations, appeared as approximating the equation. (6).

Key Words: Integro differential equation, Volterra integral equation.

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A Host-Parasite Population Dynamics Under Immigration

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ABSTRACT

In this paper, we investigated dynamics of a host-parasite model with the immigration parameter added to the host population under the constant searching efficiency.

As various internal and external parameter has an effect on living organism. In this case, to review of the model with different parameters is important. The host parasite models are one of such forms which are studied intensively in the last few decades. One of the earliest applications of discrete-time models including host-parasite interaction was formulated by Leslie and Gower in 1960. The classical host-parasite model is the Nicholson-Bailey model, except that the parasite does not necesserily kill the host. This model is based on the following assumptions :

$f(H_t, P_t)$: fraction of hosts not parasitized,

H_t : density of host species in generation t,

P_t : density of parasitoid species in generation t,

r : number of eggs laid by a host that survive through the larvae, pupae and adult stages,

e : number of eggs laid by parasite on a single host that survive through larvae, pupae and adult stages. The Nicholson-Bailey model is given as follows:

$$H_{t+1} = rH_t f(H_t, P_t)$$
$$P_{t+1} = eH_t(1 - f(H_t, P_t))$$

where, r and e are positive parameters.

It assume in this model that searching efficiency of parasitoids are limited but eggs are not limited. If the searching efficiency of parasitoid has small values, parasitoid can vanish. The eggs of parasite which stay alive are transferred to the next generations of parasitoids. A host is parasitized for once. Interference-free hosts increase their own progeny. If constant reproductive rate of host has small values, the host can vanish. Then, the positive equilibrium point of the models consisting density-dependent factor instead of a constant reproductive rate of the host can be locally stable. Even so, there are many reasons affecting the number of species such as immigration, human activities, and interaction of among species.

Briefly, firstly we examined the equilibrium points of discrete-time host-parasite model. Secondly, we analyzed the local stability of this system. So, the impact of the immigration parameter on the system is presented as mathematical consequences.

Key Words: Host-parasite model, Stability analysis, Misra-Mitra model.

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Allee Effect and Stability in a Discrete-time Host-parasitoid Model

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ABSTRACT

There is a need to study mathematical modeling for understand the nature events in the scientific manner. This models which allow to analyze the the real world phenomeon are formed by using difference equation and differantial equation. Therefore, the model of the biological system is very important to examine dynamics of populations. One of the most well-known model is Nicholson--Bailey host parasitoid model was developed in the 1930s to describe the population dynamics of a coupled host-parasitoid (or predator-prey) system. The model uses difference equations to describe the population growth in host-parasitoid populations. The model assumes that parasites search for hosts at random, and that both parasitoid and hosts are assumed to be distributed in a non-contiguous ("clumped") fashion in the environment. The selection of the function which is related to Poisson distribution generates different studies on host-parasitoid models . Addition, there are many reasons affecting the population such as immigration, human activities, and interaction of among species which depend on the factors called as Allee effect . This effect is firstly introduced by Allee in 1931 as negative density dependence when the growth rate of the population decreases in low population density. The Allee effect is important concepts in ecology and conservation biology. To know a lot of details about the biological population allows us to get more qualified results.

The purpose of this study is to investigate the local stability of fixed point of the host-parasitoid model and also to compare the local stability by adding Allee effect to the host population, parasitoid population and to both its, respectively. The results demonstrate that Allee effect decreases the stability of the fixed point the model.

Key Words: Stability Analysis, Population Model, Allee Effect, Fixed Point.

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A New Characterization of Bertrand curves Hyperbolic 3-Space and Minkowski 4-space

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ABSTRACT

In this study, firstly we review in detail the existing definitions and theorems about the concept of Bertrand curves in hyperbolic 3-space and four -dimensional Minkowski space. It is well known that an immersed curve in hyperbolic 3 -space is said to be a Bertrand curve if there exists another curve and a one-to-one correspondence between these curves which have common principal normal geodesics at corresponding points. After we define (1,3) -Bertrand curve with respect to the causal character of (1,3) -normal plane of non -degenerate special Frenet curves in four-dimensional Minkowski space. Then, we give the necessary and sufficient condition for the timelike (1,3) -Bertrand curve in four - dimensional Minkowski space which is obtained by a non -planar Bertrand curve with non-constant curvature in hyperbolic 3 -space. However, we give methods of obtaining Bertrand curve in hyperbolic 3- space by the spacelike or timelike (1,3) -Bertrand curve in four -dimensional Minkowski space. We get relationship between curvatures of these curves for the first time in this study. Finally, we show that a helix is also a Bertrand curve in hyperbolic 3- space. We draw images of the curve and its Bertrand mate in Poincare ball model of hyperbolic 3-space as an example.

Key Words: Bertrand curve, (1,3)-Bertrand curve, helix.

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Base Conformal Warped Product Manifolds with Dualistic Structures

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ABSTRACT

Dualistic structures are a fundamental mathematics concept of information geometry, specially in the investigation of the natural differential geometric structure possessed by families of probability distributions. The information geometry is nowadays applied in a broad variety of different fields and contexts which include, for instance, information theory, stochastic processes, dynamical systems and times series, statistical physics, quantum systems and the mathematical theory of neural networks. For understanding the notion of dualistic structure, we can give the following definition.

