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INTERNATIONAL CONFERENCE ON MATHEMATICS AND MATHEMATICS EDUCATION

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ABSTRACT BOOK

Mathematics in Cappadocia in memory of Alev Alatlı

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İLGİLİ MAKAMA

Matematikçiler Derneğinin 03-05 Ekim 2024 tarihleri arasında bu yıl 21.sini Nevşehir Hacı Bektaş Veli Üniversitesi ve Kapadokya Üniversitesi ev sahipliğinde Nevşehir'de düzenleyeceği Uluslararası Matematik ve Matematik Eğitimi Konferansı (International Conference on Mathematics and Mathematics Education :ICMME 2024) için **Matematikçiler Derneğinin 23/08/2024 tarih ve 24 sayılı Yönetim Kurulu Kararı** ile Kongre Başkanı Matematikçiler Derneği Yönetim Kurulu Başkanı ve Gazi Üniversitesi Öğr. Üyesi **Prof. Dr. Hasan Hüseyin SAYAN**, Düzenleme Kurulu Üyeleri, Matematikçiler Derneği Yönetim Kurulu Üyesi ve Selçuk Üniversitesi Öğr. Üyesi **Prof. Dr. Nurettin DOĞAN**, Matematikçiler Derneği Yönetim Kurulu Üyesi ve Gazi Üniversitesi Öğr. Üyesi **Prof. Dr. Mustafa ÖZKAN**, Matematikçiler Derneği Yönetim Kurulu Üyesi ve Gazi Üniversitesi Öğr. Üyesi **Prof. Dr. Fatma AYAZ** ve Matematikçiler Derneği Üyesi ve Ankara Yıldırım Beyazıt Üniversitesi Öğr. Üyesi **Doç. Dr. Abdülhamit KÜÇÜKASLAN Akademisyen Temsilcisi** olarak görevlendirilmiştir.

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INTERNATIONAL CONFERENCE ON MATHEMATICS AND MATHEMATICS EDUCATION (ICMME 2024)

ABSTRACT BOOK

International Conference
on Mathematics and Mathematics Education
(ICMME - 2024)

Nevşehir Hacı Bektaş Veli University, Nevşehir, Turkey
3-5 October 2024

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Nurettin DOĞAN
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PREFACE

International Conference on Mathematics and Mathematics Education (ICMME-2024) was held on 3-5 October 2024 in Nevşehir, Türkiye, both face to face and online via the zoom platform.

MATDER-Association of Mathematicians is an association founded in 1995 by mathematicians in Turkey. Up to now 14 national and 5 international mathematics symposium were organized by MATDER.

The last three conferences were held in Denizli (ICMME-2022), Ankara (ICMME-2021) and Konya (ICMME-2019). Each of these events has been one of the primary national symposiums. The presentations at these meetings cover nearly all areas of mathematics, mathematics education, and engineering mathematics, attracting participation from mathematicians in academia, the Ministry of Education, and engineers alike. This year, ICMME-2024 took place at Nevşehir Hacı Bektaş Veli University in Nevşehir, Türkiye, from October 3 to 5, 2024, as an international conference.

The main goal of this conference is to contribute to the advancement of mathematical sciences, mathematical education, and their applications, while bringing together members of the mathematics community, interdisciplinary researchers, educators, mathematicians, and statisticians from around the globe. The conference featured new results and future challenges through a series of invited and short talks, poster presentations, workshops, and exhibitions. All presented paper abstracts will be published in the conference proceedings. Additionally, selected and peer-reviewed articles will be published in the following journals:

- Advanced Studies: Euro Tbilisi Mathematical Journal
- International Journal of Maps in Mathematics
- Turkish Journal of Mathematics and Computer Science
- Selçuk University Journal of Engineering Sciences
- Gazi Eğitim Bilimleri Dergisi

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CONTENTS

INVITED SPEAKERS

How Will Artificial Intelligence Affect the Future?

Çetin ELMAS [1]
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ABSTRACT

Artificial intelligence (AI) technologies are considered one of the most important and transformative innovations of the 21st century, and these technologies are expected to create profound impacts in social, economic and technological fields in the future. Especially in the business world, various sectors are increasingly turning to artificial intelligence solutions to ensure automation and increased productivity. This trend allows business processes to be accelerated and operations that require less human intervention to become widespread, thus increasing the efficiency of business processes and reducing costs. Artificial intelligence applications in critical sectors such as health, finance, transportation and production offer new opportunities in terms of rapid diagnosis of problems, more effective use of resources and provision of the highest quality service at the lowest cost. Such applications provide significant contributions in a wide range of areas, from early diagnosis of diseases to predicting financial risks, from optimizing traffic flow in transportation to minimizing error rates in production processes. The effectiveness of scientific research, data analysis and mathematical modeling increases significantly with the speed and accuracy provided by artificial intelligence algorithms. In particular, algorithmic solutions of complex mathematical problems or analysis of large data sets obtained from observations can be carried out faster and more error-free through methods optimized by artificial intelligence. These developments allow new scientific discoveries and more rapid development of mathematical theories, while also increasing the automation of scientific processes. Thus, people can focus more on their creative thoughts, develop innovative solutions and take their existing knowledge further.

Norm estimates of general matrices between Lebesgue spaces

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ABSTRACT

For a large class of operators acting between weighted l^∞spaces, exact formulas are given for their norms and the norms of their restrictions to the cones of nonnegative sequences; nonnegative, nonincreasing sequences; and nonnegative, nondecreasing sequences. The weights involved are arbitrary nonnegative sequences and may differ in the domain and codomain spaces. The results are applied to the Cesàro and Copson operators, giving their norms and their distances to the identity operator on the whole space and on the cones. Simplifications of these formulas are derived in the case of these operators acting on power-weighted l^∞spaces. As an application, best constants are given for inequalities relating the weighted l^∞norms of the Cesàro and Copson operators both for general weights and for power weights.

Key Words: Operator norm, Cesàro operator, Copson operator, Best constant

REFERENCES

[1] Barza, S., Demissie, B.M. & Sinnamon, G. End-point norm estimates for Cesàro and Copson operators. *Annali di Matematica* 203, 989–1013 (2024).

[2] Boza, S., Soria, J. Weak-type and end-point norm estimates for Hardy operators. *Annali di Matematica* 199, 2381–2393 (2020).

Higher Order Elliptic Equations in Weighted Sobolev-Banach Spaces

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ABSTRACT

We consider 2m-th order linear, uniformly elliptic equations with non-smooth coefficients in Banach-Sobolev spaces generated by weighted general Banach Function Spaces on a bounded domain.

Supposing boundedness of the Hardy-Littlewood Maximal and Calderòn-Zygmund singular operators in weighted BFS we obtain local solvability in Sobolev-Banach weighted spaces and establish interior Schauder type a priori estimates for the corresponding elliptic operator.

Key Words: Elliptic equations, Sobolev-Banach function spaces, Schauder-type estimates.

REFERENCES

[1] Bilalov B.T., Sadigova S.R., On solvability in the small of higher order elliptic equations in grand-Sobolev spaces, Compl. Var. Elliptic Equ., 66, No. 12 (2021), 2117--2130.

[2] Bilalov B.T., Sadigova S.R., Softova L.G., Higher order elliptic equations in weighted Banach spaces, Annali Univ. Ferrara (2024), doi: 10.1007/s11565-024-00505-9.

Existence and Ulam Stability Results of Tempered (κ,ψ)-Hilfer Fractional Differential Problems in Banach Spaces

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ABSTRACT

The primary aim of this paper is to explore various aspects, such as the existence, uniqueness, and κ-Mittag-Leffler-Ulam-Hyers stability, pertaining to a specific class of terminal value problems within Banach spaces. These problems encompass implicit nonlinear fractional differential equations and tempered (κ,ψ)-Hilfer fractional derivatives. Our approach involves leveraging several mathematical tools, notably the Banach fixed point theorem, Monch's ¨ fixed point theorem in conjunction with the measure of noncompactness technique, and a generalized form of the well-established Gronwall inequality. Furthermore, we present illustrative examples to underscore the practical implications of our key findings.

Key Words: Tempered fractional operators, existence, uniqueness, generalized Gronwall inequality, measure of noncompactness, Mittag-Leffler function, Ulam-Hyers stability, terminal condition.

REFERENCES

[1] Ahmed, P. Kumam, F. Jarad, et al. On Hilfer generalized proportional fractional derivative. Adv. Differ. Equ. 2020, 329 (2020). https://doi.org/10.1186/s13662-020-02792-w.

[2] Ahmed, N. Limpanukorn, M. J. Ibrahim, Uniqueness of continuous solution to q-Hilfer fractional hybrid integro-difference equation of variable order. J. Math. Anal. Model. 2 (2021), 88-98. https://doi.org/10.48185/jmam.v2i3.421.

[3] Almalahi and K. Panchal, Existence results of ψ-Hilfer integro-differential equations with fractional order in Banach space, Ann. Univ. Paedagog. Crac. Stud. Math, 19 (2020), 171-192.

[4] R. Almeida, M. L. Morgado, Analysis and numerical approximation of tempered fractional calculus of variations problems, J. Comput. Appl. Math. 361 (2019), 1-12.

[5] J. Appell, Implicit functions, nonlinear integral equations, and the measure of noncompactness of the superposition operator, J. Math. Anal. Appl. 83 (1) (1981), 251-263.

[6] J. Banas and K. Goebel, Measures of noncompactness in Banach spaces. Marcel Dekker , New York, 1980.

[7] M. Benchohra, E. Karapınar, J. E. Lazreg and A. Salim, Advanced Topics in Fractional Differential Equations: A Fixed Point Approach, Springer, Cham, 2023.

Emergence and Development of Manifold Concept

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ABSTRACT

The term "manifold" comes from German Mannigfaltigkeit, by Bernhard Riemann. Riemannian geometry is thebranch of differential geometry that studies Riemannian manifolds, defined as smooth manifolds with a Riemannian metric. This gives, in particular, local notions of angle, length of curves, surface area and volume. From those, some other global quantities can be derived by integrating local contributions. Riemannian geometry originated with the vision of Bernhard Riemann (who made profound contributions to analysis, number theory and differential geometry.1826-1866) expressed in his lecture "On the Hypotheses which lie at the bases of Geometry". It is a very broad and abstract generalization of the differential geometry of surfaces in R^3.

The concept of manifold is a concept that allows us to deal with different geometries outside of Euclidean space, that is, planar (linear) spaces. The main motivation for the concept of manifold is Gaussian curvature. Gaussian curvature is the measure of deviation of a surface from its tangent plane at that point.

Keywords: Differentiable Manifolds, Complex Manifolds, Contact Manifolds.

Solipsism and Mathematics

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ABSTRACT

Solipsism is the version of the theses put forward by the school known as "sophists" in the history of philosophy. In general, they defended a thesis expressed as "we can't know anything, we can't convey it even if we know, we can't tell if we communicate it". This thesis later evolved into a conclusion that does not accept any existence other than "one-selfishness", that is, "I". This has made it the unwanted and ignored child of philosophy. However, the principles on which it puts forward and rely on remain valid even today.

In order to interpret mathematics from a solipsist point of view, it is first necessary to overcome the thesis of solipsism, which predicts "nothingness" in a sense. In the next step, it will be necessary to associate it with mathematics, that is, to interpret mathematics from a solipsist point of view.

Mathematics is also an information system after all, and this information system seems to have not entered the radar of the solipist view so far. On the other hand, mathematics is the most reliable information system in terms of the language it uses. However, the reliability and certainty it has does not protect it from the solipsist theses. In this case, what will be done will be to overhaul and reinterpret mathematics in terms of solipsist theses. In this process, geometry, which means a space representation, can help us. The space design to be mentioned here coincides with the concept of space used in geometry in a sense, as well as important differences between them. From a solipsist point of view, space representation (Ural spaces) will give us the opportunity to talk about a solipsist mathematics. For this, it will be necessary to redefine some basic concepts of mathematics.

Key Words: Solipsism, space representations, Ural spaces.

REFERENCES

[1] Ural, Ş., Solipsism. Physical Things and Personal Perceptual Space, Vernon Press, 2019.

Türkiye Century Education Model and Primary School Mathematics Curriculum

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ABSTRACT

A paradigm shift has been made in the primary school mathematics curriculum prepared within the scope of the Türkiye Century Education Model. These changes took place in the context of programs and interdisciplinary relations, as in the courses of other disciplines. Based on inter-program relationships, social emotional learning skills, values and literacy skills are included in the primary school mathematics curriculum, as in other courses. Social emotional learning skills are included in the primary school mathematics curriculum under 3 headings. These are Self Skills, Social Life Skills and Joint/Combined skills.

Another heading of inter-program components is values. In the primary school mathematics course curriculum, which was prepared by taking into account the Virtue Value Action framework, 13 of the 20 values in the common text were integrated into the program by taking into account the context of the mathematics course. The values in the program are included in learning and teaching practices.

Another title of the cross-program components is literacy skills. Literacy skills are included in the program under the title of system literacy. Under the basic title of system literacy, the most emphasized literacy skill in the Primary School Mathematics Curriculum is the "Digital Literacy" skill. Digital Literacy skills are followed by "Information Literacy", "Visual Literacy", "Data Literacy" and "Financial Literacy" skills, respectively. The fact that digital literacy skills are included in more programs than other skills reveals that the integration of technological developments into educational environments and the more frequent use of digital platforms are taken into account in the new primary school mathematics curriculum.

All 5 field skills included in the common text are included in the primary school mathematics curriculum prepared within the scope of the Turkey Century Education Model. The most used skill among the field skills is the "Working with

Mathematical Tools and Technology" skill. This skill is followed by "Mathematical Representation", "Mathematical Problem Solving", "Working with Data and Data-Based Decision Making" and "Mathematical Reasoning" skills according to the frequency of its inclusion in the program.

Alev Alatlı and Mathematics

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ABSTRACT

Alev Alatlı the founder of Cappadocia University, is considered one of the most important writers, philosophers and intellectuals in Türkiye. This presentation will focus on her thoughts and perspectives on the education system, mathematics education, logic and mathematics. Additionally, her views on the relationship between the Turkish language, logic and mathematics will be discussed.

REFERENCES

[1] Schrödinger'in Kedisi – Kabus, Alev Alatlı, Alfa Yayınları, 2001.

[2] Schrödinger'in Kedisi – Rüya, Alev Alatlı, Alfa Yayınları, 2001.

[3] Aklın Yolu da Bir Değildir, Alev Alatlı, Destek Yayınları, 2009.

[4] 4.Kelebek Söyleşileri I-II, Ayşe Böhürler, Pınar Yayınları, 2023.

[5] Eğitim Sisteminde Zihniyet Sorunu Ve Bir Teklif , Prof. Dr. Ömer Dinçer, Alev Alatlı'yı Anlamak Sempozyumu Sunumları, Mayıs 2024.

[6] Eğitimde İlk Olan Bir Şey Yoktur, Prof. Dr. İrfan Erdoğan, Alev Alatlı'yı Anlamak Sempozyumu Sunumları, Mayıs 2024.

[7] Bir Toplumsal Afazi Reçetesi Olarak Bize Yön Veren Metinler Projesi, Prof. Dr. Ömür Ceylan, Alev Alatlı'yı Anlamak Sempozyumu Sunumları, Mayıs 2024.

[8] Alev Alatlı'nın Paçozlaşma Kavramına Enerji Tarihimizden Örnekler, Prof. Dr. Volkan Ediger, Alev Alatlı'yı Anlamak Sempozyumu Sunumları, Mayıs 2024.

[9] Alev Alatlı, Mantık Ve Heyelan, Prof. Dr. Candan Gökçeoğlu, Alev Alatlı'yı Anlamak Sempozyumu Sunumları,i Mayıs 2024.

[10] Schrödingerin'in Kedisinde Bilim, Prof. Dr. İskender Öksüz, Alev Alatlı'yı Anlamak Sempozyumu Sunumları, Mayıs 2024.

[11] Fesüphanallah, Nasihatname I, Alev Alatlı, Turkuaz Yayınları, 2019.

[12] Hafazanallah, Nasihatname II, Alev Alatlı, Turkuaz Yayınları, 2019.

ABSTRACTS OF ORAL PRESENTATIONS

ALGEBRA AND NUMBER THEORY

Cholesky Factorization of the Pell-Lucas Matrix

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ABSTRACT

In recent years, the Fibonacci and Lucas numbers have drawn the attention of numerous mathematicians studying linear algebra, calculus, applied mathematics, and other areas of mathematics. The recursive relationships of the Fibonacci and Lucas number sequences are represented in a manner comparable to other relationships, and new number sequences, including the Pell and Pell-Lucas number sequences, are derived. The Pell numbers P_n and the Pell-Lucas numbers Q_n are determined by

$$P_{n+1} = 2P_n + P_{n-1}$$

for $n \geq 1$, where $P_0 = 0$ and $P_1 = 1$, and

$$Q_{n+1} = 2Q_n + Q_{n-1}$$

for $n \geq 1$, where $Q_0 = 2$ and $Q_1 = 2$, respectively.

In this study, we provide the Pell-Lucas matrix A_n and the symmetric Pell-Lucas matrix B_n. To thoroughly investigate the linear algebraic characteristics of these matrices, we analysed their mathematical properties and relationships. We give the Cholesky factorization of the symmetric Pell-Lucas matrices and create decompositions for the Pell-Lucas matrix and its inverse matrix. To do this, we first determine the inverse of the Pell-Lucas matrix A_n^{-1}, and show the factorization of it. After that, we obtain components b_{ij} from the Pell-Lucas matrix B_n and create the Cholesky factorization of it.

Moreover, by using majorization notation, we obtain several useful identities and constraints for the eigenvalues of these symmetric Pell-Lucas matrices. We get the lower and upper boundaries for those eigenvalues.

Key Words: Pell-Lucas matrix, Cholesky factorization, Majorization.

REFERENCES

[1] F. Horadam, Pell identities, Fibonacci Quarterly, (1971), 9(3), 245-252.

[2] T. Koshy, Pell and Pell-Lucas Numbers With Applications, New York: Springer, (2014).

[3] R. Brawer, M. Pirovino, The linear algebra of the Pascal matrix, Linear Algebra and Its Applications, (1992), 174, 13-23.

[4] G. Y. Lee, J. S. Kim, S. G. Lee, Factorizations and eigenvalues of Fibonacci and symmetric Fibonacci matrices, Fibonacci Quarterly, (2002), 40(3), 203-211.