Let (M, g) be a Riemannian manifold and ∇ an affine connection on *M*. A connection ∇^* is called *conjugate connection* of ∇ with respect to the metric *g* if

$$Xg(Y,Z) = g(\nabla_X Y, Z) + g(Y, \nabla^*{}_X Z) \qquad (1)$$

for arbitrary X, Y and $Z \in \chi(M$, where $\chi(M)$ is the set of all tangent vectors fields on M. The triple of a Riemannian metric and a pair of conjugates connection (g,∇, ∇^*) satisfying (1) is called a dualistic structure on M.

In this study, we show that the projection of a dualistic structure defined on a base conformal warped product manifold induces dualistic structures on the base and the fiber manifolds. Conversely dualistic structures on the base and the fiber manifolds induces a dualistic structure on the base conformal warped product manifold.

Key Words: Dualistic structure, Warped product, Affine connection.

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Characterizations of Some Special Curves in Minkowski 3-space E_1^3

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ABSTRACT

The Minkowski 3-space E_1^3 is real vector space IR^3 provided with the standart flat metric given as following

$$\langle , \rangle = dy_1^2 + dy_2^2 + dy_3^2,$$

where (y_1, y_2, y_3) is a coordinate system of E_1^3. A vector V on E_1^3 is called spacelike if $\langle V,V \rangle > 0$ or V=0, timelike if $\langle V,V \rangle < 0$ and null if $\langle V,V \rangle = 0$ and $V \neq 0$, [4].

Let M be an oriented surface in 3-dimensional Minkowski space E_1^3, and let consider a non-null curve y(s) lying fully on M. Since the curve y(s) is also in space, there exists Frenet frame {T, N, B} at each points of the curve where T is unit tangent vector, N is principal normal vector and B is binormal vector, respectively. Since the curve y(s) lies on the surface M, there exists another frame and denote by {T, g, n}. In this frame T is the unit tangent of the curve, n is the unit normal of the surface M and g is a unit vector given by g=n$\times$t. Since the unit tangent T is common in both Frenet frame and Darboux frame, the vectors N, B, g and n lie on the same plane.

Thus, the derivative formula of the Darboux frame of y(s) is given by

$$\begin{bmatrix} T' \\ g' \\ n' \end{bmatrix} = \begin{bmatrix} 0 & k_g & k_n \\ -k_g & 0 & \tau \\ k_n & \tau_g & 0 \end{bmatrix} \begin{bmatrix} T \\ g \\ n \end{bmatrix},$$

$$\langle T,T \rangle = \langle g,g \rangle = 1, \langle n,n \rangle = -1. [5]$$

In this formula, k_g, k_n, τ_g are called the geodesic curvature, the normal curvature and geodesic torsion, respectively.

The special curve pairs are the most popular subjects in curve and surface theory and involute-evolute curve couple is one of them.

In this paper, we give representation formula for spacelike curves in Minkowski 3-space E_1^3. By using this formula, we obtain some characterizations of spacelike curves according to Darboux frame in Minkowski 3-space. Besides, we find the relations between the normal curvatures, the geodesic curvatures and the geodesic torsions.

Key Words: Normal curvature, Geodesic curvature, Minkowski space.

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Determination Of The Relationship Between Mathematical Literacy Self-Sufficiency And Logical Thinking Skills

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ABSTRACT

In the last quarter of a century, developed by science and technology, it is necessary to educate individuals who are self-sufficient, who can research and find the information they want and who can do the learning work alone if necessary. In this context, individuals need to be able to make mathematical reasoning in their lives, to connect mathematics with life, and to solve the problems with the help of mental processes. Together with these changes, the attitudes towards mathematics has changed and it has not been enough only to know and translate mathematical relations around them and gain the skill of analyzing a daily problem in the frame of their mathematical logic. Considering the low success of our country's mathematical skills, the importance of mathematical literacy and logical thinking skills becomes even more evident. Furthermore, it is thought that believing in your own talents in mathematical process and skills could be one of the ways to boost academic success. Individuals' awareness of these skills is important in terms of improving themselves in this process. Taking all this into account in this study, we examined a statistically significant relationship between mathematical literacy self-efficacy and logical thinking skills by using the "Mathematical Literacy Self-Efficacy Scale" developed by Özgen and Bindak (2008) and the "Logical Thinking Skill Test" developed by Tobin and Capie.

Key Words: Literacy, Mathematical Literacy, Logical Thinking Skill.

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Efficient computation of singular and oscillatory integrals with algebraic singularities

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ABSTRACT

Phenomena of singular and high oscillation is regarded as a major computational problem occurring in Fourier analysis. The numerical calculation presents difficulties when the frequency becomes large. In this case, the integrand becomes highly oscillatory. Therefore a lot of methods have been formulated for the numerical computation of these kinds of integrals. Among these methods is the so-called Clenshaw-Curtis method. It is well known that Clenshaw-Curtis schemes exactly integrate polynomials of degree at most n but converges also for every continuous function $f(x)$, Clenshaw-Curtis quadrature can be implemented in $o(n\log n)$ operations which makes the scheme more efficient.

This work discusses the computation of selected singular oscillatory integrals with specific algebraic singularities. Integrands are truncated using Chebyshev series term by term and then their algebraic singularities type are calculated using recurrence relations. With the help of the Chebyshev expansion of f, the method used in this work is constructed with the help of the expansion of oscillatory factor $e^{i\sigma x}$. A suggested method is compared with other efficient methods in this study. Computer programming codes in MATHEMATICA are provided to judge the proficiency automatic computation of such kind of integrals. Finally, explanatory numerical examples are given to illustrate the accuracy of our study analysis.