[5] N. Irmak, C. Köme, Linear algebra of the Lucas matrix, Hacettepe Journal of Mathematics and Statistics, (2021), 50(2), 549-558.

[6] E. Kılıç, D. Taşçı, The linear algebra of the Pell matrix, Boletin de la Sociedad Matematica Mexicana, (2005), 11(3), 163–174.

[7] A. W. Marshall, I. Olkin, B. C. Arnold, Inequalities: Theory of Majorization and Its Applications, Springer Series in Statistics, (2011).

[8] A. F. Horadam, J. M. Mahon, Pell and Pell-Lucas polynomials, Fibonacci Quarterly, (1985), 23(1), 7-20.

DUAL NİL-CLEAN, UN AND UNIT-REGULAR ELEMENTS IN CERTAIN SUBRİNGS OF $\mathbb{M}_2(\mathbb{Z})$

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ABSTRACT

Let R be a ring with identity. An element $a \in R$ is called clean (respectively, unit-regular) if a is the sum (respectively, product) of an idempotent and a unit; $a \in R$ is called nil-clean if a is the sum of an idempotent and a nilpotent. A ring R is clean (respectively, unit-regular, nil-clean) if every element of R is clean (respectively, unit-regular, nil-clean). Its known that unit-regular rings and nil-clean rings are all clean. But there are questions of whether a nil-clean element is also clean and whether Jacobson's lemma holds for nil-clean elements. Let $\mathbb{Z}$ be the ring of integers and $s \in \mathbb{Z}$. In the paper under review, using the matrix $\begin{pmatrix} \mathbb{Z} & \mathbb{Z} \\ s^2\mathbb{Z} & \mathbb{Z} \end{pmatrix}$, prove the three results let $a, b \in R$. Then $1 - ab$ is nil-clean does not imply that $1 - ba$ is nil-clean, nil clean elements need not be clean, there are many unit-regular elements which are not clean.

Key Words: Nilpotent elements, idempotent elements, (nil-)clean elements, unipotent elements

REFERENCES

[1] Andrica, D. & Calugareanu, G. (2014). A nil-clean 2×2 matrix over the integers which is not clean. J. Algebra Appl. 13(6), 1450009.
[2] Calugareanu, G. & Lam, T. Y. (2015). Fine rings: A new class of simple rings, J. Algebra Appl. 15(9), 18 pages.
[3] Calugareanu, G. (2015). UU rings. Carpathian J. Math., 31(2), 157-163.
[4] Calugareanu, G., Pob, H.F. On stable range one matrices, Preprint
[5] Garcia, J.L. (1989). Properties of direct summands of modules. Commun. Algebra 17, 73–92.
[6] Khurana, D. & Lam, T.Y. (2004). Clean matrices and unit-regular matrices. J. Algebra, 280, 683-698.
[7] Wu, Y., Tang, G., Deng, G., & Zhou, Y. (2019). Nil-clean and unit-regular elements in certain subrings of $\mathrm{M}_2(\mathbb{Z})$. Czechoslovak Math. J., 69(1), 197-205.

Gaussian-Hybrid Numbers Obtained from Padovan and Jacobsthal-Padovan Numbers

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ABSTRACT

In the present paper, we first study the Gaussian Padovan hybrid numbers and Gaussian Jacobsthal-Padovan hybrid numbers. We give some new results for the Gaussian Padovan hybrid numbers and Gaussian Jacobsthal-Padovan hybrid numbers, including relations with the Padovan and Jacobsthal-Padovan numbers, and also give some new results for the Gaussian Padovan hybrid numbers and Gaussian Jacobsthal- Padovan hybrid numbers. Moreover, we obtain the recurrence relations, Binet-like formulas, Cassini-like identities for the Gaussian Padovan hybrid numbers and Gaussian Jacobsthal-Padovan hybrid numbers, respectively.

Mathematics Subject Classification: 11B39, 11B83, 11C08

Key Words : Padovan sequence, Jacobsthal-Padovan sequence, Gauss Padovan sequence, Gauss Jacobsthal-Padovan sequence.

REFERENCES

[1] Kızılateş C., A New Generalization of Fibonacci Hybrid and Lucas Hybrid Numbers, Chaos, Solitons & Fractals, 130, 2020.
[2] D. Jhala, K. Sisodiya, G.P.S. Rathore, On some identities for *k*-Jacobsthal numbers, Int. J. Math. Anal. (Ruse) 7(12),551-556, 2013.
[3] Mangueira, M. C. dos Santos, Vieira, R. P. M., Alves, F. R. V. ve Catarino, P. M. M. C., The Hybrid Numbers of Padovan And Some Identities, Annales Mathematicae Silesianae 34(2), 256–267,2020.
[4] M. Özdemir, Introduction to hybrid numbers, Adv. Appl. Clifford Algebra, 28(1), 91-98, 2018. M. Akar, S. Yüce, S. Şahin, On the dual hyperbolic numbers

Laplacian Spectrum of Some Special Graphs

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ABSTRACT

In this work, we study Laplacian spectrum of specific graph families constructed by applying join (∇) and direct product (x) operators on the complete graph (K_n), the complete bipartite graph ($K_{n,n}$), the hypercube graph (Q_n) and the cycle graph (C_n) for some integer n. In particular, we explicitly compute the complete Laplacian spectrum of the following graphs and determine for which $n_0, n_1, n_2, c_1, c_2 \in \mathbb{Z}^+$ they can be calculated:

$$\text{(i) } G = K_{n_0} \times (c_1 K_{n_1} \nabla c_2 K_{n_2}),$$

$$\text{(ii) } G = K_{n_0,n_0} \times (c_1 K_{n_1,n_1} \nabla c_2 K_{n_2,n_2}),$$

$$\text{(iii) } G = Q_{n_0} \times (c_1 Q_{n_1} \nabla c_2 Q_{n_2}),$$

$$\text{(iv) } G = C_{n_0} \times (c_1 C_{n_1} \nabla c_2 C_{n_2}).$$

We note that K_n, $K_{n,n}$, Q_n, C_n are among the few graphs whose Laplacian spectrum are known, see the monograph [1] for details. We have extended known Laplacian spectrum of families of graphs by using graph operations on these graphs by using results in [2,3]. It would be nice to have further graphs of families with known Laplacian spectrum, for instance by applying graph operators on different graph families.

Key Words: Laplacian spectrum, join operator, direct product.

REFERENCES

[1] D. M Cvetkovic, P. Rowlinson, and S. Simic. An introduction to the theory of graph spectra, volume 75. Cambridge University Press Cambridge, 2010.
[2] S. Barik, R. B. Bapat, and S. Pati. On the Laplacian spectra of product graphs. Applicable Analysis and Discrete Mathematics, 9(1):39–58, 2015.
[3] R. Merris. Laplacian matrices of graphs: a survey. Linear Algebra and its Applications, 197:143-176, 1994.

On Horadam Finite Operator Bicomplex Numbers

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ABSTRACT

In the fields of applied sciences and mathematics, particular number sequences are significant. Fibonacci, Lucas, Pell and Jacobsthal sequences are only some of the examples of particular number sequences that have numerous applications in the fields of geometry, engineering, architecture, music and art. Horadam number sequences are also a generalization of these sequences.

Bicomplex numbers are created four dimensional real vector space with a multiplicative operation. These numbers have played immensely to the fields of physical science, differential geometry and theory of relativity. Many researchers have studied bicomplex numbers whose coefficients belong to a particular number sequence.

Several famous operators including the identity operator, the forward difference operator, the backward difference operator, the means operator and the Gould operator are generalized by the finite operator. We note that there are many uses for these operators in applied mathematics, physics, and engineering. Furthermore, finite operators are extensively employed in computations by numerous researchers in different fields of study.

In this work, we define a new family of the Horadam bicomplex numbers by using finite operators. This family is a new generalization of the Horadam bicomplex numbers. These numbers are referred to as the Horadam finite operator bicomplex numbers. We give a variety of conclusions for the Horadam finite operator bicomplex numbers included Binet-like formula, generating function, exponential generating function, Catalan's identity, Cassini's identity, d'Ocagne's identity and numerous binomial-sum identities. Then we describe two new matrices. By these matrices, we

construct a matrix whose entries are Horadam finite operator bicomplex numbers and obtain the Cassini's identity.

Key Words: Bicomplex Horadam numbers, finite operator, matrix representation.

REFERENCES

[1] S. Halici, On bicomplex Fibonacci numbers and their generalization, Models and Theories in Social Systems, (2019), 179, 509-524.
[2] A.F. Horadam, Basic properties of a certain generalized sequence of numbers, The Fibonacci Quarterly, (1965), 3(3), 161-176.
[3] A.F. Horadam, Generating functions for powers of a certain generalised sequence of numbers, Duke Mathematical Journal, (1965), 32(3), 437-446.
[4] C. Kızılateş, New families of Horadam numbers associated with finite operators and their applications, Mathematical Methods in the Applied Sciences, (2021), 44(18), 14371-14381.
[5] Y. Simsek, Construction method for generating functions of special numbers and polynomials arising from analysis of new operators, Mathematical Methods in the Applied Sciences, (2018), 41(16), 6934-6954.
[6] C. Segre, Le rappresentazioni reali delle forme complesse e gli enti iperalgebrici, Mathematische Annalen, (1892), 40(3), 413-467.
[7] Y. Simsek, Some new families of special polynomials and numbers associated with finite operators, Symmetry, (2020), 12(2), 237.
[8] N. Terzioğlu, C. Kızılateş, W.S. Du, New properties and identities for Fibonacci finite operator quaternions, Mathematics, (2022), 10(10), 1719.

On Iterative Relations in Pell Subscripts

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ABSTRACT

In this study, we define iterative relations in Pell subscripts using Pell numbers, and examine the divisibility properties of these iterative relations. Furthermore, despite the computational difficulty of determining the divisibility of very large Pell numbers, the study possible to obtain results through theoretical methods.

Thus, we have successfully generalized Desmond's work on the Fibonacci sequence by using Pell numbers to develop a dissimilar approach.

Theorem 28. For $\alpha, \gamma \in \mathbb{N}$, the following equation is valid:

$$\left(P_{\alpha P_{\underbrace{\alpha P_{\alpha_{\ddots P_{\alpha P_{\alpha\gamma}}}}}_{n}}} \right)^{t} \text{ divides } P_{\alpha P_{\underbrace{\alpha_{\cdot_{\alpha P_{\alpha_{\ddots P_{\left(P_{\alpha P_{\underbrace{\alpha P_{\alpha_{\ddots P_{\alpha P_{\alpha\gamma}}}}}_{n}}} \right)}}}}}}^{t-k}}_{rn}},$$

The propositions and results obtained in the study allow us to prove this theorem. Through the analysis of iterative relations and their divisibility properties, we have extended the theoretical framework to include Pell numbers, effectively overcoming the computational challenges associated with large terms in the sequence.

Key Words: Pell numbers, divisibility.

REFERENCES

[1] T. Koshy, Pell and Pell-Lucas Numbers, Springer, New York, 2014.
[2] J. E. Desmond, On Iterative Fibonacci Subscripts, The Fibonacci quarterly, 9(1971),35.
[3] J. Vinson, The relation of the period modulo *m* to the rank of apparition of *m* in the Fibonacci sequence, the Fibonacci quarterly, 1 (1963),37-45.

On Split Quaternions

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ABSTRACT

Quaternions theory and characterizations have an important place in terms of algebraic, geometric and application areas. Kinematics, a branch of mechanics, only examines the displacement of a point or point system depending on time. In terms of examining movements in kinematics; rotational motion, sliding motion and general motions can be expressed with the theory of quaternions. Quaternions are a number system that is increasingly used in many fields such as geometry, physics, kinematics, vector analysis, medicine, computers, animation, image enhancement and robotics. Quaternions, a 4-dimensional number system obtained by generalizing complex numbers, were defined by Irish mathematician Sir William Rowan Hamilton (1805-1865) in 1843. Hamilton, who tried to find an algebraic representation to perform all operations corresponding to complex numbers, achieved the desired result by giving up the mutability of multiplication. The set of real quaternions given as an expansion of 4-dimensional complex numbers forms a quaternion algebra. Hamılton defines quaternions that have properties different from complex and real numbers; he showed that division was possible for two vectors and introduced a new multiplication operation into vector algebra.

By defining a new product on the quaternions set, examining movements in Euclidean space has been made easier. Quaternions have many applications in spherical geometry, and the most common ones in the literature are real, dual and split quaternions.

Split quaternions, defined by James Cockle in 1849, are a type of quaternion obtained by taking $e_1^2 = -1, e_2^2 = e_3^2 = 1$ in the real quaternions set. The algebra of split quaternions, which is associative but not commutative, is not a quotient algebra because it has zero divisors. The presence of zero divisors divides this type of quaternion into classes. In particular, rotations in 3-dimensional Lorentzian space are expressed by unit time-like split quaternions. Thus, similar to spherical

geometry, movements on unit hyperboloids are studied with split quaternions and provide many application areas.

In this study; based on the studies in the literature, split quaternions on the real number field will be examined. By reviewing the basic definitions and operations on these types of quaternions; the given theory will be expressed and proven with new methods and applied to examples.

Key Words: Quaternion, quaternion algebra, split quaternion, split quaternion algebra, split quaternion matrices.

REFERENCES

[1] Cockle, J. (1849). On Systems of Algebra Involving More Than One Imaginary, Philosophical Magazine 35(3).
[2] O' Neil, B. (1983). Semi Reimannian Geometry. Academic Press, New York, London.
[3] Erdoğdu, M. (2015). Split Quaternion Matrices, Akdeniz University, Graduate School of Natural and Applied Sciences, Ph.D. Thesis, Antalya.
[4] Jafari, M. and Yaylı, Y. (2014). Matrix Theory Over the Split Quaternions, Int.J. of Geometry, 3.2, 57-69.
[5] Kula, L. and Yaylı, Y. (2007). Split Quaternions and Rotations in Semi Euclidean Space, Journal of Korean Mathematical Society, 44, 1313-1327.
[6] Hacısalihoğlu, H.H. (1983). Motion Geometry and Quaternions Theory, Gazi University Faculty of Arts and Science Publications, Math. No.2, Ankara.
[7] Özdemir, M. (2020). Quaternions and Geometry, Golden Point Publishing, 1st Ed., İzmir.
[8] Kula, L. (2003). Split Quarternions and Geometrical Applications. Ankara University, Graduate School of Natural and Applied Sciences, Ph.D. Thesis, 116 s., Ankara.
[9] Agrawal, O.P. (1987). Hamilton Operators and Dual Number-Quaternions in Spatial Kinetics., Mechanisms and Machine Theory, Vol.22, 569-575.
[10] Ata, E. and Yaylı, Y. (2009). Split Quaternions and Semi-Euclidean Projective, Chaos, Solitons & Fractals, 41, 1910-1915.
[11] Koru Yücekaya, G. (2014). Matrix Representation on Quarternion Algebra, Turkish Journal of Mathematics and Computer Science (TJMCS), Article ID 20140039, 7 pages.
[12] Çetinkaya, H., Koru Yücekaya, G. (2022). On Matrixes of Commutative Quaternions, ICMME 2022, Denizlİ
[13] Koru Yücekaya, G., Orman F. (2023). Algebra of Elliptical Quarternion, Supplement Issue 4, Vol.16, ASETMJ.

On the spectrum and energy of normalized Seidel Laplacian matrix

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ABSTRACT

Let G be a simple, undirected graph with n vertices and m edges. Let $v_1, v_2, \dots, v_n$ be the vertices of G. The degree of a vertex v_i is the number of edges incident to it and is denoted by d_i. The adjacency matrix of a graph G is the square matrix $A(G) = (a_{ij})$, in which $a_{ij} = 1$ if v_i is adjacent to v_j and $a_{ij} = 0$ otherwise. The energy of a graph is introduced in [2] as $E(G) = \sum_{i=1}^{n} |\alpha_i|$, where α_i are the eigenvalues of adjacency matrix $A(G)$.

Let $D(G) = diag(d_1, d_2, \dots, d_n)$ be the diagonal matrix of vertex degrees. The Laplacian matrix of G is defined as $L(G) = D(G) - A(G)$. Let $\beta_1, \beta_2, \dots, \beta_n$ be its eigenvalues. Then the Laplacian energy of G is introduced as $LE(G) = \sum_{i=1}^{n} \left|\beta_i - \frac{2m}{n}\right|$ [3].

The normalized Laplacian matrix of graph G is defined as $\mathcal{L}(G) = D(G)^{-1/2} L(G) D(G)^{-1/2}$. Let $\theta_1, \theta_2, \dots, \theta_n$ be its eigenvalues. Then the normalized Laplacian energy of G is $E_{\mathcal{L}}(G) = \sum_{i=1}^{n} |\theta_i - 1|$.

The normalized Laplacian, a matrix popularized by Fan Chung in her book Spectral Graph Theory, has applications in random walks [1].

The Seidel matrix of a graph G is the real symmetric matrix $S(G) = (s_{ij})$, where $s_{ii} = 0$ on the diagonal, $s_{ij} = -1$ if v_i and v_j adjacent and 1 otherwise. Note that $S(G) = A(\bar{G}) - A(G)$, where $\bar{G}$ is the complement of the graph G. Let $\mu_1, \mu_2, \dots, \mu_n$ be the eigenvalues of Seidel matrix. The Seidel energy of G is introduced as [4] $E_S(G) = \sum_{i=1}^{n} |\mu_i|$.

It is noteworthy that Seidel matrices were introduced in [5] as a tool for studying equiangular line systems in Euclidean spaces.

Let $D_S(G) = diag(n-1-2d_1, n-1-2d_2, \dots, n-1-2d_n)$ be a diagonal matrix.

Then the Seidel Laplacian matrix of G is defined as [6] $S_L(G) = D_S(G) - S(G)$. Also, $D_S = D(\bar{G}) - D(G)$ and $S_L(G) = L(\bar{G}) - L(G)$. The eigenvalues $\mu_1^L, \mu_2^L, \dots, \mu_n^L$ of $S_L(G)$ are called as the Seidel Laplacian eigenvalues of G. Further, the Seidel Laplacian energy of G is $E_{S_L}(G) = \sum_{i=1}^{n} \left|\mu_i^L - \frac{n(n-1)-4m}{n}\right|$.

In this study, we firstly define the normalized Seidel Laplacian matrix and normalized Seidel Laplacian energy of G as $S^{\mathcal{L}}(G) = D_S^{-1/2}(G) S_L(G) D_S^{-1/2}(G)$ and $E_{S^{\mathcal{L}}}(G) = \sum_{i=1}^{n} |\partial_i^{\mathcal{L}} - 1|$, where $\partial_i^{\mathcal{L}}$'s are the eigenvalues of normalized Seidel Laplacian matrix $S^{\mathcal{L}}(G)$, respectively. These are analogous to the normalized Laplacian matrix and normalized Laplacian energy. The connections between normalized Seidel Laplacian matrix $S^{\mathcal{L}}(G)$ and normalized Laplacian matrix $\mathcal{L}(G)$ are also given. We use twin vertices to determine eigenvalues of normalized Seidel Laplacian matrix $S^{\mathcal{L}}(G)$ and establish the some properties of the eigenvalues of normalized Seidel Laplacian matrix $S^{\mathcal{L}}(G)$ and of normalized Seidel Laplacian energy $E_{S^{\mathcal{L}}}(G)$.

Key Words: normalized Seidel Laplacian matrix, normalized Seidel Laplacian eigenvalues, normalized Seidel Laplacian energy.

REFERENCES

[1] F.R.K. Chung, Spectral Graph Theory, American Math. Soc., Providence, 1997.
[2] I. Gutman, The energy of a graph, Ber. Math. Stat. Sekt. Forschungsz. Graz, 103 (1978), 1–22.
[3] I. Gutman, B. Zhou, Laplacian energy of a graph, Linear Algebra Appl. 414 (2006) 29–37.
[4] W. H. Haemers, Seidel switching and graph energy, MATCH Commun. Math. Comput. Chem., 68 (2012) 653–659.
[5] van Lint JH, Seidel JJ. Equilateral point sets in elliptic geometry. Indag Math. 1966;28:335–348
[6] H. S. Ramane, R. B. Jummannaver and I. Gutman, Seidel Laplacian energy of graphs, Int. J. Appl. Graph Theory, 1(2) (2017), 74–82.
[7] C. Reinhart, The normalized distance Laplacian, Spec. Matric. 9 (2021), 1–18.
[8] S. Tarhan, A.D. Maden, On the spectrum and energy of normalized Seidel Laplacian matrix, submitted.

On the Quaternion Padovan Numbers

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ABSTRACT

In the present work, we explore the concept of quaternion Padovan sequences, extending the classical Padovan sequence into the realm of quaternions. Quaternions, introduced by Hamilton in 1843, are a number system that generalizes complex numbers into four dimensions, with applications in various fields such as quantum physics, computer graphics, and three-dimensional rotations. The study begins by reviewing the definition and properties of quaternions, including their non-commutative multiplication rules and the formulation of quaternion numbers.

The Padovan sequence is defined by the recurrence relation $P_{n+3}=P_{n+1}+P_n$ with initial conditions $P_0=P_1=P_2=1$. This sequence is generalized to quaternions by defining quaternion Padovan numbers, denoted $Q_P(a,b,c,d)$, which follow a similar recurrence relation for each coordinate of the quaternion. The study provides detailed recurrence relations and initial conditions for these quaternion Padovan numbers and derives several new identities and plastic-like ratio. Furthermore, the study introduces the concept of quaternion Padovan sequences in four dimensions and investigates their algebraic properties, including the calculation of the norm and conjugate of these quaternion sequences.

Key Words: Padovan numbers, quaternions.

REFERENCES

[1] C. J. Harman, Complex Fibonacci numbers, The Fibonacci Quarterly, 19(1) (1981) 82–86.
[2] A.F. Horadam, Complex Fibonacci Numbers and Fibonacci Quaternions, Amer. Math. Monthly, 70 (1963) 289–291.
[3] A. F. Horadam, Quaternion recurrence relations. Ulam Quarterly, 2(2) (1993), 23–33.

On Wiener Indices of Circular Graphs

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ABSTRACT

The concept of Wiener index was introduced by H. Wiener (Wiener, 1947) for the study of physical, chemical and some pharmacological properties of molecules. It is defined as the sum of the distances between all pairs of nodes in a graph, with the ordering of node pairs being irrelevant. Recent studies have shown that the Wiener index has become an important parameter for some special graphs. Please refer to the references given for details. (Walikar vd., 2004), (Cohen vd., 2010), (Li ve Song, 2014), (Knor vd., 2015)

In this study, Wiener indices of circular graphs are analyzed and the limits of Wiener indices of circular graphs are determined. Machine learning, which has recently replaced artificial neural networks (YSA) with artificial intelligence, has transformed the clustering algorithm into two-cluster graphs called input-output layers. Thus, circular graphs and their indices, which are defined by considering the number of common neighborhoods of finite bipartite graphs, will shed light on many academic studies.

Key Words: Circular Graph, Wiener İndex, Bipartite Graph

REFERENCES

[1] Cohen, N., Dimitrov D., Krakovski R., Skrekovski R. ve Vukašinović V, 2010, On Wiener index of graphs and their line graphs, Match Communications in Mathematical and in Computer Chemistry, 64(3): 683–698.
[2] Knor, M., Luzar, B., Skrekovski, R., Gutman, I., 2014, On Wiener index of common neighborhood graphs, MATCH Commun. Math. Comput. Chem., 72(1) : 321–332.
[3] Li, S., Song, Y., 2014, On the sum of all distances in bipartite graphs, Discrete Applied Mathematics, 169: 176-185.
[4] Walikar, H.B., Shigehalli V.S., Ramane H.S., 2004, Bounds on the Wiener number of a graph, Match-Communication In Mathematical And In Computer Chemistry, 50: 117–132.
[5] Wiener, H., 1947, Structural determination of paraffin boiling points, Journal of the American chemical society, 69(1): 17–20.

q-Analogues of Some Leonardo Polynomials

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ABSTRACT

Leonardo Pisano numbers are defined by the following recurrence relation for $n \geq 2$

$Le_n = Le_{n-1} + Le_{n-2} + 1$

with the initial conditions $Le_0 = Le_1 = 1$ in [4]. There is a relation between the Leonardo numbers and the Fibonacci numbers for $n \geq 0$ such that $Le_n = 2F_{n+1} - 1$. The Leonardo Numbers are generalized in a variety of ways by different researchers see [1-11].

In this presentation, we introduced the q-analogues of Leonardo polynomials called the q-Leonardo Pisano polynomials. Also we define q-Leonardo Lucas polynomials of first and second kind. We obtained explicit formulas of these polynomials, generating functions, some summations properties of these polynomials. We show that there are connections between the q-Leonardo Pisano polynomials and q-Leonardo Lucas polynomials.

Key Words: Leonardo polynomials, Fibonacci polynomials, q -analogue.

REFERENCES

[1] Y. Alp and Y. E.G. Koçer, Hybrid Leonardo numbers. Chaos Solitons Fract. 150 (2021), 111-128.
[2] Y. Alp and Y. E.G. Koçer, Some properties of Leonardo numbers, Konuralp J. Math. 9(1) (2021), 183–189.
[3] Y. Alp, Some properties of the generalized Leonardo numbers, Journal of New Theory 47 (2024), 52-60.
[4] P. Catarino and A. Borges, On Leonardo numbers, Acta Math. Univ. Comenian. 89(1) (2019), 75–86.
[5] P. Catarino and A. Borges, A note on incomplete Leonardo numbers, Integers 20 (2020), A43.
[6] F. Kuruz, A. Dagdeviren, P. Catarino, On Leonardo Pisano hybrinomials. Mathematics 9(22) (2021).
[7] M. Mangueira, R. Vieira, F. Alves and P. Catarino, A generalização dos duais e sedenions de Leonardo, C.Q.D.-Rev. Eletrôn. Paul. Mat. 20 (2021), 13–27.

[8] H. Özimamoğlu, A new generalization of Leonardo hybrid numbers with q-integers, Indian J Pure Appl. Math. 55 (2024), 325–334.
[9] A.G. Shannon, A note on generalized Leonardo numbers, Notes Number Theory Discrete Math. 25(3) (2019), 97–101.
[10] R. Vieira, F. Alves and P. Catarino, Relações bidimensionais e identidades da sequência de Leonardo, Rev. Sergipana Mat. Educ. Mat. 4(2) (2019), 156–173.
[11] E. Tan, and H.H. Leung, On Leonardo p-numbers. Integers 23 (2023), A7.

The Degree of *M*-ambiguity of the Parikh Matrix

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ABSTRACT

The Parikh matrix mapping was introduced by Mateescu et al. [3] as generalization of Parikh mapping. The Parikh matrix of a word is an upper triangular matrix which gives information on the number of occurrences of certain subwords of that word. In general, a Parikh matrix does not uniquely determine a word. Words having the same Parikh matrix *M* constitute the class of *M*-equivalent words. *M*-equivalent binary words are studied in [1, 2]. The degree of *M*-ambiguity of a Parikh matrix *M* is the number of *M*-equivalent words. In [4], an upper bound polynomial for the degree of *M*-ambiguity of an arbitrary word is investigated.

In this talk, we will give the degree of M-ambiguity of some Parikh matrices for the binary alphabet.

Key Words: Parikh matrix, *M*-equivalent words, degree of *M*-ambiguity.

REFERENCES

[1] Atanasiu, C. Martin-Vide and A. Mateescu, On the injectivity of the Parikh matrix mapping, Fundamenta Informaticae, 49 (4) (2002), 289-299
[2] A. Atanasiu, Binary amiable words, Int. J. Found. Comput. Sci. 18 (2007), 387-400
[3] A. Mateescu, A. Salomaa, K. Salomaa and S. Yu, A sharpening of the Parikh mapping, Theoretical Informatics and Applications, 35 (2001), 551-564
[4] A. Salomaa, Parikh matrices: Subword indicators and degrees of ambiguity, In Adventures between lower bounds and higher altitudes, Lectures Notes in Computer Science, 11011, Springer, Cham (2018), 100-112.

Tripartite Number Set (Space Numbers): Birth of New Number Set/ New concept to Infinity & Math Limits

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ABSTRACT

A new number set called "***Tripartite Number Set***" lead to "**Space numbers set**" works as third dimension number set above and beyond the ***real*** and ***imaginary*** in complex number sets that can solve the problem of division by zero (Infinity) and make clear understanding for the "Infinity", supports the concept of "Math's Limits". it's an extension to the existing number sets, which includes the complex numbers as subset for it.

Tripartite Numbers are numbers whose division by zero are exist. They are existing in a third line in space above and beyond the Real Line numbers and the Imaginary line numbers (Plane of Complex numbers), where each complex number in its plane divided by zero find its place in the third-dimension line, and written as: **(x+ yi) *zt**, where x, y and z are real numbers and i is the imaginary unit (i= **√−1)**. & t is the tripartite/infinite unit (t =1/0).

So, we have the number sets *N* → Z → Q → R → C, and now

S:*N* → Z → Q → R → C → S

Natural numbers Integers Rationales Real numbers complex numbers Space numbers

(on Line) (on Line) (on Line) (complete Line) (2D Plane) (3D Space)

A Tripartite number T ={(**at**): **a** is real number., **t** =1/0}, a/0=at, and a (x+ yi)/0 ≡ (x +yi) *at.

A **Space number S** = {(x + yi + zt), x, y, & z are real, i= √**−1**. t =1/0}.

Keywords: Tripartite numbers, Space numbers, Infinite numbers, Division by zero.

Writing the product of consecutive Lucas numbers as repdigits

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ABSTRACT

Let $(F_n)_{n\geq 0}$ be Fibonacci sequence given by the relation $F_n = F_{n-1} + F_{n-2}$ with $F_0 = 0$ and $F_1 = 1$. Lucas sequence $(L_n)_{n\geq 0}$ that satisfies the same recurrence relation with the initial conditions $L_0 = 2$ and $L_1 = 1$. There are lots of combinatorial identities about them.

It is proven by Luca that 55 is the largest repdigit Fibonacci number and 11 is the largest repdigit Lucas number. Redigits have the form $a(10^m - 1)/9$, for some $m \geq 1$ and $1 \leq a \leq 9$. Moreover, Marques and Togbe proved that there is no repdigit number written as the product of Fibonacci numbers with at least two digits. We find repdigit number written as the product of Lucas numbers with at least two digits. Namely, we solve the following equation

$$L_n \dots L_{n+k-1} = a\left(\frac{10^m - 1}{9}\right),$$

for some $m \geq 1,\ k \geq 2$ and $1 \leq a \leq 9$ are integers..

Key Words: Repdigits, Lucas numbers, Diophantine equations.

REFERENCES

[1] F. Luca, Fibonacci and Lucas numbers with only one distinct digit, Potugal. Math. 50, 243-254, 2000.

[2] D. Marques, A. Togbe, On repdigits as product of consecutive Fibonacci numbers. Rendiconti dell'Istituto di Matematica dell'Universita di Trieste, v. 44, p. 393-397, 2012

ANALYSIS

A Survey on Reproducing Kernel Hilbert Spaces

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ABSTRACT

This study is a master's thesis. The content of this thesis consists of general information about reproducing kernel Hilbert spaces, which are widely used in various fields such as Mathematics, Statistics, and machine learning. In this study, we first introduced the concept of a reproducing kernel Hilbert space (shortly RKHS) and provided the definition of a reproducing kernel. We also discussed the characteristic property of the reproducing kernels, gave the statement and a brief proof of the Moore-Aronszajn Theorem, which is one of the classical theorems in the theory of reproducing kernel Hilbert spaces. Next, we explored how to construct a reproducing kernel Hilbert space from a given kernel function in some concrete cases. Finally, we briefly discussed several applications of reproducing kernel Hilbert spaces, including their use in interpolation-approximation theory, statistics, and machine learning.

Key Words: reproducing kernel Hilbert space, positive semidefinite kernel, interpolation- approximation.

REFERENCES

[1] Paulsen, Vern I., and Mrinal Raghupathi. An introduction to the theory of reproducing kernel Hilbert spaces. Vol. 152. Cambridge university press, 2016

[2] Kreyszig, Erwin. "Introductory functional analysis with applications. Jonh Wiley & Sons." Inc., New York (1978).

B-fractional maximal operator in the B-local Morrey-Lorentz spaces

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ABSTRACT

The harmonic analysis theory consists of important operators such as singular integral, maximal function, sharp maximal function, fractional maximal operator, Riesz potential, convolution type operators and approximate identities. In various function spaces, the boundedness of these operators and their versions which are generated by the Laplace-Bessel differential operator have been examined and the boundedness problems play a significant role in this theory.

Lorentz spaces, which are very useful in the theory of interpolation, have first been introduced by Lorentz [3]. These spaces are Banach spaces and generalizations of Lebesgue spaces. The Lorentz space $L_{p,q}(\mathbb{R}^n)$, $0 < p, q \leq \infty$, is known as the set of all measurable functions f such that

$$\|f\|_{L_{p,q}(\mathbb{R}^n)} = \left\| t^{\frac{1}{p}-\frac{1}{q}} f^*(t) \right\|_{L_q(0,\infty)} < \infty,$$

where, by f^* we denote the nonincreasing rearrangement of f and defined as

$$f^*(t) = \inf \{ \lambda > 0 : |\{ y \in \mathbb{R}^n : |f(y)| > \lambda \}| \leq t \}, \qquad t \in (0, \infty).$$

Local Morrey-Lorentz spaces $M_{p,q}^{loc}(\mathbb{R}^n)$, which are generalizations of the Lorentz spaces, have first been introduced by Aykol et al. [1]. Also, they have obtained that the maximal operator is bounded in the local Morrey-Lorentz spaces [1].

In this talk, we present the necessary and sufficient conditions for the boundedness of the fractional maximal operator generated by the Laplace-Bessel differential operator (B-fractional maximal operator) M_γ^α in the B-local Morrey-Lorentz spaces with the use of generalized Hardy type operators and sharp rearrangement inequalities. Also, it is easy to observe that $M_\gamma^\alpha = M_\gamma f$ for $\alpha = 0$ [2]. Consequently, we give the boundedness of the B-maximal operator M_γ in the B-local Morrey-Lorentz spaces $M_{p,r,\lambda,\gamma}^{loc}(\mathbb{R}_{k,+}^n)$.

Key Words: Local Morrey-Lorentz spaces, fractional maximal operator, Hardy type operator, $\Upsilon-$rearrangement.

REFERENCES

[1] C. Aykol, V.S. Guliyev and A. Serbetci, Boundedness of the maximal operator in the local Morrey-Lorentz spaces, J. Inequal. Appl., 2013(1) (2013), 1-11
[2] V.S. Guliyev, On maximal function and fractional integral, associated with the Bessel differential operator, Math. Inequal. Appl., 6(2) (2003), 317-330
[3] G.G. Lorentz, Some new function spaces, Ann. Math., 51(1) (1950), 37-55.

Compactness of the Weyl type operator for $p \leq q$

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ABSTRACT

In this scientific paper, the results of the necessity and sufficiency conditions of the fact that the operator T for the case $p \leq q$ is compactness from the weighted Lebesgue Space $L_{p,w} = L_{p,w}(I)$ to weighted Lebesgue space $L_{q,v} = L_{q,v}(I)$ were obtained, where $L_{p,w} = L_p(w, I)$ is the set of all measurable functions f in the interval $I = (a, b)$, $0 \leq a < b \leq \infty$ for the functions f in the space $L_{p,w} = L_p(w, I)$ the norm is finite:

$$\|f\|_{p,w} := \left(\int_a^b |f(x)|^p w(x) dx \right)^{\frac{1}{p}} < \infty.$$

$I = (a, b)$, $0 \leq a < b \leq \infty$, $0 < \alpha < 1$ and u and v almost everywhere let be locally integrable and positive functions on the interval I. Also, let be, $1 < p < \infty$,

$0 < q < \infty$ and $\frac{1}{p} + \frac{1}{p'} = 1$.

Let us denote all functions $f: I \to R$ -measurable in the interval I.

Moreover, $W: I \to R$ is non-negative, strictly increasing and let I be a locally absolutely continuous function on the interval. For all $x \in I$ where $\frac{dW(x)}{dx} = w(x)$.

Accordingly, we map the operator T from the space $L_{p,w} = L_{p,w}(I)$ to the space $L_{q,v} = L_{q,v}(I)$ such that:

$$Tf(x) := \int_x^b \frac{\left(ln\left(\frac{W(s)}{W(s) - W(x)} \right) \right)^{\beta} u(s) W^{\gamma}(s) f(s) w(s) ds}{\left(W(s) - W(x) \right)^{1-\alpha}},$$

consider, where $x \in I, 0 < \alpha < 1, \gamma \leq \beta \leq 0$. The evaluation of this operator in the

interval (a,x) with $\gamma = 0$ is considered in the scientific paper [1].