Key Words: Oscillatory integrals, Recurrence relations, Chebyshev expansion.

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Fuzzy Logic Examinations of Relationships between ^{222}Rn and Earthquake Magnitudes

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ABSTRACT

Fuzzy logic applications have a fast increase in past few years. In general, they are used in decision making systems for machine learning and for complex dynamics. Fuzzy logic is offer opportunities for sub-division branch of a system, which plays an important role look at the system a more detailed. In practice, fuzzy logic applications are most preferred, because in combination with neurocomputing and genetic algorithms. Fuzzy logic provides ease of implementation because of its properties, such as; easy to understand, flexible, tolerant of imprecise data and very simple. Fuzzy logic can be a very powerful explains about internal structure of dynamic system. Earthquake occurrences show quite complicated features. The most commonly used indicator in earthquake prediction studies is soil radon gas and its emission of concentration measurements. The fuzzy logic applies of time series with various tools is a powerful tool to understand of the complex systems. Therefore, in this study, we have tried to explain relationships between Radon gas (^{222}Rn) and earthquakes using fuzzy logic. In study, 365 (for one year) soil ^{222}Rn gas measurements are analysed with the fuzzy logic methodologies. The application is performed for data of Nurdağı region near the East Anatolian Fault Zone, Turkey.

Key Words: Fuzzy Logic, Expert System, Radon, Earthquake.

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Generalized Fermi-Walker Derivative

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ABSTRACT

To interpret the universe, it needs to be observed. An observer needs an appropriate frame construction for the definition of its location and its geometric analysis at a proper time. Rest spaces of an observer are transported through Levi-Civita parallelism when the observer γ is freely falling, so a fix direction has a null covariant derivative. If γ is not freely falling, the rest space also is not transported by Levi-Civita parallelism anymore. So in order to define "constant" directions a new derivative was defined for accelerated observers [1,2,6,8]. This derivative which is called Fermi-Walker derivative is an isometry between tangent spaces along the curve. But the Fermi-Walker derivative is only relevant for accelerating observers. Starting from this point, many scientists have given extensions for the Fermi-Walker transport with several physical motivations [7]. Then Pripoae [4,5] enlarged the context by defining a rich class of generalized Fermi-Walker connections which are relevant for both accelerating and non-accelerating observers. According to the new derivative, the observer must be able to choose between several parallel transports and not resume itself to the Fermi-Walker one.

In this study generalized Fermi-Walker derivative, generalized Fermi-Walker parallelism concepts are given and by using the definition of non-rotating frame, generalized non-rotating frame is defined along any curve in Euclidean space with the choice of tensor field.

Key Words: Fermi-Walker derivative, generalized Fermi-Walker derivative, Fermi-Walker parallelism, generalized Fermi-Walker parallelism, non-rotating frame, generalized non-rotating frame, tensor field.

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Hypergeometric integrals and Arrangements of Hyperplanes

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ABSTRACT

In integral representations of Euler type of classical hypergeometric functions of several variables or of hypergeometric functions which are studied these days, integrals of the product of powers of polynomials appear. We will establish a framework to treat such integrals, and after that, we will study hypergeometric functions of several variables as an application of the theory. Since ordinary theory of integrals of single-valued functions is formalized under the name of the de Rham theory, by modifying this theory, we will constuct a theory suitable for our purpose in this poster. As the key to the de Rham theory is Stokes theorem, we will show by posing the question how to formulate Stokes theorem for integrals of multi-valued functions. So this is an outline of the Aomoto–Gelfand theory of multivariable hypergeometric integrals and Varchenko's formula for the determinant of the period matrix of the hypergeometric pairing. A signifıcant feature of this work is the use of the theory of arrangements of hyperplanes to transform a problem in analysis into one in combinatorics.

The theory of hypergeometric functions is a venerable subject with three centuries of history. These functions have been particularly important in applied mathematics and physics. The last 20 years have seen the emergence of the Aomoto–Gelfand multivariable theory of hypergeometric functions [1,7]. These generalizations also have deep connections with recent work in theoretical physics, in particular with the representation theory of quantum groups and with conformalfıeld theory [9]. A signicant feature of this work is the use of the theory of arrangements of hyperplanes [8] to transform a problem in analysis into one in combinatorics. We outline this geometric connection here.

Key Words: Arrangement of hyperplanes, Local system homology and cohomology, Hypergeometric pairing

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Investigation of the Relationship Between Number Perceptions Skills and Mathematical Achievement of 8th Grade Students About the Square-rooted Numbers

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ABSTRACT

Sense of number is known as the ability understand the number is lessness, partial-whole, the relation to real quantities and the measurements in environment. The pupils having the number sense reach mathematical judgements using different reasonable thinking strategies more easily while interpreting expressions including numbers and mathematical operation. It not only makes the topic understandable but also improves students problem solving skills. Therefore that the pupils have “The Ability of Number Sense” is crucial for them in understanding Maths and giving meaning to what they learnt. In this research it’s aimed to determine the relation between 8^{th} grade students’ number sense achievement in root numbers and academic success in maths classes. The sample of the research is formed by total number of 59 8^{th} grade students from three state secondary schools subject to Isparta and Antalya Directorates of National Education. As data collection instrument, Number sense test of 12 questions developed by researchers in root numbers , and as a method, descriptive scan method were used in research. The strategies applied in the number sense test by 7th class students and their mathematics grades were compared. As a result , Between the pupils’ mathematics success and their using ability of number sense we observe a meaningful positive relation for the pupils having high maths success.