If $u \equiv 1, \beta = 0, \gamma = 0,$ the independent condition of our operator T, the operator K

becomes a fractional order integral operator of the function f with respect to the function W:

$$Kf(x) := \int_x^b \frac{f(s)w(s)ds}{(W(x) - W(s))^{1-\alpha}},$$

where $x \in I$. The fact that this operator K is measured from the space $L_{p,w}$ to the space $L_{q,v}$ was obtained in the scientific paper [2].

If the operator K contains $W(x) \equiv x$, this operator becomes the Weyl operator:

$$I_\alpha^* f(x) := \int_x^b \frac{f(s)ds}{(s-x)^{1-\alpha}}, x \in I$$

this operator

$$I_\alpha g(s) := \int_a^s \frac{g(x)dx}{(s-x)^{1-\alpha}}, x \in I$$

The Riemann-Liouville operator becomes a dual operator.

Theorem. Let $\frac{1}{\alpha} < p \leq q < \infty,$ $0 < \alpha < 1, \gamma \leq \beta \leq 0,$ and let u be a non−decreasing (positive) function in the interval I. Then the operator T is compact from the space $L_{p,w}$ to the space $L_{q,v}$ if and only if $A = \sup_{z \in I} A(z) < \infty$ and $\lim_{z \to a^+} A(z) = \lim_{z \to b^-} A(z) = 0$, where:

$$A(z) = \left(\int_a^z W^{q\beta}(x)v(x)dx \right)^{\frac{1}{q}} \left(\int_z^b u^{p'}(s)W^{p'(\gamma+\alpha-\beta-1)}(s)w(s)ds \right)^{\frac{1}{p'}}.$$

Key words: Compactness; weight function; Logarithmic singularity.

REFERENCES

[1] A.M. Abylayeva, R. Oinarov and B.Seilbekov, Boundedness and compactness of a class of integral operators with power and logarithmic singularity when $p \leq q$. // Journal of Inequalities and Applications – 2022, – №.23.

[2] A.M. Abylayeva Boundedness and compactness for a class of fractional integration operators of Weyl type. // Eurasian Mathematical. J. – Volume 7, – 2016, – No.1, – C. 9–27.

Convergence Theorems of New Iterative Scheme for Nonexpansive Mappings in Banach Space

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ABSTRACT

Fixed point theory is one of the most powerful and fruitful tools in of modern mathematics. It also acts as a bridge between Analysis and Topology and provides a very productive area of interaction between the two. Thus, this theory has been investigated extensively in past two decades by numerous researchers.

Our main focus in this paper is to consider a new iterative sheme for finding the fixed point as follows:

Let Ebe a real uniformly convex Banach space, Kbe a closed convex nonempty subset of E which is also a nonexpansive mappings of$K$ into K. Let $\{a_n\},\{b_n\},\{c_n\},\{d_n\}$ be sequences in $(0,1)$. Let $\{x_n\}$ be a sequence generalized by

$$\begin{cases} x_1 \in K, \\ x_{n+1} = T[(1-a_n)u_n + a_n x_n], \\ u_n = (1-b_n)z_n + b_n T z_n, \\ z_n = (1-c_n)T x_n + c_n T y_n, \\ y_n = (1-d_n)x_n + d_n T x_n, n \geq 1. \end{cases}$$

Let $F(T) \neq \emptyset$. In this work, weak and strong convergence theorems of the iteration method given above for nonexpansive mapping in real uniformly convex Banach space were established. Moreover, weak convergence theorems making use of Opial's condition and Fréchet differentiable norm is proved. We also show some strong convergence theorems for the class of nonexpansive mappings. Our results improve and generalize these announced by Mann [Mean value methods in iteration, Proc. Am. Math. Soc.], Ishikawa [Fixed points by a new iteration method, Proc. Am. Math. Soc.], Agarwal et al. [Iterative construction of fixed points of nearly asymptotically nonexpansive mappings, J. Nonlinear Convex Anal] and Jubair et al. [Estimating Fixed Points via New Iterative Scheme with an Application. Journal of Function Spaces].

Key Words: Fixed point, nonexpansive mapping, Banach space.

REFERENCES

[1] E. Picard, Memoire sur la theorie des equations aux derivees partielles et la methode des approximations successives, J. Math. Pures Appl. 6 (1890) 145-210
[2] W. R. Mann, Mean value methods in iteration, Proc. Am. Math. Soc. 4 (1953) 506-510
[3] S. Ishikawa, Fixed points by a new iteration method, Proc. Am. Math. Soc. 44 (1974) 147-150
[4] R. P. Agarwal, D. O'Regon, D. R. Sahu, Iterative construction of fixed points of nearly asymptotically nonexpansive mappings, J. Nonlinear Convex Anal., 8 (2007), 61-79
[5] Jubair, M., Ali, J., & Kumar, S, Estimating Fixed Points via New Iterative Scheme with an Application. Journal of Function Spaces, 2022(1), 3740809.
[6] Z. Opial, Weak convergence of the sequence of successive approximations for nonexpansive mappings, Bull. Amer. Math. Soc. 73 (1967), 591-597.

Estimates of approximation numbers and completeness of root vectors of a singular operator generated by the linear part of the Korteweg-de Vries operator

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ABSTRACT

We study a linear operator generated by the linear part of the Korteweg-de Vries operator in the case of an unbounded domain with unbounded coefficients. The paper proves the existence of the resolvent, the separability of the operator (maximum regularity of solutions), the compactness of the resolvent. Also two-sided estimates of the distribution function of approximation numbers are obtained. As is known, that the estimates of approximation numbers show the speed of the best approximation of the resolvent of an operator by finite-dimensional operators.

In addition to these questions, the paper examines the question of the completeness of root vectors. As is known, the completeness of root vectors can be applied in the study of a nonlinear operator.

Consider the differential operator

$$(L+\mu I)u=\frac{\partial u}{\partial t}+\frac{\partial^3 u}{\partial x^3}+q(x)u+\mu u\,, \tag{1.1}$$

initially defined on $C^{\infty}_{0,\pi}(\overline{\Omega})$, where $\overline{\Omega}=\{(t,x): -\pi\le t\le\pi,\ -\infty<x<\infty\},\ \mu\ge 0$. $C^{\infty}_{0,\pi}$ is a set of infinitely differentiable functions satisfying the condition

$$u(-\pi,x)=u(\pi,x). \tag{1.2}$$

and compactly supported with respect to the variable x.

Further, we assume that the coefficient $q(x)$ satisfies the conditions:

i) $q(x)\ge\delta_0>0$ is a continuous function in R;

ii) $c_0=\sup\limits_{|x-y|\le 1}\dfrac{q(x)}{q(y)}<\infty$.

Note here that $q(x)$ can be an unbounded function at infinity.

The operator $L+\mu I$ admits closure in the space $L_2(\Omega)$, which we also denote by $L+\mu I$. For this operator in an unbounded domain with an unbounded coefficient, the following questions will be studied:

- the existence of the resolvent;
- separability (maximum regularity of solutions);
- compactness of the resolvent;
- estimates of singular (s-numbers) numbers.

Key Words: Korteweg-de Vries operator, resolvent, separability, singular (s-numbers) numbers.

REFERENCES

[1] M.B. Muratbekov, A.O. Suleimbekova. Compactness, estimates of eigenvalues and s-numbers of a resolvent for a linear singular operator of the Korteveg-de Vrise type. Filomat 36:11 (2022), 3691-3702. (Arial 10)

[2] M.B. Muratbekov, M.M. Muratbekov, K.N. Ospanov. Coercive solvability of an odd-order differential equation and its applications. Reports of the Academician of Sciences (RAS) 2010, vol. 435, No. 3, pp.1-3.

Hermite-Hadamard and Ostrowski type Inequalities via (m, n)-Exponential type Convex Functions with Applications

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ABSTRACT

Numerous scholars have placed a growing emphasis on the study of convex functions due to their wide-ranging and adaptable nature. This area of research has evolved and expanded in multiple directions, reflecting its broad applicability across various mathematical and applied fields. In recent years, an impressive volume of research papers has emerged, primarily focusing on integral inequalities linked to convex functions. The key aim of this paper is to present the notion of a convex function of (m, n)-exponential type and to derive enhanced versions of the well-known Hermite-Hadamard and Ostrowski-type inequalities by leveraging this newly introduced class of convex functions. Moreover, we delve into fresh special cases stemming from our analysis, showcasing how our findings not only generalize but also further develop and extend the results previously established in the literature. By offering refined versions of classical inequalities, our study contributes to the ongoing discourse on convex functions and their associated properties. These improvements, grounded in the (m, n)-exponential type convexity, open up novel avenues for exploring relationships within the realm of convex analysis. Additionally, the new cases highlighted in our work provide deeper insights, underscoring the versatility and utility of this generalization. Thus, our results serve as a bridge, connecting past research with potential future inquiries, and enhancing the theoretical framework for convex functions and their applications.

Key Words: Hermite Hadamard inequality, exponential type convexity, Ostrowski inequality, convex function, Holder's inequality.

REFERENCES

[1] Dragomir, S. S., and Pearce, C. (2003). Selected topics on Hermite-Hadamard inequalities and applications. Science direct working paper, (S1574-0358), 04.

[2] You, X. X., Ali, M. A., Budak, H., Agarwal, P., Chu, Y. M. (2021). Extensions of Hermite-Hadamard inequalities for harmonically convex functions via generalized fractional integrals. Journal of Inequalities and Applications, 2021 (1), 102.

[3] Khurshid, Y., Adil Khan, M., Chu, Y. M., Khan, Z. A. (2019). Hermite-Hadamard-Fejér Inequalities for Conformable Fractional Integrals via Preinvex Functions. Journal of Function Spaces, 2019(1), 3146210.

[4] Latif, M., Chu, Y. M. (2022). New weighted Hermite-Hadamard type inequalities for differentiable strongly convex and strongly quasi-convex mappings. J. Math. Inequal, 16, 1413-1428. AİMS matematik Feng, B., Ghafoor, M., Chu, Y. M., Qureshi, M. I., Feng, X., Yao, C., Qiao, X. (2020). Hermite-Hadamard and Jensen's type inequalities for modified (p, h)-convex functions. AIMS Math, 5(6), 6959-6971.

[5] Zhou, S. S., Rashid, S., Noor, M. A., Noor, K. I., Safdar, F., Chu, Y. M. (2020). New Hermite-Hadamard type inequalities for exponentially convex functions and applications. AIMS Math, 5(6), 6874-6901.

[6] Zhang, X. M., Chu, Y. M., Zhang, X. H. (2010). The Hermite-Hadamard type inequality of GA-convex functions and its application. Journal of Inequalities and Applications, 2010, 1-11.

[7] Khan, M. A., Chu, Y., Khan, T. U., Khan, J. (2017). Some new inequalities of Hermite-Hadamard type for s-convex functions with applications. Open Mathematics, 15(1), 1414-1430.

[8] Awan, M. U., Akhtar, N., Iftikhar, S., Noor, M. A., Chu, Y. M. (2020). New Hermite–Hadamard type inequalities for n-polynomial harmonically convex functions. Journal of Inequalities and Applications, 2020, 1-12.

[9] Latif, M. A., Hussain, S., Chu, Y. (2021). Generalized Hermite–Hadamard type inequalities for differentiable harmonically-convex and harmonically quasi-convex functions. J. Math. Inequal, 15(2), 755-766.

[10] Latif, M. A., Rashid, S., Dragomir, S. S., Chu, Y. M. (2019). Hermite–Hadamard type inequalities for co-ordinated convex and qausi-convex functions and their applications. Journal of Inequalities and Applications, 2019(1), 1-33.

[11] Maden, S., Kadakal, H., Kadakal, M., and İşcan, İ. (2017). Some new integral inequalities for n-times differentiable convex and concave functions. J. Nonlinear Sci. Appl, 10(12), 6141-6148.

[12] Özcan, S., and İşcan, İ. (2019). Some new Hermite–Hadamard type inequalities for s-convex functions and their applications. Journal of inequalities and applications, 2019, 1-11.

Hermite-Hadamard Inequalities for Conformable Fractional Integrals

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ABSTRACT

This study is on new midpoint-type and trapezoidal-type inequalities for Hermite-Hadamard inequalities for Ψ-Hilfer fractional integrals and for Hermite-Hadamard inequalities for Conformable fractional integrals, respectively. The study, consisting of four sections, begins in the first part with some necessary inequalities, definitions, and theorems. Additionally, in the second part, the proof of the Hermite-Hadamard inequality, inequalities for Riemann-Liouville fractional integrals, some inequalities for Ψ-Hilfer fractional integrals, and finally, some inequalities for Conformable fractional integrals are presented. In the second section, using the boundedness of the second derivative of the function f, new midpoint-type and trapezoidal-type Hermite-Hadamard inequalities are provided for Ψ-Hilfer fractional integrals. Similarly, in the third part, new midpoint-type and trapezoidal-type Hermite-Hadamard inequalities for Conformable fractional integrals are presented, supported by examples, again using the boundedness of the second derivative of the function f. In the fourth and final section, conclusions and some suggestions for future studies are provided.

Key Words: Hermite-Hadamard inequality, Fractional integrals, Bounded function, Midpoint type inequalities, Trapezoidal type inequalities, Conformable Fractional integrals, Ψ-Hilfer Fractional integrals.

REFERENCES

[1] S.S. Dragomir and R. P. Agarwal, *Two inequalities for differentiable mappings and applications to special means of real numbers and to trapezoidal formula*, Appl. Math. Lett. 11 (5) (1998) 91–95.

[2] A. Hyder, A.H. Soliman, *A new generalized θ-conformable calculus and its applications in mathematical physics*, Physica Scripta, 96, 2020, 015208.

[3] F. Jarad, E. Uğurlu, T. Abdeljawad, and D. Baleanu, *On a new class of fractional operators*, Advances in Difference Equations, (2017), 2017, 247.

[4] R. Khalil, M. Al Horani, A. Yousef, and M. Sababheh, *A new definition of fractional derivative*, Journal of Computational and Applied Mathematics, 264, 2014, 65-70.

[5] U. S. Kirmaci, *Inequalities for differentiable mappings and applications to special means of real numbers to midpoint formula*, Appl. Math. Comput., 147 (5), 2004, 137–146

[6] E. Set, J. Choi, and A. Gözpınar, *Hermite-Hadamard type inequalities for new conformable fractional integral operator*, ResearchGate Preprint, 2018, https://www.researchgate.net/publication/322936389

[7] D. Zhao, M. Luo, *General conformable fractional derivative and its physical interpretation*, Calcolo, 54, 2017, 903-917

[8] *Bernoulli Inequality* by Chris Boucher, Wolfram Demonstrations Project.

[9] D.K. Callebaut, *Generalization of the Cauchy–Schwarz inequality*, J. Math. Anal. Appl. 12 (3) (1965) 491–494. doi:10.1016/0022-247X(65)90016-8.

[10] S.S. Dragomir and C. Pearce, *Selected topics on Hermite-Hadamard inequalities and applications*, Mathematics Preprint Archive, vol. 2003, no. 3, pp. 463–817, 2003.

[11] D. S. Mitrinovic and P. M. Vasic, *Analytic inequalities*. New York, USA: Springer, 1970, vol. 61.

[12] B.G. Pachpatte, *Mathematical Inequalities*, vol. 67, Amsterdam, Holland: Elsevier Science, 2005, pp. 11-45.

[13] H. Minkowski, *Geometrie der Zahlen*, Chelsea, 1953.

[14] J.E. Pecaric and Y.L. Tong, *Convex functions, partial orderings, and statistical applications*, San Diego, USA: Academic Press, 1992.

[15] M.Z. Sarikaya and H. Yildirim, *On Hermite-Hadamard type inequalities for Riemann-Liouville fractional integrals*, Miskolc Mathematical Notes, vol. 17, no. 2, pp. 1049–1059, 2016.

[16] S.S. Dragomir, *Some inequalities of Hermite-Hadamard type for symmetrized convex functions and Riemann-Liouville fractional integrals*, RGMIA Res. Rep. Coll., vol. 20, p. 15, 2017.

[17] B.G. Pachpatte, *Analytic inequalities: recent advances*. Amsterdam, Paris: Atlantis Press, 2012, vol.

[18] A. Gözpınar, *Some Hermite-Hadamard type inequalities for convex functions via new fractional conformable integrals and related inequalities*, AIP Conf. Proc., 2018, 1991, 020006.

[19] A.A. Hyder, A.A. Almoneef, H. Budak, and M.A. Barakat, *On new fractional version of Generalized Hermite-Hadamard Inequalities*, Mathematics, 10(18), 2022, 3337. M. U. Awan, M. A. Noor, T. S. Du, and K. I. Noor, *New refinements of fractional Hermite-Hadamard inequality*, RACSAM, 113(1), (2019), 21-29.

[20] H. Budak, H. Kara, M. Z. Sarikaya, and M. E. Kiriş, *New extensions of the Hermite-Hadamard inequalities involving Riemann-Liouville fractional integrals*, Miskolc Mathematical Notes, 21(2), 2020, 665-678.

[21] H. Budak, E. Pehlivan, and P. Kosem, *On new extensions of Hermite-Hadamard inequalities for generalized fractional integrals*, Sahand Communications in Mathematical Analysis, 18(1), 2021, 73-88.

[22] F. X. Chen, *Extensions of the Hermite-Hadamard inequality for convex functions via fractional integrals*, J. Math. Inequal., 10(1), (2016), 75-81.

[23] F. X. Chen, *On the generalization of some Hermite-Hadamard inequalities for functions with convex absolute values of the second derivatives via fractional integrals*, Ukrainian Mathematical Journal, 12(70), (2019), 1953-1965.

[24] S. S. Dragomir, C. E. M. Pearce, *Selected topics on Hermite-Hadamard inequalities and applications*, RGMIA Monographs, Victoria University, 2000. Online: http://www.sta.vu.edu.au/RGMIA/monographs/hermite_hadamard.html.

[25] S. S. Dragomir, P. Cerone, and A. Sofo, *Some remarks on the midpoint rule in numerical integration*, Studia Univ. Babes-Bolyai, Math., XLV(1), (2000), 63-74.