Key Words: Number emotion, strategies of number emotion square root numbers.

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Journey to Mathematics World with Pattern Blocks

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ABSTRACT

The concrete models have been given emphasis on school mathematics curricula for long years in Turkey [i.e. 1, 2, 3, 4]. The state has been producing and sending physical manipulatives to some of the public schools in our country.

Physical instructional objects are recommended in many school mathematics curricula in the different countries to develop psychomotor skills, spatial abilities and creativity. Moreover, Sowell states that the participants from kindergarten to university are positively effected in terms of achievement in and attitudes toward mathematics by using the physical instructional objects [5, 489].

In the elementary school mathematics curricula there are some examples on how to use pattern blocks in teaching and learning process [1, 3, 4]. It is explicitly stated that students should be able use pattern blocks effectively [4, p.22]. It is recommended that they should be used in mathematics courses because of the advantages of instructional materials related to mathematics achievement, attitudes toward mathematics, belief about mathematics, spatial ability, psychomotor skills, creativity and so on. They can be utilized in teaching/learning fractions, tessellations, patterns, and transformation geometry.

In the poster there will be various hints for the activities with pattern blocks including examples in the school mathematics curricula and games/puzzles.

Key Words: Pattern blocks, instructional material, manipulatives

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Magnetic Null Curves in E_1^3

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ABSTRACT

Let E_1^3 be a 3-dimensional Minkowski space with a linear connection ∇ corresponding to its Minkowski metric g given by $g(x,y) = x^0y^0 + x^1y^1 - x^2y^2$. Here, there are three categories of vector fields, namely,

spacelike if $g(X,X) > 0$ or $X = 0$,

timelike if $g(X,X) < 0$,

lightlike (or null) if $g(X,X) = 0, X \neq 0$.

Now, let the curve α be a null curve. Then, all its tangent vectors are null. To derive the Frenet type equations of a null curve α, defined by $\alpha: [a,b] \to E_1^3$, Cartan has shown that, with respect to an affine parameter, say p, and a positively oriented set $\{\alpha'(p), \alpha''(p), \alpha'''(p)\}$, $\forall p \in [a,b]$, there exist a local frame $F = \{\xi = \alpha', N, W\}$, called *Cartan frame* satisfying

$$g(\xi,\xi) = g(N,N) = 0, \qquad g(W,W) = 1, \qquad (1)$$

$$g(W,\xi) = g(W,N) = 0, \qquad g(\xi,N) = 1, \qquad (2)$$

with the vector product $\times$ given by

$$\xi \times W = -\xi, \ \xi \times N = -W, \ W \times N = -N. \qquad (3)$$

The Cartan equations are given by

$$\begin{aligned} \xi' &= \nabla_\xi \xi = \kappa W, \\ N' &= \nabla_\xi N = -\tau W, \\ W' &= \nabla_\xi W = -\tau\xi + \kappa N, \end{aligned} \qquad (4)$$

where κ and τ are the curvature and torsion functions of α with respect to F, respectively. Here, p is called *distinguished parameter* of α. Also, the vector fields N and W define the line bundles $ntr(\alpha)$ and $S(T\alpha^\perp)$ over α, respectively. The line

bundle $S(T\alpha^{\perp})$ is called the *screen vector bundle* and $ntr(\alpha)$ the *null transversal vector bundle* of α with respect to $S(T\alpha^{\perp})$.

Also, a divergence-free vector field defines a magnetic field in a three-dimensional semi-Riemannian manifold M. It is known that, $V \in \chi(M^n)$ is a Killing vector field if and only if $L_V g = 0$ or, equivalently, $\nabla V(p)$ is a skew-symmetric operator in $T_p(M^n)$, at each point $p \in M^n$. It is clear that, any Killing vector field on (M^n, g) is divergence-free. In particular, if $n = 3$, then every Killing vector field defines a magnetic field that will be called *Killing magnetic field*.

If (M, g) is an n-dimensional semi-Riemannian manifold, then a *magnetic field* is a closed 2-form F on M and the *Lorentz force* Φ of the magnetic field F on (M, g) is defined to be a skew-symmetric operator given by

$$g(\Phi(X), Y) = F(X, Y), \ \forall X, Y \in \chi(M).$$

The *magnetic trajectories* of F are curves α on M that satisfy the *Lorentz equation*

$$\nabla_{\alpha'}\alpha' = \Phi(\alpha').$$

In this study, firstly we define the notions of ξ-magnetic null curve, N-magnetic null curve and W-magnetic null curve in Minkowski 3-space. Also, we investigate the existence of a magnetic vector field V of a curve α to be α is a ξ-magnetic, N-magnetic or W-magnetic null trajectory of V by obtaining the Lorentz force according to the Cartan frame of these curves.

Key Words: Magnetic curves, null curves, Killing vector fields, Lorentz force.

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On Lie And Jordan Products In Prime Near-Rings

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ABSTRACT

An additively written group $(N,+)$ equipped with a binary operation $.: N \to N, (x,y) \to xy$, such that $x(yz) = (xy)z$ and $x(y+z) = xy + xz$ for all $x, y, z \in N$ is called a left near-ring. A near-ring N is said to be $3-$prime if $xNy = \{0\}$ implies $x = 0$ or $y = 0$. For any $x, y \in N$, as usual $[x,y] = xy - yx$ and $xoy = xy + yx$ will denote the well-known Lie and Jordan products respectively. An additive map $d: N \to N$ is called a derivation if $d(xy) = xd(y) + d(x)y$ holds for all $x, y \in N$. An additive mapping $F: N \to N$ is said to be a generalized derivation on N if there exists a derivation d such that $F(xy) = xF(y) + d(x)y$ for all $x, y \in N$. Inspired by the definitions derivation and generalized derivation, the notion of $(\sigma,\tau)-$derivation and a generalized $(\sigma,\tau)-$derivation were extended. Let σ and τ be endomorphisms of N. An additive mapping $d: N \to N$ is said to be a $(\sigma,\tau)-$derivation if $d(xy) = d(x)\sigma(y) + \tau(x)d(y)$ holds for all $x, y \in N$. An additive mapping $F: N \to N$ is called a generalized $(\sigma,\tau)-$derivation if there exists a $(\sigma,\tau)-$derivation d such that $F(xy) = \sigma(x)F(y) + d(x)\tau(y)$ holds for all $x, y \in N$. Note that if $d = F$, then a generalized $(\sigma,\tau)-$derivation F is just a (σ,τ)-derivation. If $\sigma = \tau = 1$, the identity map on N, then a generalized $(\sigma,\tau)-$derivation F is simply a generalized derivation. If $\sigma = \tau = 1$ and $d = F$, then a generalized $(\sigma,\tau)-$derivation F is a derivation. Hence the concept of generalized $(\sigma,\tau)-$derivations includes those of derivations, generalized derivations and $(\sigma,\tau)-$derivations.

There has been angoing interest concerning the relationship between the commutativity of prime rings or near-rings and behavior of derivations. Some

comparable results on near-rings have also been derived, (see refererences for a partial bibliography). In [5], M. N. Daif and H. E. Bell proved that R is semiprime ring, U is a nonzero ideal of R and d is a derivation of R such that $d([x,y]) = \pm[x,y]$, for all $x,y \in U$, then $U \subseteq Z$. This theorem is extended by several ways. In the present study, we shall prove that 3-prime near-ring N is commutative ring, if any one of the following conditions are satisfied: (i) $d([x,y]) = \pm\sigma(x^m[x,y]x^n)$, (ii) $d(xoy) = \pm\sigma(x^m(xoy)x^n)$, (iii) $d([x,y]) = \pm\sigma(x^m(xoy)x^n)$, (iv) $d(xoy) = \pm\sigma(x^m[x,y]x^n)$,for all $x,y \in N,\ m,n \in \mathbb{N}$ where d is a (σ,τ) $-$derivation. Also, these results were extended for generalized (σ,τ) $-$derivations.

Key Words: Near ring, derivation, (σ,τ)-derivation

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On light-like Mannheim D-curves with Mannheim partner curves lying on surfaces in Minkowski 3-spaces

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ABSTRACT

In the differential geometry of a regular curve in the ambient space, it is well-known that one of the important problem is the characterization of a regular curve [1]. First way is to use the curvature functions of a regular curve playing an important role to determine the shape and size of the curve. Another way to classification and characterization of curves is the relationship between the frames of the curves.

In this direction, many mathematicians obtained many interesting results on relations between the curvature and torsion of the curves from now on (see [5, 6, 7, 8]). One of the curves is the Mannheim curve. By the definition, space curves whose principal normals are the binormals of another curve are called Mannheim curves. The notion of Mannheim curves was discovered by A. Mannheim in 1878. These curves in Euclidean 3-space are characterized in terms of the curvature and torsion as follows:

A space curve is a Mannheim curve if and only if its curvature κ and torsion τ satisfy the relation $\kappa = c(\kappa^2 + \tau^2)$ for some constant c.

Also, R. Blum studied a remarkable class of Mannheim curves in [2]. O. Tigano obtained general Mannheim curves in the Euclidean 3-space in [4]. Recently, H. Liu and F. Wang studied the Mannheim partner curves in Euclidean 3-space and Minkowski 3-space. They obtained the necessary and sufficient conditions for the Mannheim partner curves in [3]. This work is motivated by [5].

In the present paper, we give some new characterizations for light-like Mannheim D-curves with space-like (time-like) Mannheim partner curves lying on surfaces in Minkowski 3-spaces. Also, we obtain there is no light-like Mannheim D-curves with space-like Mannheim partner D-curves being a principal line.

Key Words: Mannheim partner curve, Light-like curve, Space-like surface, Lorentzian surface, Minkowski space.

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On Semigroup Ideals of Prime Near-Rings with Generalized Semiderivation

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ABSTRACT

An additively written group $(N,+)$ equipped with a binary operation $.: N \to N, (x,y) \to xy$, such that $x(yz) = (xy)z$ and $x(y+z) = xy + xz$ for all $x,y,z \in N$ is called a left near-ring. A near-ring N is said to be $3-$prime if $xNy = \{0\}$ implies $x = 0$ or $y = 0$. For any $x,y \in N$, as usual $[x,y] = xy - yx$ will denote the well-known Lie product. A nonempty subset U of N will be called a semigroup right ideal (resp. semigroup left ideal) if $UN \subseteq U$ (resp. $NU \subseteq U$) and if U is both a semigroup right ideal and a semigroup left ideal, it will be called a semigroup ideal. An additive mapping $d: N \to N$ is called a semiderivation if there exists an additive function $g: N \to N$ such that (i) $d(xy) = d(x)g(y) + xd(y) = d(x)y + g(x)d(y)$ and (ii) $d(g(x)) = g(d(x))$ hold for all $x,y \in N$. An additive mapping $F: N \to N$ is said to be a generalized semiderivation of N if there exists a semiderivation $d: N \to N$ associated with a map $g: N \to N$ such that (i) $F(xy) = F(x)y + g(x)d(y) = d(x)g(y) + xF(y)$ and (ii) $F(g(x)) = g(F(x))$ hold for all $x,y \in N$.