[26] S. S. Dragomir, P. Cerone, and A. Sofo, *Some remarks on the trapezoid rule in numerical integration*, Indian J. Pure Appl. Math., 31(5), (2000), 475-494.

[27] M. Jleli, B. Samet, *On Hermite-Hadamard type inequalities via fractional integrals of a function with respect to another function*, Journal of Nonlinear Sciences and Applications, 9(3), 2016, 1252-1260.

[28] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*, North-Holland Mathematics Studies, 204, Elsevier Sci. B.V., Amsterdam, 2006.

[29] İ. Mumcu, E. Set, A. O. Akdemir, F. Jarad, *New extensions of Hermite-Hadamard inequalities via generalized proportional fractional integral*, Numerical Methods for Partial Differential Equations, 2021, 1-12

[30] S. G. Samko, A. A. Kilbas, O. I. Marichev, *Fractional integrals and derivatives*, Yverdon-les-Bains, Switzerland: Gordon and Breach Science Publishers, 1993, (Vol. 1), Yverdon.

[31] M. Z. Sarikaya, E. Set, H. Yaldiz, and N. Basak, *Hermite-Hadamard's inequalities for fractional integrals and related fractional inequalities*, Mathematical and Computer Modelling, DOI:10.1016/j.mcm.2011.12.048, 57 (2013) 2403-2407.

[32] X. X. You, M. A. Ali, H. Budak, P. Agarwal, and Y. M. Chu, *Extensions of Hermite-Hadamard inequalities for harmonically convex functions via generalized fractional integrals*, Journal of Inequalities and Applications, 2021(1), 2021, 1-22

[33] R. P. Agarwal, *Certain fractional q-integrals and q-derivatives*, Mathematical Proceedings of the Cambridge Philosophical Society, Vol. 66, No. 2, pp. 365-370, Cambridge University Press, 1969.

[34] B. Li, *Refinements of Hermite-Hadamard's type inequalities for operator convex functions*, Int. J. Contemp. Math. Sci., 8(9-12), 2013, 463-467.

[35] M. V. Mihai, M. U. Awan, M. A. Noor, J. K. Kim, and K. I. Noor, *Hermite-Hadamard inequalities and their applications*, Journal of Inequalities and Applications, 2018(1), 2018, 1-9.

[36] H. Budak, *New Hermite-Hadamard type inequalities for convex mappings utilizing generalized fractional integrals*, Filomat, 33(8), 2019, 2329-2344.

[37] A. A. Abdelhakim, *The flaw in the conformable calculus: It is conformable because it is not fractional*, Fractional Calculus and Applied Analysis, 22, 2019, 242-254.

Integral Generalization of Brass-Stancu Operators

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ABSTRACT

In this presentation, we deal with Kantorovich type generalization of the Brass-Stancu operators. For the sequence of these operators, we study L^p-convergence and give some upper estimates for the L^p-norm of the approximation error via first-order averaged modulus of smoothness and the first-order K-functional. Moreover, we show that the Kantorovich generalization of each Brass-Stancu operator satisfies variation detracting property and is bounded with respect to the norm of the space of functions of bounded variation on $[0,1]$. Finally, we present graphical and numerical examples to compare the convergence of these operators to given functions under different parameters.

Key Words: Stancu-type operators, L^p-convergence, averaged modulus of smoothness.

REFERENCES

[1] H. Brass, Eine verallgemeinerung der Bernsteinschen operatoren, Abh. Math. Sem. Univ. Hamburg, 38 (1971),111-122.

[2] M. Bodur, T. Bostancı and G. Başcanbaz-Tunca, On Kantorovich variant of Brass-Stancu operators, Demonstr. Math., 57 (1) (2024), 1-14.

[3] D. D. Stancu, Approximation of functions by means of a new generalized Bernstein operator, Calcolo, 20 (2) (1983), 211-229.

Mohand Transform And Hyers-Ulam Stability Of Fractional Linear Differential Equation Involving Riemann-Liouville Fractional Derivative

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ABSTRACT

The concept of fractional-order derivatives has a history as old as that of integer-order derivatives. The concept of fractional-order derivatives emerged in 1695 with a letter from L'Hospital to Leibniz. Fractional-order derivatives have been used in various fields of physics, biology, mathematics, and engineering. Recently, many studies have been conducted on the Hyers-Ulam stability of fractional differential equations.

Hyers-Ulam stability is among the main topics regarding the theory of functional equations. Firstly, in 1940, Ulam posed a problem related to the stability of functional equations at the University of Wisconsin. Hyers [1] answered Ulam's question for Banach spaces. Subsequently, differential equations began to be used instead of functional equations.

In 2015, Wang and Xu [4] solved the Hyers-Ulam stability problem of the following two types of fractional linear differential equations by using the Laplace transform:

$$({}^{C}D^{\alpha}_{0+}y)(t) - \lambda y(t) = f(t),$$

and

$$({}^{C}D^{\alpha}_{0^+}y)(t) - \lambda({}^{C}D^{\beta}_{0^+}y)(t) = g(t),$$

where $\lambda \in \mathbb{R}, t > 0, n-1 < \alpha \leq n, m-1 < \beta \leq m, 0 < \beta < \alpha, m, n \in \mathbb{N}, m \leq n,$ $f(t)$ and $g(t)$ are real functions defined on $\mathbb{R}_+$ and ${}^{C}D_{0+}^{\alpha}$ is the Caputo fractional derivative.

In 2016, Shen and Chen [3] solved the stability problem of the following linear fractional differential equations with constant coefficients by using the Laplace transform method:

$$(D_{0+}^{\alpha} y)(t) - \lambda y(t) = f(t),$$

where D_{0+}^{α} denotes the following Riemann-Liouville fractional derivative:

$$(D_{0^+}^{\alpha} f)(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_0^t \frac{f(u)du}{(t-u)^{\alpha-n+1}} \quad . \qquad (1)$$

In 2020, Jarad and Abdeljawad [2] studied the solution of the following fractional differential equations by using generalized the Laplace transform method:

$${}_{a}D_{g}^{\alpha} y(t) - \lambda y(t) = f(t), t > a, 0 < \alpha \leq 1, \lambda \in \mathbb{R},$$

$$({}_{a}I_{g}^{1-\lambda} y)(a^+) = c, c \in \mathbb{R},$$

where ${}_{a}D_{g}^{\alpha}$ is the generalized Riemann-Liouville fractional derivatives defined by

$$({}_{a}D_{g}^{\alpha} f)(t) = \left(\frac{1}{g'(t)}\frac{d}{dt}\right)^n ({}_{a}I_{g}^{n-\alpha} f)(t)$$
$$= \frac{\left(\frac{1}{g'(t)}\frac{d}{dt}\right)^n}{\Gamma(n-\alpha)} \int_a^t (g(t)-g(u))^{n-\alpha-1} f(u) g'(u) du \ ,$$

where $n = [\Re(\alpha)] + 1,\ \Re(\alpha) > 0, g^i \neq 0, i = 2, \dots, n,\ g(t)$ a strictly increasing function with continuous derivative g' on the interval (a, b).

Motivated by the ideas by Wang and Xu [4] and Jarad and Abdeljawad [2] (for $g(t) = t$ and $a = 0$), in this paper we proved the Hyers-Ulam stability of the following linear fractional differential equations by using the Mohand transform method:

$$D_{0+}^{\alpha} y(t) - \lambda y(t) = f(t),$$

where $\lambda \in \mathbb{R}, t > 0, n-1 < \alpha \leq n,\ m, n \in \mathbb{N}, m \leq n,\ f(t)$ is real functions defined on $\mathbb{R}_+$ and $D_{0^+}^{\beta}$ is defined in (1). Mohand transform is defined for a function of exponential order in the following A set:

$$A = \{f(t): \exists M_1, k_1, k_2 > 0. |f(t)| < M_1 e^{\frac{|t|}{k_j}}, t \in (-1)^j \times [0, \infty)\},$$

where M_1 is a finite number, k_1, k_2 can be finite or infinite and $f(t)$ is a function defined for every $t \geq 0$. Mohand transform is defined by

$$M\{f(t)\} = v^2 \int_0^\infty f(t) e^{-vt} dt\,, \qquad k_1 \leq v \leq k_2.$$

Key Words: Hyers-Ulam stability, Mohand transform, Riemann-Liouville fractional derivative.

REFERENCES

[1] D. H. Hyers, On the stability of the linear functional equations, Proc. Nati. Acad. Sci. U. S. A. 27(1941), 222-224.

[2] F. Jarad and T. Abdeljawad, Generalized fractional derivatives and Laplace transform, Discrete and continuous dynamical systems series, 13(3)2020,709-722.

[3] Y. Shen, W. Chen, Laplace transform method for the Ulam stability of linear fractional differential equations with constant coefficients, Mediterr.J.Math.14(2017),1-17.

[4] C. Wang, T. Z. Xu, Hyers-Ulam stability of fractional linear differential equations involving Caputo fractional derivatives, Applications of Mathematics, 60(2015), 383-393.

New results for the weighted Hardy inequality inequalities involving weak Riesz--Fischer functionals

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ABSTRACT

We establish a set of relations between several quite diverse types of weighted inequalities involving various integral operators and fairly general quasinorm-like functionals. The main result enables one to solve a specific problem by transferring it to another one for which a solution is known. The main result is formulated in a rather surprising generality, involving previously unknown cases, and it works even for some nonlinear operators such as the geometric or harmonic mean operators. Proofs use only elementary means.

Key Words: Weighted inequalities, integral operators, weak Riesz--Fischer property.

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On New Estimates for the B-Riesz transform

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ABSTRACT

In this paper, we study the maximal truncated B-Riesz transform $R_{\gamma}^{(k)*}$produced by the B-Riesz transform $R_{\gamma}^{(k)}$ with the convolution-type, that is smooth homogeneous, and generated by a generalized translate operator T^{y}, $y \in \mathbb{R}_{+}^{y}$. We obtained a new Cotlar's inequality for this operator. The point to be noted here is the kernel of the operator which depends on the homogeneous polynomial, and the generalized translate operator to which the operators are related. However, the case of the kernel and the generalized shift operator should be considered to play an important role in this study.
Here we deal specifically with the case of odd kernels.

Key Words: Generalized shift operator, B-Riesz transform, Cotlar's inequality.

REFERENCES

[1] J. Mateu and J. Verdera, L_{p} and weak L_{1} estimates for the maximal Riesz transform and the maximal Beurling transform, Math. Res. Lett. 13(5-6) (2006), 957-966.

[2] Joan Mateu, Joan Orobitg, Carlos P\'erez, Joan Verdera, New Estimates for the Maximal Singular Integral, International Mathematics Research Notices

On optimal embedding of the space of generalized fractional-maximal functions in rearrangement-invariant spaces

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ABSTRACT

In this paper, the spaces of generalized fractional maximal functions are considered. Embeddings of such spaces into rearrangement-invariant spaces are studied, and descriptions of the optimal rearrangement-invariant space (in short: RIS) for such an embedding are given.

Criteria for embedding spaces of generalized fractional-maximal functions into rearrangement-invariant spaces are obtained.

Definition 1. [1] Let $n \in N$ and $R \in (0, \infty]$. We say that a function $\Phi: (0, R) \to R_+$ belongs to the class $A_n(R)$ if:

(1) Φ is decreases and is continuous on $(0, R)$;

(2) the function $\Phi(r)r^n$ is quasi-increasing on $(0, R)$.

For example, $\Phi(t) = t^{\alpha-n} \in A_n(\infty)$, $0 < \alpha < n.$

Definition 2. [2] Let $n \in N$ and $R \in (0, \infty]$. A function $\Phi: (0, R) \to R_+$ belongs to the class $B_n(R)$ if the following conditions hold:

(1) Φ is decreases and is continuous on $(0, R)$;

(2) there exists $C = C(\Phi, n) > 0$ such that

$$\int_0^r \Phi(\rho)\rho^{n-1}d\rho \le C\Phi(r)r^n, r \in (0, R).$$

For example, $\Phi(\rho) = \rho^{\alpha-n} \in B_n(\infty)(0 < \alpha < n)$, $\Phi(\rho) = \ln\frac{eR}{\rho} \in B_n(R), R \in R_+$.

Definition 3. [2] Let $R \in (0, \infty]$. We say that $\Phi: (0, R) \to R_+$ belongs to the class $\mathrm{D}(R)$ if for some $C = C(\Phi) > 0$

$$\int_0^r \frac{dt}{\Phi(t)t} \leq \frac{C}{\Phi(r)};\ r \in (0;R).$$

Definition 4. Let $\Phi: (0,\infty) \to (0,\infty)$. The generalized fractional-maximal function $M_\Phi f$ is defined for the function $f \in L^1_{Loc}(R^n)$ by

$$(M_{\Phi}f)(x) = \sup_{r>0} \Phi(r) \int_{B(x,r)} |f(y)|dy,$$

where $B(x,r)$ is a open ball with the center at the point $x \in R^n$ and radius $r > 0$. In the case $\Phi(r) = r^{\alpha-n}$, $\alpha \in (0,n)$ we obtain the classical fractional maximal function $M_\alpha f$.

Let $T \in (0,\infty]$, $\mathfrak{I}_T = \{M(T)\}$ the set of cones consisting measurable function on $(0,\ T)$ equipped with a positive homogeneous functionals $\rho_{M(T)}$. Let $X(\mathbb{R}^n)$ is the rearrangement invariant space, $\tilde{X}(R^+)$ is their Luxemburg representation.

*Definition 5.***1** Let $M(T) \in \mathfrak{I}_T$. The embedding $M(T) \mapsto \tilde{X}(0,T)$ means that $M(T) \subset \tilde{X}(0,T)$ and there exists a constant $C_{M(T)}(T) \in R_+$ such that

$$\|\mathrm{h}\|_{X(0,T)} \leq C_{M(T)}\rho_{M(T)}(h), \qquad h \in M(T).$$

Definition 6. **2** Let E - RIS. By an optimal RIS for embedding

$$M_E^{\Phi}(\mathbb{R}^n) \hookrightarrow X(\mathbb{R}^n),$$

we mean an RIS $X_0 = X_0(R^n)$ such that $M_E^{\Phi}(\mathbb{R}^n) \mapsto X_0(\mathbb{R}^n)$ and if $M_E^{\Phi}(\mathbb{R}^n) \hookrightarrow X(\mathbb{R}^n)$ is valid for another rearrangement-invariant space X, then $X_0 \subset X$. Such an optimal RIS is called a rearrangement-invariant envelope of the space of fractional-maximal functions.

Theorem 1. **3** Let $\Phi \in B_n(\infty)$. The optimal RIS $X_0 = X_0(\mathbb{R}^n)$ for embedding

$$M_E^{\Phi}(\mathbb{R}^n) \hookrightarrow X(\mathbb{R}^n)$$

has an equivalent norm (in sense of Luxemburg representation):

$$\|f\|_{X_0(0,\infty)} = \sup_{g^*} \{ \int_0^\infty f^*(\tau) g^*(\tau) d\tau : g \in L_0(0,\infty),$$

$$\sup_{\int_0^t \mathrm{h}(s)ds \le \int_0^t g^*(s)ds} \| \int_t^\infty \Phi(s^{1/n}) \mathrm{sh}(s) ds \|_{E'} \le 1 \}.$$

The proof of this theorem uses the following result of Sinnamon [3].

*Theorem 2.***4** Suppose $\mathrm{f}, \mathrm{w} \in \mathrm{L}_0^+$. Then

$$\sup_{g: \int_0^x g \le \int_0^x f} \int_0^\infty g(x) w(x) dx = \int_0^\infty f(x) \left(\sup_{t \in [x,\infty)} w(t) \right) dx.$$

The generalized Riesz potential was considered in [4]. In [2] was proved that the generalized fractional maximal function $(M_{\Phi} f)(x)$ is estimated by the generalized Riesz potential: $(M_{\Phi} f)(x) \le (I_G |f|)(x)$, $x \in R^n$.

Key Words: Optimal embedding, generalized fractional maximal functions, rearrangement-invariant spaces, non-increasing rearrangements.

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REFERENCES

[1] N.A. Bokayev, A.Gogatishvili, A.N. Abek, Cones generated by a generalized fractional maximal function, Bulletin of the Karaganda University Mathematics series, 110(2) (2023), 53-62.

[2] N.A. Bokayev, A.Gogatishvili, A.N. Abek, On estimates of non-increasing rearrangement of generalized fractional maximal function, Eurasian Math. Journal, 14(2) (2023), 13-23.

[3] G.Sinnamon, Transferring monotonicity in weighted norm inequalities, Collect. Math, 54(2) (2003), 181-216.

[4] M.L. Goldman, Optimal Embeddings of Generalized Bessel and Riesz Potentials, Proceedings of the Steklov Institute of Mathematics, 269(1) (2010), 85-105.

On the behavior of bi-parametric potential-type operators in Lebesgue Spaces

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ABSTRACT

Let $f \in L_p(\mathbb{R}^n)$, $(1 \leq p \leq \infty)$ and $(0 < \beta, t < \infty)$. Denote

$$(B_t^{(\beta)} f)(x) = \int_{\mathbb{R}^n} w_t^{(\beta)}(y) f(x-y)\, dy, \quad (x \in \mathbb{R}^n),$$

where

$$w_t^{(\beta)}(y) = F^{-1}_{x \to y} (\exp(-t|x|^\beta)),$$

F^{-1} is the inverse Fourier transform and $|x| = \sqrt{x_1^2 + \cdots x_n^2}$.

The function $w_t^{(\beta)}(y)$ generalizes the classical Poisson and Gauss kernels and turns into them in the case of $\beta = 1$ and $\beta = 2$, respectively.

In paper [1], the following family of bi-parametric potential type operators was introduced:

$$(J_\beta^\alpha f)(x) = \frac{1}{\Gamma(\alpha/\beta)} \int_0^\infty e^{-\tau} \tau^{\alpha/\beta - 1} (B_t^{(\beta)} f)(x) dt.$$

It should be reminded that, in the case of $\beta = 2$ the classical Bessel potential and $\beta = 1$ the classical Flett potential [2,3] are obtained.

In this study, we investigate the behavior of the integral operators $J_\beta^\alpha f$ in Lebesgue spaces $L_p(\mathbb{R}^n)$.

Key Words: Fractional integral, Bessel potential, Flett potential, semigroup, convolution, Fourier transform.

MSC: 26A33, 47G10, 45P05

REFERENCES

[1] I.A. Aliev, Bi-parametric potentials, relevant function spaces and wavelet-like transforms. Integral Equations Operator Theory, 2009, 65: 151-167.

[2] I.A. Aliev, S. Sezer, M. Eryigit, An integral transform associated to the Poisson integral and inversion of Flett potentials. Journal of mathematical analysis and applications, 2006, 321(2), 691-704.

[3] I.A. Aliev, B. Rubin, S. Sezer, S. Uyhan, Composite wavelet transforms: applications and perspectives, Contemporary Mathematics, AMS, 2008, 464, 1-27.

On the characterization of weighted inequalities for some quasilinear operators

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ABSTRACT

In this paper we characterize weighted inequalities for a quasilinear operators involving a superposition of the three operators, Copson, Hardy and Tandori operators.

Let $\mathfrak{M}^+(0,\infty)$ the set of all non-negative measurable functions on $(0,\infty)$.

Let $1 \le p < \infty, 0 < q < \infty$, u, v and w are weights (i.e., locally integrable non-negative functions on $(0,\infty)$), φ is strictly increasing function on $(0,\infty)$, and $\frac{\varphi}{U}$ be decreasing on $(0,\infty)$, where

$$U(s) = \int_0^s u(t)dt.$$

Our goal in this paper is to characterize the following inequality

$$\left(\int_0^\infty \left(\sup_{t<s<\infty} \frac{1}{\varphi(s)} \left(\int_0^s U(\tau)h(\tau)d\tau + U(s)\int_s^\infty h\right)\right)^q w(t)dt\right)^{\frac{1}{q}} \le C\left(\int_0^\infty h^p v\right)^{\frac{1}{p}} \qquad (1)$$

for all $h \in \mathfrak{M}^+(0,\infty)$.

The following theorem provides a criterion for the embedding of inequality (1) for some values of the parameters p and q.

Theorem 1. Let $q \in (0,\infty)$, $p \in (1,\infty)$, $r = \frac{pq}{p-q}$, $\frac{1}{p} + \frac{1}{p'} = 1$ and u, v, w be weight functions on $(0,\infty)$, φ be a U-quasi-concave function on $(0,\infty)$. Then there exists a constant $C > 0$ such that inequality (1) holds for all $f \in \mathfrak{M}^+(0,\infty)$ if and only if one of the following conditions holds:

(i) $1 < p, q < p$

$$C_1 := \left(\int_0^\infty \left(\int_0^\infty \frac{w(s)ds}{(\varphi(s)+\varphi(t))^q}\right)^{\frac{r}{q}} d\nu_p(t)\right)^{\frac{1}{r}} < \infty,$$

where ν_p is the measure of representation of the following expression

$$\varphi^r(t) \sup_{s\in(t,\infty)} \frac{1}{\varphi^r(s)} \left(\int_0^\infty \frac{U^{p'}(\tau)U^{p'}(s)}{U^{p'}(s) + U^{p'}(\tau)} v^{1-p'}(\tau)d\tau\right)^{\frac{r}{p'}},$$

i.e.

$$\varphi^r(t) \sup_{s\in(t,\infty)} \frac{1}{\varphi^r(s)} \left(\int_0^\infty \frac{U^{p'}(\tau)U^{p'}(s)}{U^{p'}(s) + U^{p'}(\tau)} v^{1-p'}(\tau)d\tau\right)^{\frac{r}{p}} = \int_0^\infty \frac{\varphi^r(t)}{\varphi^r(s)+\varphi^r(t)} \nu_p(s)ds.$$

(ii) $q < p = 1$

$$C_2 := \left(\int_0^\infty \left(\int_0^\infty \frac{w(s)ds}{(\varphi(s)+\varphi(t))^q}\right)^{\frac{r}{q}} d\nu_\infty(t)\right)^{\frac{1}{r}} < \infty,$$

where ν_∞ is the measure of representation of the following expression

$$\varphi^r(t) \sup_{s\in(t,\infty)} \frac{1}{\varphi^r(s)} \left(\sup_{s\in(0,\infty)} \frac{U(t)U(s)}{(U(s)+U(t))v(s)}\right)^r$$

i.e.

$$\varphi^r(t) \sup_{s\in(t,\infty)} \frac{1}{\varphi^r(s)} \left(\sup_{s\in(0,\infty)} \frac{U(t)U(s)}{(U(s)+U(t))v(s)}\right)^r = \int_0^\infty \frac{\varphi^r(t)}{\varphi^r(s)+\varphi^r(t)} \nu_\infty(s)ds.$$

Moreover, the best constant in inequality (1) satisfies the following:

$$C \approx \begin{cases} C_1 & \text{in the case (i)} \\ C_2 & \text{in the case (ii).} \end{cases}$$

The proof of the Theorem 1 is based on the so called discretization and untidiscretization techniques develop in the paper [1], [2] and in the monograph [3].

Similar close inequalities were previously considered in [4] and [5].

Key Words: Characterization, weighted inequalities, quasilinear operator, embedding.

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REFERENCES

[1] A.Gogatishvili, L.Pick, Discretization and anti-discretization of rearrangement-invariant norms, Publ. Mat., Bare, 47(2) (2003), 311-358.

[2] A.Gogatishvili and L. Pick. Embeddings and duality theorems for weak classical Lorentz spaces, Canad. Math. Bull. 49(2006), no. 1, 82 - 95.

[3] W.D. Evans, A.Gogatishvili, B.Opic, Weighted Inequalities Involving *p*-quasiconcave Operators, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ (2018), xv+140 pp.

[4] A.Gogatishvili, R.CH. Mustafayev, L.E. Persson, Some New Iterated Hardy-Type Inequalities, Journal of Function Spaces and Applications, (2012), 1-30.

[5] A.Gogatishvili, R.Mustafayev, L.E. Persson. Some new iterated Hardy-type inequalities: the case $\theta = 1$, J. Inequal. Appl., 515 (2013), 1-29.

On The Convexity and Starlikeness of Certain Subclasses of Analytic Functions

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ABSTRACT

AMS American Mathematical Society 30. c. 45.

There are three main problems in Geometric Function Theory.

1. Distortion Theorems
2. Coefficient bounds or estimates
3. Convexity and starlikeness of certain subclasses of analytic functions. We define the following classes

$$CCV(0,\alpha) = \{f\epsilon H: Re\frac{f'(z)}{g'(z)} > 0, g\epsilon CV(\alpha)\}$$

$$CST(0,\alpha) = \{f\epsilon H: Re\ \frac{f(z)}{g(z)} > 0, g\epsilon ST(\alpha)\}$$

Where f and g are normalized analytic functions in the unit disk, $0 \leq \alpha < 1$.

In this work we obtain the radius of convexity of the $\alpha - type - close - to\ convex\ functions$ and the radius of starlikeness of the $\alpha - type - close - to\ star\ functions$ and Corollaries of the theorems

Key Words: Analytic function, starlike function, convex function, close-to-convex function and close-to-star function, radius convexity and starlikeness.

REFERENCES

[1] Altıntaş O. Starlike,convex and close-to-convex functions of complex order. Hac. Bull of Nat. Sciences and Engineering . Series, 1. (29), 37-49, (1999).

[2] Ratti J. S. The Radius of convexity of certain analytic functions, Internat. J. Mathematics and Mathematics Sciences, Vol.3, No.3, 483-489 (1980)

Some Approximation Results for Nonlinear Operators in Modular Space

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ABSTRACT

The key moment in the development of approximation theory is Weierstrass' theorem.
The problem has been studied by many famous mathematicians. Bernstein has succeeded to give the most elegant and short proof of this theorem via Bernstein polynomials. Another important instrument in approximation theory by positive linear operators is the Korovkin theory. Korovkin theory enables a very simple and useful criterion in the approximation theory of functions, i.e. the classical Korovkin theorem states the uniform convergence of a sequence of positive linear operators in $C[a,b]$, the space of continuous real valued functions defined on [a, b], by providing the convergence only on three test functions$\{1, x, x^2\}$.
On the other hand, modular spaces are mathematical structures that generalize vector spaces by relaxing certain linear constraints, making them suitable for studying a wide range of functional analysis problems. Modular spaces have been instrumental in the study of Orlicz spaces, Musielak-Orlicz spaces and other spaces where classical norms are insufficient, enabling the advancement of both theoretical and applied fields in mathematics. The development of modular convergence has opened up new avenues for research in functional analysis
In this talk, we present Korovkin type theorem for nonlinear operators in modular spaces. An example that satisfies our theorem is also provided.

Key Words: Nonlinear operator, Korovkin theory, modular spaces.

REFERENCES

[1] C. Bardaro and I. Mantellini, Korovkin theorem in modular spaces, Comment. Math. 47 (2007), 239-253.

[2] C. Bardaro and I. Mantellini, Multivariate moment type operators: approximation properties in Orlicz spaces, J. Math. Inequal 2 (2008), 247-259.

Some Classical Operators on Vanishing Morrey Spaces with Mixed Homogeneity

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ABSTRACT

This study investigates the boundedness and continuity properties of classical singular operators in the context of vanishing Morrey spaces with mixed homogeneity, building upon the foundational works on Calderón-Zygmund operators (e.g. [1], [3]). By exploring the anisotropic behaviour of Calderón-Zygmund operators and maximal operators under various dilation metrics [4], we aim to extend the classical results to more general quasi-norms on homogeneous metric spaces [2]. Utilizing anisotropic quasi-norms [5], we establish new estimates for these operators and their interactions with vanishing Morrey spaces, offering insight into potential extensions of classical results in this framework [7]. Our preliminary results suggest that these operators retain continuity and boundedness properties under suitable conditions. This work has potential applications in harmonic analysis and partial differential equations, particularly in non-Euclidean spaces [6].

Key Words: Singular operators, Calderón-Zygmund, Vanishing Morrey spaces, Mixed homogeneity.

REFERENCES

[1] M. Bramanti and M.C. Cerutti, Commutators of Singular Operators on Homogeneous Spaces, Boll. Un. Mat. Ital. B, 10-7 (1996), 843-883.

[2] E.B. Fabes and N.M. Rivière, *Commutators of singular integrals*, Studia Math, 27 (1966), 9-17.

[3] D. Gilbarg and N.S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer-Verlag, (1983).

[4] N. Samko, *Maximal, potential and singular operators in vanishing generalized Morrey spaces,* J. Global Optim, 57 (2013), 1385–1399.

[5] L. Softova, *Singular integrals and commutators in generalized Morrey spaces*, Acta Math. Sin. (Engl. Ser.), 22-3 (2006), 757–766.

[6] E. Nakai, *Hardy-Littlewood maximal operator, singular integral operators and the Riesz potentials on generalized Morrey spaces*, Math. Nachr, 166 (1994), 95–103.

[7] C.T. Zorko, *Morrey space*, Proc. Of The Amer. Math. Soci., 98 (1986), 586–592.

SOME SPECIAL INEQUALITIY AND APPLICATIONS FOR m-EARTHQUAKE CONVEX FUNCTION

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ABSTRACT

In this paper, we give a new m-earthquake convex function definition. Moreover, we demonstrate Hermite-Hadamard inequality and practices for m-earthquake convex function. Furthermore, we obtained some algebraic properties and refinements of Hermite-Hadamard for m-earthquake convex function.

Key Words: Convexity; Earthquake convex; Hermite-Hadamard inequality.

REFERENCES

[1] M. Kadakal, I. İşcan, and H. Kadakal, Earthquake convexity and some new related inequali- · ties, Journal of Applied Analysis, 2023. https://doi.org/10.1515/jaa-2023-0046.

[2] İ . İşcan, Hermite-Hadamard type inequalities for p-convex functions, Int. J. Anal. Appl., 11 (2) ; (2016), 137-145.

[3] S. Varošanec. On hconvexity, J. Math. Anal. Appl., 326; (2007), 303-311.

[4] İ. İşcan, Ostrowski type inequalities for p-convex functions, New Trends in Mathematical Sciences, 4 (3); (2016) 140-140.

[5] K.S. Zhang and J.P. Wan, p-convex functions and their properties, Pure Appl. Math., 23 (1) ; (2007), 130-133

[6] M. Vivas-Cortez, A. Kashuri, S.I. Butt, M. Tariq and J. Nasir, Exponential Type p-Convex Function with Some Related Inequalities and their Applications, Appl. Math. Inf. Sci., 15; No. 3; (2021), 253-261 .

[7] H. Kadakal, Hermite-Hadamard type inequalities for trigonometrically convex functions, Sci. Stud. Res. Ser. Math. Inform. 28 (2018), no:2; 19-28.

[8] A. Akkurt, M. Z. Sarıkaya, H. Budak and H. Yıldırım, On the Hadamard's type inequalities for co-ordinated convex functions via fractional integrals, Journal of King Saud UniversityScience, 29; (2017), 380-387 .

[9] H. F. Gidergelmez, A. Akkurt and H.Yıldırım, Hermite-Hadamard Type Inequalities for Generalized Fractional Integrals via Strongly Convex Functions, Konuralp Journal of Mathematics, 7(2), (2019), 268-273.

[10] A. Akkurt and H. Yıdırım, Hermite-Hadamard Type Inequalities for (n; m; h1; h2; ') Convex Functions Via Fractional Integrals, Malaya J. Mat. 4(2); (2016), 23237.

[11] A. Akkurt and H. Yıldırım, On some Fractional Integral Inequalities of Hermite-Hadamard type for rPreinvex Functions, Khayyam J. Math., no:2; (2016), 120-127; 2 .

[12] I. I¸scan, New refinements for integral and sum forms of Hölder inequality, J. Inequal. Appl. · 2019 (2019) ; Paper No. 304.

[13] M. Kadakal, I. I¸scan, H. Kadakal and K. Bekar, On improvements of some integral inequali- · ties, Honam Math. J. 43 (2021) ; no:3; 441-452.

[14] S.S. Dragomir and R.P. Agarwal, Two Inequalities for Diferentiable Mappings and Applications to Special Means of Real Numbers and to Trapezoidal Formula, Appl. Math. Lett. 11(1998). no. 5, 91-95.

Superposition of *(X,X;1,1,∞)*-admissible Hardy-Littlewood Maximal Operator on Generalized Morrey-Banach Spaces

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ABSTRACT

The aim of this talk is to give the definitions of *(X,Y;Φ,r,s)*-admissible quasilinear operators and the generalized Morrey-Banach spaces. We define these spaces for two Banach subspaces *X* and *Y*, non-increasing function *Φ*, and a wide range of the numerical parameters *r,s*, $1 \leq r < \infty$, $0 < s \leq \infty$. And also, we define the operator $S_{\Phi,r,s}$ and we show that it is the generalization of all classical operators of harmonic analysis.

Furthermore, we show the boundedness of *(X,X;1,1,∞)*-admissible Hardy-Littlewood maximal operators on the generalized Morrey-Banach spaces and also including weak estimates. The definition of *(X,Y;Φ,r,s)*-admissible quasilinear operator covers all previously given definitions of admissible operators. These conditions satisfy almost all the classical operators in harmonic analysis. Hence, we show that the Hardy-Littlewood maximal operator *M* is an applications of *(X,Y;Φ,r,s)*-admissible quasilinear operators.

Key Words: Admissible operators, Hardy-Littlewood maximal operator, generalized Morrey-Banach spaces.

REFERENCES

[1] D.R. Adams, A note on Riesz potentials, Duke Math., vol. 42, pp. 765-778, 1975.

[2] F. Chiarenza and M. Frasca, Morrey spaces and Hardy-Littlewood maximal function, Rendiconti di Matematica e delle sue Applicazioni, vol. 7, no. 3-4, pp. 273-279, 1987.

[3] E. Nakai, Hardy-Littlewood maximal operator, singular integral operators and Riesz potentials on generalized Morrey spaces, Math. Nachr., 166 (1994), 95-103.

The Riesz Potential in Musielak-Orlicz-Morrey Spaces

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ABSTRACT

Musielak-Orlicz spaces, also known as generalized Orlicz spaces, are a current area of study of increasing importance in the theory of function spaces. The structure of these spaces has been investigated in detail in [1]. In addition to being a natural generalization of results obtained in both Lebesgue and Orlicz spaces with variable exponents, these spaces have wide applications in areas such as nonlinear differential equations, image processing and fluid dynamics. For results on the boundedness of classical operators of harmonic analysis in Musielak-Orlicz spaces, we refer to [1, 3, 4].

Generalized Orlicz-Morrey spaces, defined as a structure combining Orlicz and generalized Morrey spaces, have recently been the subject of intensive research. Nakai [5] first defined these spaces in 2004, which were introduced as three different types in the literature. Later, Sawano et al. [6] defined a different type of generalized Orlicz Morrey space in 2012. What is now called the third type of Orlicz-Morrey space, a structure combining Morrey and Orlicz spaces, was defined by Deringöz et al. [2] in 2014.

In this study, Musielak-Orlicz-Morrey spaces are introduced from the point of view of the third type Orlicz-Morrey space and the boundedness of the Riesz potential operator, one of the classical operators of harmonic analysis, is investigated in these spaces.

Key Words: Riesz Potential, Musielak-Orlicz-Morrey Spaces, Generalized Orlicz Spaces

REFERENCES

[1] P. Harjulehto and P. Hästö, Orlicz Spaces and Generalized Orlicz Spaces, Springer, Cham, 2019.

[2] F.Deringoz, V.S Guliyev, S.Samko, Boundedness of maximal and singular operators on generalized Orlicz-Morrey spaces, Operator Theory, Operator Algebras and Applications, Birkhäuser, Basel, 2014.

[3] L.Diening, Maximal function on Musielak–Orlicz spaces and generalized Lebesgue spaces, Bulletin des Sciences Mathématiques, 2005.

[4] P. Harjulehto and P. Hästö, The Riesz potential in generalized Orlicz spaces, Forum Mathematicum. Vol. 29. No. 1. De Gruyter, 2017.

[5] E. Nakai, Generalized fractional integrals on Orlicz-Morrey spaces, Banach and Function Spaces 2004.

[6] Y. Sawano, S. Satoko , and H.Tanaka, Orlicz–Morrey spaces and fractional operators, Potential Analysis, 2012.