Let N be a $3-$prime near-ring with a nonzero generalized semiderivation F associated with a semiderivation d a map g and U be a nonzero semigroup ideal of N. In this study, it is shown that N is commutative ring if any one of the following conditions are satisfied: (i) $F(U) \subseteq Z$, (ii) $F([u,v]) = 0$, (iii) $F([u,v]) = \pm[u,v]$, (iv) $F([u,v]) = [F(u),v]$, (v) $[F(u),v] \in Z$, for all $u,v \in U$.

Key Words: Prime near ring, derivation, semiderivation.

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On Slant Helices in Hyperbolic 3-Space and de Sitter 3-Space

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ABSTRACT

It is well known that there are two kinds of non - flat Riemannian or Lorentzian space form in Minkowski 4 - space. One of them is a non - flat Riemannian space form with negative constant curvature which is called hyperbolic 3 - space, the other is a non - flat Lorentzian space form with positive constant curvature which is called de Sitter 3 - space. Curves theory is important field in the differential geometry. There are a lot of special curves such as geodesics, circles, Bertrand curves, circular helices, general helices, slant helices etc. To study characterizations of these special curves play important role in the view of application to other fields (i.e. kinematic, mechanic, fractal geometry, computer aided design). Slant helix and its geometric properties is studied by many authors in different ambient spaces.

In this study, we define non - degenerate immersed slant helices in non - flat 3 - dimensional (Riemannian or Lorentzian) space forms. We use the Killing vector field with constant length, which is an axis of the slant helix, along the curve. Moreover, we give characterization of a slant helix in hyperbolic 3 - space and de Sitter 3 - space with respect to the Sabban frame of the curve.

Key Words: Hyperbolic 3-space, de Sitter 3-space, Killing vector field, slant helix.

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On Special Curves of Constant Precession

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ABSTRACT

The theory of curves has been one of the fascinating subject because of having many application area from geometry to the various branch of science. As a consequence of this fact, many different type of curves are defined by using Frenet Serret vector fields in numerous spaces. Among these, Bertrand, Mannheim, involute evolutes, Smarandache and special Smarandache curves have drawn great attention by researchers for a long time and have studied in detailed [1-6].

On the other, a constant precession curve is a curve which has property that is transversed with a unit speed, its centrodes (Darboux vector field)

$$w = \tau T + \kappa B$$

revolves about a fixed axis with constant angle and speed. If one describes this Darboux vector field in terms of an alternative moving frame, this vector provides the following conditions

$$D\Lambda N = N', \qquad D\Lambda C = C', \qquad D\Lambda W = W'$$

Then we call it C-constant precession curve [7].

Simply a Smarandache curve is a regular curve in Minkowski space time whose position vector is composed by Frenet vectors on another regular curve (see [1]). The authors have specialized Smarandache curves and obtained Frenet-Serret invariants in various spaces.

In the present work, we have dealt with special curves of constant precession and focus on the characterizations of some special curves of constant precession.

Key Words: : C-constant precession curve, Darboux vector fields, Serret-Frenet vector fields.

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On The Gauss Map Of Tubular Surfaces in Pseudo-Galilean 3-Space

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ABSTRACT

The spaces of Galilean and Pseudo Galilean are different spaces from the Euclidean space. The Pseudo-Galilean geometry is one of the real Cayley-Klein geometries (of projective signature (0, 0, +, -). The absolute of the Pseudo Galilean geometry is an ordered triple {w, f, I} where w is the ideal (absolute) plane, f is line in w and I is the fixed hyperbolic involution of points of f. There are two kinds of vector called isotropic and non isotropic vectors in these spaces.

In current literature, translation surfaces, surfaces of revalution, canal surface etc. are defined in Galilean and Pseudo Galilean spaces and many mathematicians studied those surfaces in different respects. Laplacien transformations of the position surface and the Gauss map are examples of that studies. Tubular surface also called pipe surface is the subclass of the canal surfaces. In the present paper, we defined tubular surfaces in Pseudo-Galilean 3-space G^1_3 and we gave the Laplacien transformations of the position vector and the Gauss map. We classified tubular surfaces in terms of Laplacien of the position vector and the Gauss map of the surface satisfying $\Delta X=0$, $\Delta X=AX$, $\Delta G=0$, $\Delta G=\lambda G$ and $\Delta G=AG$. Also we gave some examples.

Key Words: Tubular surface, Gauss map , Pseudo-Galilean 3-Space.

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On The Timelike Surface With Constant Angle in Hyperbolic-3 Space

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ABSTRACT

In this paper , we study constant timelike angle surface whose unit normal vector field make constant timelike with a fixed spacelike axis in R_1^4 in Hyperbolic space H^3 . Let $x: M \to H^3$ be a spacelike immersion and let ξ be a unit normal vector field to M. If there exists spacelike direction U such that timelike angle $\theta(\xi,U)$ is constant on M , then M is called constant timelike angle surfaces with spacelike axis in H^3. Also, conditions being a constant angle surface in H^3 have been
determined and invariants of these surfaces have been investigated.

Constant timelike and spacelike angle surface have not been investigated in hyperbolic space H^3. Constant angle spacelike surface in hyperbolic space H^3 and constant angle spacelike surface in de-Sitter space S_1^3 are developed in our papers. In this paper, a special class of surfaces which is called the constant timelike angle surfaces is given in hyperbolic space H^3. A constant timelike angle surface in hyperbolic space H^3 is a surface whose tangent planes make a constant timelike angle with a fixed spacelike vector field on R_1^4. In Minkowski space R_1^4, due to the variety of causal character of a vector field, there is a natural concept of variable angle between two arbitrary vector fields. Since x spacelike immersion into H^3 , ξ is unit spacelike normal vector field to M .

Key Words: Constant Angle Surface, Hyperbolic Space, Timelike Surface.

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Qi Type Integral Inequalities

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ABSTRACT

Integral inequalities have been frequently employed in the theory of functional analysis, differential equations and applied sciences such as probability and statistics. There are a lot of types integral inequalities such as Hermite - Hadamard type inequalities, Opial type inequalities, Lyapunov type inequalities, Halanay type inequalities, Wirtinger type inequalities, Hardy type inequalities.

In the last two decades, they have been the focus of attention in many papers. Especially an integral inequality which is called Qi Inequality by mathematics community, has been studied by many authors.

Qi proposed an open integral problem at the end of [1] which was actually posed by himself in the preprint version [2]:

Problem 1: Under what conditions does the inequality

$$\int_a^b [f(x)]^t dx \geq \left(\int_a^b f(x) dx \right)^{t-1}$$

hold for $t > 1$?

Since then, this Problem 1 has been stimulating much interest of many mathematicians and affirmative answer to it has been established. Also, some other Qi type integral inequalities were proposed by researchers; some of them are given as follows:

Problem 2: Under what conditions does the inequality

$$\int_a^b [f(x)]^{n+2} dx \geq \left(\int_a^b f(x) dx \right)^{n+1}$$

hold for $n \in N$?

Problem 2 is special case of Problem 1.

The following two problems are extention of Problem 1 and 2.

Problem 3: Under what conditions does the inequality

$$\int_a^b [f(x)]^\alpha dx \geq \left(\int_a^b f(x)dx\right)^\beta$$

hold for α and β?

Problem 4: Under what conditions does the inequality

$$\int_a^b [f(x)]^\alpha dx \geq \left(\int_a^b f^\gamma(x)dx\right)^\beta$$

hold for α, β and γ?

Researchers investigated some sufficient conditions for the above integral inequalities using a lot of techniques including analytic method, mathematical induction, convexity criteria.

We survey the literature answering the proposed questions.

Key Words: Qi type integral inequality, Integral inequality.

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Some Algebraic Operations on Normed Rings

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ABSTRACT

In this study, we study normed ring sets and well-known results of some study are given. And then, we construct patching data over fields *K*(*x*), where *K* is a complete ultrametric valued field. The 'analytic' fields P_i will be the quotient fields of certain rings of convergent power series in several variables over *K*. At a certain point in a proof by induction we consider a ring of convergent power series in one variable over a complete ultrametric valued ring. So, we start by recalling the definition and properties of the latter normed rings. In this study, we study normed ring sets and well-known results of some study are given. And then, a notion of normed rings is given and various properties are studied. As a result of this examination, it is wondered whether the normative convergent power series are related to the normed ring. In this study, definitions such as topological ring on the normed rings are made by defining the norm and the ring, and definitions such as evaluation homomorphism on convergent power series on the normalled rings are examined. In addition, headings such as the properties of the normed ring and convergent power series have been examined.

Key Words: Norm, Rings, Normed Rings, Algebraic Operations

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Symmetric biderivation on hyperrings

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ABSTRACT

The theory of hyperstructures was introduced in 1934 by Marty [5] at the 8th congress Scandinavian Mathematicians. Then several researchers have worked on this new field and developed it. Mittas [7] introduced the notion of canonical hypergroups. Corsini [2] studied the Canonical Hypergroups, Feebly Canonical Hypergroups, Quasi-Canonical Hypergroups. Krasner [3] introduced the notion of hyperrings and hyperfields. Asokkumar studied the idempotent elements of Krasner hyperrings. The notion of derivations of rings plays a significant role in algebra. Derivations in prime rings firstly initiated by Posner [8] and it is considered a fundamental construction in the theory of centralizing maps on prime rings. Gy. Maksa defined symmetric biderivation on ring. A great deal of work in this context are available in the literature (see, for example [4] and [5]). Asokkumar [1] presented derivations in hyperrings and hyperrings.

In this study, R will be represent Krasner hyperring. A map $D: R \times R \to R$ is said to be a symmetric bi-derivation of R if D satisfies: (i) $D(x+z,y) \subseteq D(x,y) + D(z,y)$ and (ii) $D(xz,y) \in D(x,y)z + xD(z,y)$ for all $x,y,z \in R$ and a map $d: R \to R$ defined by $d(x) = D(x,x)$ is called the trace of D. The trace of D satisfies the relation

$$d(x+y) = D(x+y,x+y) \subseteq d(x) + D(x,y) + D(x,y) + d(y).$$

In here after giving definition of symmetric biderivation and its examples will investigate some properties of symmetric biderivation and its trace in hyperrings.

Key Words: Krasner hyperrings, symmetric biderivation, trace of symmetric biderivation.