Weighted inequalities for bilinear discrete Hardy operator with a matrix

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ABSTRACT

Let $0 < q, p, s < \infty$, $u = \{u_i\}_{i=1}^{\infty}$, $v = \{v_i\}_{i=1}^{\infty}$ and $w = \{w_i\}_{i=1}^{\infty}$ be weight sequences, i.e., positive sequences of real numbers. We denote by $l_{p,v}$ and $l_{s,w}$, respectively, the spaces of sequences $f = \{f_i\}_{i=1}^{\infty}$ and $g = \{g_i\}_{i=1}^{\infty}$ of real numbers and the following quasinorms are finite:

$$\| f \|_{p,v} = \left(\sum_{i=1}^{\infty} | v_i f_i |^p \right)^{\frac{1}{p}} < \infty,$$

$$\| g \|_{s,w} = \left(\sum_{i=1}^{\infty} | w_i g_i |^s \right)^{\frac{1}{s}} < \infty .$$

We consider the following bilinear discrete operator with matrix

$$(Agf)_n := \sum_{i=1}^{n} g_i \sum_{j=1}^{n} a_{n,j} f_j \quad , \qquad (1)$$

and the bilinear Hardy inequality

$$\left(\sum_{n=1}^{\infty} u_n^q \left| \sum_{i=1}^{n} g_i \right|^q \left| \sum_{j=1}^{n} a_{n,j} f_j \right|^q \right)^{\frac{1}{q}} \le C \left(\sum_{i=1}^{\infty} | v_i f_i |^p \right)^{\frac{1}{p}} \left(\sum_{i=1}^{\infty} | w_i g_i |^s \right)^{\frac{1}{s}}, \qquad (2)$$

where C is a positive constant independent of f, g. $(a_{k,i})$, $k \ge i \ge 1$, is a matrix, whose non-negative entries satisfy the discrete Oinarov condition: there exist $d \ge 1$, entries $a_{k,i}$ are almost non-decreasing in k and almost non-increasing in i, such that

$$\frac{1}{d}(a_{k,j}+a_{j,i}) \le a_{k,i} \le d(a_{k,j}+a_{j,i}), \qquad (3)$$

or, equivalently, the relation $a_{k,i} \approx (a_{k,j}+a_{j,i})$ hold for all $k \ge j \ge i \ge 1$.

In this work, the necessary and sufficient conditions for the fulfillment of inequality (2) were obtained for the case $1 < \min\{p,s\} \le q < \max\{p,s\} < \infty$. Recently, Hardy-type inequalities for classes of matrix operators have been intensively studied. In peppers [1] and [2], the bilinear discrete Hardy-type inequality (2) when $a_{k,i}=1$ for all $k \ge i \ge 1$ already has been explored for all possible cases of parameters $0 < q, p, s < \infty$.

$$M_1 = \sup_{j\ge1}\left(\sum_{m=1}^{j} w_m^{-s'}\right)^{\frac{1}{s'}}\left(\sum_{k=j}^{\infty}\left(\sum_{i=1}^{k} a_{k,i}^{p'} v_i^{-p'}\right)^{\frac{q(p-1)}{p-q}}\left(\sum_{n=k}^{\infty} u_n^q\right)^{\frac{pq}{p-q}} u_k^q\right)^{\frac{p-q}{pq}},$$

$$M_2 = \sup_{j\ge1}\left(\sum_{m=1}^{j} w_m^{-s'}\right)^{\frac{1}{s'}}\left(\sum_{k=1}^{j}\left(\sum_{n=j}^{\infty} a_{n,k}^q u_n^q\right)^{\frac{p}{p-q}}\left(\sum_{i=1}^{k} v_i^{-p'}\right)^{\frac{p(q-1)}{p-q}} v_k^{-p'}\right)^{\frac{p-q}{pq}},$$

$$M_3 = \sup_{j\ge1}\left(\sum_{m=1}^{j} w_m^{-s'}\right)^{\frac{1}{s'}}\left(\sum_{k=j}^{\infty}\left(\sum_{n=k}^{\infty} a_{n,k}^q u_n^q\right)^{\frac{p}{p-q}}\left(\sum_{i=1}^{k} v_i^{-p'}\right)^{\frac{p(q-1)}{p-q}} v_k^{-p'}\right)^{\frac{p-q}{pq}},$$

$$M_4 = \sup_{j\ge1}\left(\sum_{n=j}^{\infty} a_{n,j}^q u_n^q\right)^{\frac{1}{q}}\left(\sum_{i=1}^{j} v_i^{-p'}\right)^{\frac{1}{p'}}\left(\sum_{m=1}^{j} w_m^{-s'}\right)^{\frac{1}{s'}},$$

$$M_5 = \sup_{j\ge1}\left(\sum_{m=1}^{j} v_m^{-p'}\right)^{\frac{1}{p'}}\left(\sum_{n=j}^{\infty}\left(\sum_{k=n}^{\infty} a_{k,j}^q u_k^q\right)^{\frac{s}{s-q}}\left(\sum_{i=1}^{n} w_i^{-s'}\right)^{\frac{s(q-1)}{s-q}} w_n^{-s'}\right)^{\frac{s-q}{sq}},$$

$$M_6 = \sup_{j\ge1}\left(\sum_{k=1}^{j} a_{j,k}^{p'} v_k^{-p'}\right)^{\frac{1}{p'}}\left(\sum_{n=j}^{\infty} u_n^q\right)^{\frac{1}{q}}\left(\sum_{i=1}^{j} w_i^{-s'}\right)^{\frac{1}{s'}},$$

$$M_7 = \sup_{j\ge1}\left(\sum_{m=1}^{j} a_{j,m}^{p'} v_m^{-p'}\right)^{\frac{1}{p'}}\left(\sum_{n=j}^{\infty}\left(\sum_{k=n}^{\infty} u_k^q\right)^{\frac{s}{s-q}}\left(\sum_{i=1}^{n} w_i^{-s'}\right)^{\frac{s(q-1)}{s-q}} w_n^{-s'}\right)^{\frac{s-q}{sq}}.$$

Theorem 1. Let $1 < \min\{p, s\} \le q < \max\{p, s\} < \infty$. Let the elements of the matrix $(a_{n,j})$ satisfy discrete Oinarov condition. Then the inequality (2) for the operator (1) holds if and only if

(a) $M = \max\{M_1, M_2, M_3\} < \infty$ in case $1 < s \le q < p < \infty$.

(b) $\mathrm{M} = \max\{M_4, M_5, M_6, M_7\} < \infty$ in case $1 < p \le q < s < \infty$.

Moreover, $C \approx M$ in case (a) and $C \approx \mathrm{M}$ in case (b), where C is the best constant in (2).

Key Words: Inequality, Hardy-type operator, bilinear operator, weights, weighted sequence space.

REFERENCES

[1] M. Krepela, Iterating bilinear Hardy inequalities, J. Math. Ineq. 60 (2017), 955-971.

[2] P. Jain, S. Kanjilal, G.E. Shambilova and et al., Bilinear weighted Hardy-type inequalities in discrete and q-calculus, Math. Ineq.Appl. 23 (2020),1279-1310.

Weighted Norm Inequalities for Matrix Operators with $1 < q < p < \infty$

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ABSTRACT

Let $1 < p, q < \infty$ and $u = \{u_i\}_{i=1}^{\infty}$, $v = \{v_i\}_{i=1}^{\infty}$ be positive sequences of real numbers. Let $(a_{i,j})$ be a non-negative triangular matrix with entries $a_{i,j} \geq 0$ when $i \geq j \geq 1$ and $a_{i,j} = 0$ when $i < j$. Let $l_{p,v}$ denote the space of sequences of real numbers $f = \{f_i\}_{i=1}^{\infty}$ such that $\|f\|_{pv} = (\sum_{i=1}^{\infty} |v_i f_i|^p)^{\frac{1}{p}}$, $1 \leq p < \infty$.

We consider the weighted estimate

$$\|A^{\pm} f\|_{q,u} \leq \|A^{\pm}\|_{pv \to qu} \|f\|_{p,v}, \quad \forall f \in l_{p,v} \tag{1}$$

for a general class of matrix operators

$$(A^{+} f)_i = \sum_{j=1}^{i} a_{ij} f_j, \quad i \geq 1, \tag{2}$$

$$(A^{-} f)_j = \sum_{i=j}^{\infty} a_{ij} f_j, \quad j \geq 1, \tag{3}$$

where $a_{ij} > 0, i \geq j \geq 1$. Note the validity of inequality (1) that the validity of inequality (1) is equivalent to the boundedness of matrix operators (2) and (3) from l_{pv} into l_{qu}. In the paper [1], the wide classes O_n^{+} and O_n^{-}, $n \geq 0$, of matrices were introduced. These classes are defined by conditions on a matrix $(a_{i,j})$ that are weaker than the conditions are defined before in [2] - [5]. The criteria for the boundedness of operators (2) and (3) for $1 < p \leq q < \infty$ were obtained, where the matrices belong to the class $O_n^{+} \cup O_n^{-}$, $n \geq 1$. Moreover, according to [1], these classes form extension systems $O_0^{\pm} \subset O_1^{\pm} \subset \ldots \subset O_n^{\pm} \subset \ldots$, and it was shown that the system of classes $\{O_n^{\pm}, n \geq 0\}$ is closed with respect to iteration, i.e., the iteration matrix of operators (2) and (3) from classes $O_n^{\pm}$ and $O_m^{\pm}$ belongs to the class $O_{n+m+1}^{\pm}$. This property makes it possible to study various problems that involve iterations of

matrix operators, in particular, the problem of weighted estimates for quasilinear operators with iteration of operators (2) and (3).

The problem of boundedness of operators (2) and (3) with matrices from the classes O^+_n and O^-_n, $n > 1$, has not been proved for the case $1 < q < p < \infty$.

In the present work, we consider the case $1 < q < p < \infty$ and find the criteria for the boundedness of operators (2) and (3) from l_{pv} into l_{qu}, when their matrices belong to the classes $O^{\pm}_{n}$ for any $n \geq 2$.

Key Words: Boundedness, matrix operator, weighted inequality.

REFERENCES

[1] R. Oinarov and Z. Taspaganbetova, Criteria of boundedness and compactness of a class of matrix operators, Journal of Inequalities and Applications, no. 53(2012), 1-18.

[2] R. Oinarov, L-E.Perssonand A. Temirkhanova, Weighted inequalities for a class of matrix operators: the case $p \leq q$.Math. Inequal.Appl. 12 (2009), no. 4, 891-903.

[3] Zh. Taspaganbetovaand A. Temirkhanova, Boundedness and compactness criteriaof a certain class of matrixoperators. Mathematical J. Almaty, 11 (2011), no. 2, 73-85.

[4] Zh. Taspaganbetova, A. Temirkhanova, Criteria of boundedness of matrix operators in weighted spaces of sequences and their applications, Ann. Funct. Anal., 1 (2011), 114–127.

[5] A. Temirkhanova Weighted inequality for one class of matrix operators for $1 < q < p < \infty$, Eurasian Math. J., 2 (2008), 117–127 (in Russian).

APPLIED MATHEMATICS

A Collocation Method fort he Numerical Solutions of the Generalized Oskolkov Equation

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ABSTRACT

Using the quintic B-spline collocation finite element method, this article focuses on creating numerical schemes for the generalized Oskolkov equation. By utilizing the von-Neumann theory, it can be demonstrated that the suggested approach exhibits marginally unconditional stability.Three model problems, involving the evolution of an undular bore, Gaussian initial condition, and a single solitary wave, were used to assess the method's accuracy and efficiency. By computing invariant I and error norms $L_2$and L_∞, the performance of the proposed method is illustrated [1]. Both graphical and numerical results are shown. The method's correctness and robustness, which can be further applied to solve similar situations, are validated by numerical experiments.

Keywords : Generalised Oskolkov equation; Gaussian Initial Condition; finite element method; collocation; quintic B-splines.

REFERENCES

[1] K., Seydi Battal Gazi, B., Samir Kumar, Y. S., Derya. A Novel Scheme Based on Collocation Finite Element Method to Generalised Oskolkov Equation. *Journal of Science and Arts*, 2021, Vol. 21, No. 4(57), pp. 895-908.

A fractional prey-predator model with functional carrying capacity

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ABSTRACT

In our ecosystem, predation is the primary interaction between different species. The relationship between prey and predators plays a crucial role in species survival and biodiversity conservation. To understand the long-term behavior of populations, researchers study the dynamics of predator-prey models. As a result, various models have been proposed to incorporate different biological phenomena. Typically, prey growth is modeled using a logistic equation that includes a growth rate and carrying capacity. In population biology, carrying capacity is defined as the maximum load an environment can support, and it is often treated as a constant value. However, due to environmental changes, carrying capacity can vary over time, and a species may even influence its own intrinsic carrying capacity [1]. Some models have been developed and analyzed to address this, often using delay differential equations [2,3]. In this work, we use fractional differential equations to introduce a general memory effect into the system. Additionally, we consider a model where prey's growth is subject to the Allee effect, a phenomenon that describes a positive correlation between average individual fitness and population size [4]. So, at low population densities, the presence of conspecifics can increase the per capita growth rate. We will conduct stability and bifurcation analyses and provide numerical simulations to support our findings.

Key Words: Prey-predator, Varying carrying capacity, Allee, Fractional order

REFERENCES

[1] Yukalov, V. I., Yukalova, E. P., & Sornette, D. (2012). Extreme events in population dynamics with functional carrying capacity. The European Physical Journal Special Topics, 205(1), 313-354.

[2] Pal, S., Gupta, A., Misra, A. K., & Dubey, B. (2023). Complex dynamics of a predator–prey system with fear and memory in the presence of two discrete delays. The European Physical Journal Plus, 138(11), 984.

[3] Pati, N. C., & Ghosh, B. (2022). Delayed carrying capacity induced subcritical and supercritical Hopf bifurcations in a predator–prey system. Mathematics and Computers in Simulation, 195, 171-196.

[4] B. Dennis, Allee Effects: Population Growth, Critical Density, and the Chance of Extinction, Nat. Resource Model. 3 (1989), 481–538.

A Fresh Look To Numerical Solution of a Model Problem by Hermite Galerkin Method

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ABSTRACT

Various physical and natural phenomena around us both in engineering and other scientific fields are generally modelled with the help of ordinary, partial or fractional order differential equations. Those equations are widely encountered but in such a complex way that it is almost impossible to be solved by classical analytical methods. Of course, first of all scientists try to find the analytical solutions of those encountered differential equations used for modelling the physical phenomena around us **[1]**. But, because of the intricate natural designs and structure characteristics of those natural phenomena, it is generally difficult to be able to obtain the analytical solutions of those differential equations that we encounter in nature and most of the time it becomes almost impossible under the conditions of the present day. Thus, the scientists resort to numerical methods to find approximate solutions to replace exact solutions of those equations. Among those methods, the most frequently used ones in the literature are finite difference, variational, quadrature differential and finite element methods. In the present study, the approximate solutions of a model problem constructed by a differential equation with definite given boundary conditions will be obtained by Galerkin finite element method based on Cubic Hermite B-spline bases **[2]**. As of the knowledge of the authors, this is the first time to apply presented Galerkin finite element method with the given Cubic Hermite B-spline basis functions to the presented prototype model problem. To give a clear explanation of the subject, the problem used in the present study is selected as a fundamental prototype problem having an exact solution. By this way, it becomes possible to be

able to test how close the applied numerical scheme produces approximate solutions to the exact solution by using widely used error norms L2 and L∞. The numerical results found out by the application of the method are presented in tables together with the error norms L2 and L∞. By the application of the computer program, newly obtained results will be illustrated in graphs and tables for some values obtained as a result of the computer simulation, and finally a clear comparison is going to be made with those of some other studies available in the literature.

Key Words: Cubic Hermite B-spline; Galerkin method; Shifted Legendre and Chebyshev polynomials Roots, Finite Element Method.

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REFERENCES

[1] J.N. Reddy, An introduction to the finite element method, McGraw-Hill, New York, 1993.

[2] S. B. G. Karakoç, Y. Uçar and N.M. Yağmurlu, Numerical Solutions of the MRLW Equation by Cubic B-spline Galerkin Finite Element Method, Kuwait J. Sci. 42(2) (2015),141-159.

Analysis of selected factors on construction finance index with artificial neural networks and their ODE model

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ABSTRACT

Construction cost index is a price index that measures the changes in construction costs in terms of labor and material. There are many factors affecting this index. In this study, inputs (x) given as the USD opening value, consumer price index (CPI Annual % Change) and Bank housing loan TL Interest rate and output (y) given as construction finance index residential purpose total obtained from TUİK were analyzed with artificial neural networks (ANN). The data between 01.01.2015-01.05.2024 were handled monthly. The data were subjected to min/max normalization before analysis. Then, a linear activation function in ANN was found and thus the weights of the inputs on the output were shown. Then, the coefficients were obtained with the MATLAB R2023b program by considering the activation function in the form of y=b2+LW*tanh(b1+IW*x). In this way, the error was further reduced compared to the linear function. In the other part of the study, the total of 4 variables given above was presented along the linear differential equation system depending on time. The coefficients in the system were calculated with minimum error by means of real values. In this way, the change of each variable depending on time was given.

Key Words: Construction cost index, ANN, Activation function, linear differential equation system.

REFERENCES

[1] Elfahham, Y. (2019). Estimation and prediction of construction cost index using neural networks, time series, and regression. Alexandria Engineering Journal, 58(2), 499-506.

[2] Aydınlı, S. (2022, October). Causal Relationships between Construction Cost Index and Macroeconomics in Turkey. In 7th International Project and Construction Management Conference (IPCMC2022) (pp. 525-33).

[3] Aslam, B., Maqsoom, A., Inam, H., Basharat, M. U., & Ullah, F. (2023). Forecasting Construction Cost Index through Artificial Intelligence. Societies, 13(10), 219.

Analysis of some partial differential equation as a model of nonlinear oscillation

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ABSTRACT

In this work we study the flexure of thin plates, which are caused by the expansive soil movements [1, 2, 3]. Experimental data indicates that the expansive soil can be very dangerous to underground thin plates [4].

We study the well-known equation of small vibration of thin plates [5]:

$$\frac{\partial^2}{\partial x^2}\left[D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)\right]+ \quad (1)$$

$$2(1-\nu)\frac{\partial^2}{\partial x\partial y}\left(D\frac{\partial^2 w}{\partial x\partial y}\right)+\frac{\partial^2}{\partial y^2}\left[D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)\right]-q=0,$$

where *E* is Young's modulus (*E* > 0), ν is Poisson's ratio $(0<\nu<\frac{1}{2})$, q is the distributed load, and *D* is flexural rigidity of the plate, which is determined by formula

$$D=\frac{Eh^3}{12(1-\nu^2)}, \quad (2)$$

where *h* is the thickness of the plate.