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The Evaluation of the Effects of the Four Operations in Mathematics with the Sign Language That Developed By Pairing the Numbers One By One with Our Body

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ABSTRACT

Old civilizations used many different methods to show numbers.In British Papua New Guinea for example, the New Guinean people used parts of their bodies in counting. Old civilizations used many different methods to show numbers. In British Papua New Guinea for example, the New Guinean people used parts of their bodies in counting. The aim of this study is to provide an effective and permanent learning by applying the four operations in mathematics by using an alternative method by matching the numbers with our body parts.

The sample of the study was composed of 150 fifth grade students in a secondary school in Bingöl province. As a method, we used exercises to use the body parts instead of numbers and figures for a week to ensure that the numbers were matched correctly thus, effective and permanent learning has been tried to be achieved. After sufficient learning was achieved with these studies made with the students exercise papers, which required four operation skills were prepared and applied to the students. Learning environment was visualized with photographs. The answers given by the students were controlled by the grade pointing keys and the correct and incorrect answers were grouped according to gender, the arithmetic averages were found and frequency tables were formed.

It has come out that female students have more accurate answers than male students. It has also been observed that female students were more willing and more engaged in the studies than the males. It has been observed that such practices have an effect on students' attitudes towards mathematics and their learning.

Key Words: Numbers, Our body, four operations

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^{222}Rn Concentration Dependence on Seasonal Changes with Spatial and Statistics Modelling

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ABSTRACT

The analysis of temporal and spatial variations on the flux of soil gases across the soil–air interface is a useful tool to study geo-dynamical processes associated with seismic activity. One of these gases is radon, ^{222}Rn, a naturally occurring radioactive noble element with a half-life of 3.82 days in the disintegration chain of ^{238}U, with ^{226}Ra as its immediate parent nuclide. ^{222}Rn atoms are continuously generated in the rock matrix and emanate into the air filled pore space, from where some of them reach the ground surface and escape into the atmosphere. It that spreads to environment as closely associated with the geological structure of the geographical region, so amount of ^{222}Rn demonstrates vary greatly from position to position. Geostatistics methods are the most important tools in modeling the variables that show spatial difference. The semivariogram (SV) is a special branch of geoistatistic developed by Georges Matheron and commonly used in different areas of geology, meteorology, transport of radionuclides and other natural sciences. It is developed for investigation spatial similarity and changing of any regionalized variable (ReV) and it expresses the rate of change of a regionalized variable along a specific orientation. It is a measure of the degree of spatial dependence between observations. In this study, we used semivariogram method and investigated changing of ^{222}Rn concentration with seasonal and spatial in the East Anatolian Fault System.

Key Words: Semivariogram, Radon, Earthquake, Spatial Prediction.

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^{222}Rn Prediction using the Multiple Regression Model in Eastern Anatolian Fault Zone (EAFZ)

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ABSTRACT

Regression analysis is used to determine the relationship between two or more variables and to make estimations about the subject by making use of this relationship, aiming to determine the nature of the relationship, assuming that there is a causal relationship between variables [1-4]. In nature, it is possible to find cause-effect relationships in many cases. In accordance with this purpose, regression models were obtained using the technique called multiple regression analysis to explain the relationship between soil ^{222}Rn (Radon gas), earthquake and meteorological variables. It was possible to make ^{222}Rn prediction from this regression model [2]. The application of model was made for the Pütürge (Malatya) station located on the Eastern Anatolian Fault Zone (EAFZ). The regression coefficients of earthquake magnitudes from a radius of 30 km radius around the research station and the meteorological variables (maximum solar radiation per minute within a day (cal/cm²), 5 cm, 10 cm, 20 cm and 50 cm deep soil temperature (°C), vapour pressure (hPa), wet and dry thermometer temperature (°C)) were calculated using the least squares method. The concentration of soil ^{222}Rn gas was selected as the dependent variable. ^{222}Rn gas predictions can be made using the results of the mathematical models.

Key Words: Radon, Regression analysis, Earthquake, Meteorological variables, Prediction.

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WORKSHOP

The Nature Of Mathematical Literacy: How To Write Mathematical Literacy Problems?

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ABSTRACT

In this study, learning outcomes are the focus point. Learning outcomes are the written statements that are about what a student can be able to do after a learning activity. These statements show expected knowledge levels, skills, values and attitudes.

In Turkey, there are two main components that help to evaluate learning outcomes. One of them is international assessments i.e. PISA and TIMSS. The other one is national assessments i.e. TEOG, YGS, LYS. But, in this study we pay attention to PISA, with the concept of "mathematical literacy" [1]. As a result of all these assessments, Turkish students seem to be very unsuccessful at using mathematical concepts in real life situations. According to PISA results, Turkish students are mostly able to solve direct and easy mathematical problems, while very few of them can solve complicated real life problems [2]. Mathematical literacy is worth to study on, because, it does not evaluate theoretical knowledge, it evaluates theoretical knowledge's real life implementations [3].

The results of PISA show that Turkish students fall behind most of OECD countries. When we examine the questions of PISA, we realised that Turkish students are not familiar with this type of questions. Because, Turkish students study according to Turkish national exams and they do not include real life problems.

In this regard, we thought that it is important to know the nature of mathematical literacy problems. In this study, we will discuss about mathematical literacy problems' nature. In addition, we will try to work on writing mathematical literacy problems.

Key Words: Learning outcomes, mathematical literacy, primary education.

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