In the case of constant cross section, we have $h=h_0=const$, $D=D_0=const$. And Eq. (1) can be rewritten in the following form

$$\Delta^2 w=\frac{q}{D}. \quad (3)$$

Considering upward positive, then $w(x,y)$ and swelling loads are positive, while the weight of the plate is negative. Consequently, Eq. (3) should have the following form:

$$\Delta^2 w=Q_g(x,y,w)-Q_s(x,y), \qquad \forall(x,y)\in\Omega\subset\mathrm{R}^2, \quad (4)$$

where Q_s is the distributed weight load, and Ω is the middle plane of the undeformed plate, and Q_g is the distributed load of the expansive soil. Further, the thin plate was assumed to be clamped along the edges:

$$w|_{\partial\Omega}=0, \qquad \frac{\partial}{\partial\vec{n}}w|_{\partial\Omega}=0, \quad (5)$$

where $\frac{\partial}{\partial \vec{n}}$ is the normal derivative along the boundary $\partial\Omega$.

We denote by $w_0(x, y)$ the solution of the following problem

$$\Delta^2 w_0 = Q_s(x, y), \qquad \forall (x, y) \in \Omega \subset \mathrm{R}^2, \tag{6}$$

$$w_0|_{\partial\Omega} = 0, \qquad \frac{\partial}{\partial \vec{n}} w_0|_{\partial\Omega} = 0. \tag{7}$$

It is well-known [6], that there exists an unique solution of the linear problem (6), (7). Let $\tilde{u}(x, y) = w(x, y) - w_0(x, y) \quad \forall (x, y) \in \Omega$. Then $\upsilon(x, y)$ is the solution of the following nonlinear boundary value problem.

Problem 1.

$$\Delta^2 \upsilon = \alpha k(x, y) e^{-\upsilon(x,y)}, \qquad \forall (x, y) \in \Omega \subset \mathrm{R}^2, \tag{8}$$

$$\upsilon|_{\partial\Omega} = 0, \qquad \frac{\partial}{\partial \vec{n}} \upsilon|_{\partial\Omega} = 0, \tag{9}$$

where $\upsilon(x, y) = \alpha \tilde{u}(x, y)$, $\alpha = \frac{1}{H_\alpha C_b}$ and $k(x, y) = \sigma_0 D_p e^{-\alpha w_0(x,y)}$.

In the following, $C(\Omega)$ denotes the class of continuous real valued functions in Ω.

We denote by $C^{(4)}(\Omega)$ the set of functions $u(x, y)$ whose partial derivatives, up to and including order four, are all continuous on Ω. And $C^{(4)}_{w0}(\Omega)$ denotes the set of functions $\upsilon(x, y) \in C^{(4)}(\Omega)$ that satisfy inequality $0 \leq \upsilon(x, y) \leq w_0(x, y)$ for all $(x, y) \in \Omega$.

Further only the circular plate is studied, i.e. $\Omega = \{(x, y) \in \mathrm{R}^2 : x^2 + y^2 < 1\}$.

Theorem 1. *There exists an unique solution $\upsilon(x, y)$ of the nonlinear problem (1) in $C^{(4)}_{w0}(\Omega)$.*

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Key Words: Partial differential equation, model of nonlinear oscillation, theory of oscillation equations.

REFERENCES

[1] J.M. Makar, R. Desnoyers, S.E. McDonald, Failure modes and mechanisms in gray cast iron pipe, Ontario, Canada, (2001) 1-10.

[2] B. Rajani, C. Zhan, S. Kuraoka, Pipe-soil interaction analysis of jointed water mains, Can. Geotech. J. 33(3) (1995) 393 404.

[3] Y.E.A. Rjeily, M.F. Khouri, Longitudinal Stress Analysis of Buried Pipes under Expansive Soils, Inter. J. Sci. Res. 3(11) (2014) 2592-2599.

[4] Y. Hu, H.Q. Vu, Field performance of water mains buried in expansive soil, First International Structural Specialty Conference, Calgary, Alberta, (2006).

[5] V. Komkov, Optimal control of vibrating thin plates, SIAM J. Contr. 8(2) (1970) 273-304.

[6] V.S. Vladimirov, Equations of mathematical physics, Mir, Moscow,1984.

Analysis of the Travelling Wave Solutions of the Estevez-Mansfield-Clarkson Equation

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ABSTRACT

In this study, the Estevez–Mansfield–Clarkson equation is considered to be analyzed for its wave solutions. To implement this purpose the exponential function method is used and ultimately new hyperbolic, trigonometric and rational forms of the exact solutions are obtained. Besides this, the two and three dimensional graphics together with the contour and density plots are presented. Chemical engineers use the Estevez–Mansfield–Clarkson equation extensively for studying surface reactions and gas adsorption. The behavior of gases adsorbed onto solid surfaces may be modeled and analyzed using this equation, which is essential for process design and optimization in a variety of industries, including materials science, environmental control, and catalysis. In order to create effective catalytic processes, it is necessary to comprehend how gases interact with surfaces, which is something that the equation assists with. Reactor and other chemical process performance can be improved with accurate adsorption modeling. Understanding the kinetics of surface reactions is essential for creating new materials and enhancing those that already exist. Everything from industrial catalysts to sensors may be impacted by this. The equation can be used in environmental science to investigate and enhance techniques for ensnaring and eliminating contaminants from gases, such as in water and air treatment systems. It facilitates the creation of novel materials with particular adsorption qualities that find utility in filtration and separation technologies, among other fields. In general, the Estevez–Mansfield–Clarkson equation is a useful resource for comprehending and refining gas-solid interaction processes.

Key Words: The Estevez–Mansfield–Clarkson equation, Exponential function method, Mathematica software.

REFERENCES

[1] N. Raza and A. H. Abdel-Aty, Traveling wave structures and analysis of bifurcation and chaos theory for Biswas-Milovic Model in conjunction with Kudryshov's law of refractive index, *Optik* (2023), 171085.

[2] A. Saha, B. Pradhan and S. Banerjee, Bifurcation analysis of quantum ion-acoustic kink,anti-kink and periodic waves of the Burgers equation in a dense quantum plasma, *Eur Phys J Plus* 135(2) (2020), 216.

[3] L. Tang, Bifurcations and dispersive optical solitons for the cubic–quartic nonlinear Lakshmanan–Porsezian–Daniel equation in polarization-preserving fibers, Optik 270 (2022), 170000.

[4] L. Tang, Bifurcations and dispersive optical solitons for the cubic–quartic nonlinear Lakshmanan–Porsezian–Daniel equation in polarization-preserving fibers, Optik 270 (2022), 170000.

[5] Y. Zhen-Ya, Abundant symmetries and exact compacton-like structures in the twoparameter family of the Estevez–Mansfield–Clarkson equations. *Commun Theor Phys 37(1)* (2002), 27.

[6] H. K. Barman, M. E. Islam and M. A. Akbar, A study on the compatibility of the generalized Kudryashov method to determine wave solutions, *Propuls Power Res*. 10(1) (2021), 95–105.

[7] M. M. Al-Sawalha, S. Noor, S. Alshammari, A. H. Ganiem and A. Shafee, Analytical insights into solitary wave solutions of the fractional Estevez-Mansfield-Clarkson equation, *AIMS Math*. 9(6) (2024), 13589–13606.

Analytical and Numerical Solutions of Nonlinear Differential Equations Based on Power-Law Model

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ABSTRACT

This work presents a comprehensive study of the generalized viscous Burgers' equation under the framework of a power-law fluid model. The Burgers' equation, widely used in modelling various physical phenomena such as fluid dynamics is traditionally based on Newtonian fluid assumptions. However, many real-world materials exhibit non-Newtonian behavior, necessitating more generalized approaches for solving nonlinear differential equations governed by power-law models. Power-law models have been widely used to describe systems with scaling properties, non-uniform growth, and self-similarity. In this work, we provide some analytical and numerical solutions of a viscous Burgers' equation for the isothermal flow of power-law non-Newtonian fluids:

$$\rho \frac{\partial u}{\partial t}(x,t) = \mu \frac{\partial}{\partial x}\left(\left|\frac{\partial u}{\partial x}\right|^{m-1} \frac{\partial u}{\partial x}\right),$$

where ρ is the density, μ the viscosity, u the velocity of the fluid in x direction and m the power-law index. The equation augmented with the initial condition $u(0,x) = u_o(x) = \sin \pi x,\ 0 < x < 1$ and boundary condition $u(t,0) = u(t,1) = 0$.

We study the existence and uniqueness of solutions of the Burgers equation in Sobolev space. Moreover, we construct numerical solutions to the problem using the high-level modelling and simulation package COMSOL Multiphysics. We explore the effects of different power-law indices on the evolution of the solution and compare the efficiency and accuracy of the proposed numerical method with the exact analytical solutions.

Key Words: non-Newtonian fluids, Analytic solutions, Numerical solutions, Burger's equation, Power-law, Sobolev space, COMSOL Multiphysics.

REFERENCES

[1] D. Wei and H. Borden, "Traveling wave solutions of Burgers' equation for power-law non-Newtonian flows," *Appl. Math. E-Notes*, vol. 11, pp. 1-11, 2011. [Online]. Available: https://www.researchgate.net/publication/265241427_Traveling_Wave_Solutions_of_Burgers'Equation_for_Power-Law_Non-Newtonian_Flows

[2] F. N. YILMAZ and B. Karasözen, "Solving optimal control problems for the unsteady Burgers equation in COMSOL Multiphysics," *JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS*, pp. 4839–4850, 2011, Accessed: 00, 2020. [Online]. Available: https://hdl.handle.net/11511/32104.

[3] P. Orlandi, "Fluid Flow Phenomena: A Numerical Toolkit". Dordrecht, Netherlands: Springer, 2000. doi: 10.1007/978-94-011-4281-6.

[4] Y. Benia and B.-K. Sadallah, "Existence of solutions to Burgers equations in domains that can be transformed into rectangles," *Electron. J. Differential Equations*, vol. 2016, pp. 1–13, 2016.

Analytical study on a new conformable (3+1)-dimensional extended Korteweg-de Vries equation

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ABSTRACT

Because of its broad applicability, fractional partial differential equations have drawn much interest from academics lately. These equations are used to model various phenomena that occur in real-world situations. One of these equations are the well-known Korteweg-de-vries (KdV) and Boussinesq equations. This study introduces new analytical solutions to the (3+1)-dimensional extended KdV equation. These extension include a second-order time derivative term similar to Boussinesq equation. For this purpose, sub-equation method is used to obtain new analytical solutions. This method is based on the Riccati equation and its solutions. The present equation has been used to model a wide range of nonlinear phenomena occurring in plasma physics, fluid mechanics, tsunami events, and other scientific disciplines. It can also be used to analyse the properties of the above-mentioned equation. Numerous analytical solutions that were not discovered in the literature are acquired, and a few of the solutions' graphical representations are provided.

Key Words: Sub-equation method, analytical solution, extended Korteweg-de Vries equation.

REFERENCES

[1] Wazwaz, A. M., Alhejaili, W., Matoog, R. T., & El-Tantawy, S. A. (2024). Painlevé integrability and multiple soliton solutions for the extensions of the (modified) Korteweg-de Vries-type equations with second-order time-derivative. *Alexandria Engineering Journal*.

[2] Akinyemi, L., Şenol, M., & Iyiola, O. S. (2021). Exact solutions of the generalized multidimensional mathematical physics models via sub-equation method. *Mathematics and Computers in Simulation*, *182*, 211-233.

[3] Alhojilan, Y., Ahmed, H. M., & Rabie, W. B. (2023). Stochastic solitons in birefringent fibers for Biswas–Arshed equation with multiplicative white noise via Itô calculus by modified extended mapping method. *Symmetry*, *15*(1), 207.

[4] Gencyigit, M., Şenol, M., & Koksal, M. E. (2023). Analytical solutions of the fractional (3+ 1)-dimensional Boiti-Leon-Manna-Pempinelli equation. *Computational Methods for Differential Equations*, *11*(3), 564-575.

A New Generalization of the Gaussian k-Pell Numbers and Their Polynomials

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ABSTRACT

Fibonacci and Lucas numbers have received increasing attention in the last few decades. It is extensively studied in several fields of mathematics, such as linear algebra, calculus, and applied mathematics. In addition, Gaussian numbers, for which various studies have been carried out, were introduced as complex Fibonacci numbers by Horadam in 1963. The Pell numbers, another significant and noteworthy numbers, similar to the Fibonacci and Lucas numbers, is determined by the recurrence

$$P_n = 2P_{n-1} + P_{n-2}$$

for $n \geq 2$, with initial values $P_0 = 0$ and $P_1 = 1$. Using the Pell recurrence relation as an analogy, the set of Gaussian Pell numbers is defined based on this definition.

In this study, we make generalizations of the well-known Gaussian Pell numbers and indicate this new class as generalized Gaussian k-Pell numbers. Furthermore, we provide relationships between the Gaussian Pell numbers and the generalized Gaussian k-Pell numbers. In the same manner, as an extension of the Gaussian Pell polynomials in Yağmur's work in 2019, we identify a novel class of generalized Gaussian k-Pell polynomials and explain the relationships between generalized Gaussian k-Pell polynomials and Gaussian Pell polynomials. Then, we determine the matrix representation for new generalization of these numbers and polynomials, and demonstrate and prove Cassini's identity of them.

Key Words: Gaussian Pell numbers, Gaussian Pell polynomials, Cassini's identity.

REFERENCES

[1] A. F. Horadam, Complex Fibonacci numbers and Fibonacci quaternions, American Mathematical Monthly, (1963), 70, 289–291

[2] G. Berzsenyi, Gaussian Fibonacci numbers, The Fibonacci Quarterly, (1977), 15(3), 233–236.

[3] S. Halıcı and S. Öz, On Gaussian Pell polynomials and their some properties, Palestine Journal of Mathematics, (2018), 7(1), 251–256.

[4] S. Halıcı and S. Öz, On Some Gaussian Pell and Pell–Lucas numbers. ¨ Ordu University Journal of Science and Technology, (2016), 6(1), 8–18.

[5] M. Taştan and E. Özkan, On the Gauss k-Fibonacci polynomials, Electronic Journal of Mathematical Analysis and Applications, (2021), 9(1), 124–130.

[6] T. Yağmur, Gaussian Pell–Lucas polynomials, Communications in Mathematics and Applications, (2019), 10(4), 673–679.

A Novel Method for Exact Solutions of the Generalized Huxley-Burgers' Family

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ABSTRACT

The objective of this study is to investigate the analytical solutions of fractional-order partial differential equations categorized as generalized equations. This study uses the extended Kudryashov method to address the fractional-order generalized Burgers-Huxley equation, which provides insight into the evolutionary aspects of nerve impulse propagation, considering the interaction between reaction mechanisms and diffusion transports, the fractional-order generalized Huxley equation, which deals with nerve impulse propagation in nerve fibers and the motion of liquid crystals in walls, the fractional-order generalized Fisher equation, which helps us to analyse population dynamics, taking into account factors such as natural selection and migration, the fractional-order generalized Burgers equation, which studies the behaviour of weakly non-linear acoustic waves travelling in one direction through gas-filled pipes. Through this approach, exact solutions to these equations are derived, and we visualise the obtained results in 3D using Maple. The results underline the effectiveness and reliability of the extended Kudryashov method as a clear and efficient tool for solving the specified equations.
A significant advantage of this method is its capacity to simultaneously yield multiple exact solutions for equations presented in a generic format. The clarity, efficiency, and reliability of this approach establish it as a robust tool for generating novel exact solutions.

Key Words: The fractional order generalized nonlinear differential equations, extended Kudryashov method, conformable fractional derivative.

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REFERENCES

[1] Ş. M. Ege and E. Mısırlı, Extended Kudryashov method for fractional nonlinear differential equations, Mathematical Sciences and Applications E- Notes 6.1 (2018), 19-28.

[2] A. R. Hadhoud, F. E. Abd Alaal, A. A. Abdelaziz, T. Radwan, Numerical treatment of the generalized time-fractional Huxley- Burgers' equation and its stability examination, De Gruyter, 54(1), (2021), 436-451.

[3] S. E. Tutam, M. Akar, Exact solutions of the generalized Huxley-Burgers' equations, Modern Physics Letters B, (2024), 2450452.

Application of Laplace Transforms in Cryptographic Technique with Bessel Function

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ABSTRACT

In this research, we present a novel Laplace transform-based encryption method. The recommended encryption technique uses the inverse Laplace transform to decrypt data and the Laplace transform of Bessel functions to encode plaintext. This approach highlights the significant relationship between mathematical theory and practical applications of cryptography by demonstrating the potential of Bessel functions and Laplace integral transformations in the field of encryption. Although they are not frequently associated with conventional cryptographic methods, Bessel functions and Laplace transforms offer valuable insights into the development and evaluation of encryption algorithms. Bessel functions are widely used in communications, signal processing, and error-correcting codes. They are obtained by solving Bessel's differential equation.

Their relevance in encryption technology stems from their ability to represent intricate waveforms and their use in modulation and demodulation procedures. Bessel functions in particular can be used to build encryption techniques that depend on waveform manipulation and signal modification. Moreover, a strong basis for investigating novel cryptographic methods is provided by the transformational qualities of Laplace transforms combined with the mathematical attributes of Bessel functions, such as orthogonality and completeness. Innovative encryption techniques

that strengthen security characteristics and boost computational efficiency can be created by looking at these qualities in the context of theoretical cryptography. Hence, this work advances knowledge about the use of sophisticated mathematics in the creation of safe and effective cryptographic systems. Similarly, some other integral transformations have also been used in encryption problems, such as the Sumudu, Natural, Kamal transforms, etc.

Key Words: Bessel function, Laplace transform, cryptography, encryption, decryption

REFERENCES

[1] Bodkhe, D. S., & Panchal, S. K. Application of Sumudu transform in cryptography. In *International Conference on Mathematical Sciences, Elsevier* (2014). (pp. 928-931).

[2] Briones, R. P. (2018). Modification of an encryption scheme based on the laplace transform. *International Journal of Current Research*, *10*(7), 71759-71763.

[3] Debnath, Lokenath, and Dambaru Bhatta. *Integral transforms and their applications*. Chapman and Hall/CRC, 2016.

[4] Hiwarekar, A. P. New mathematical modeling for cryptography. *Journal of Information Assurance and Security*, *9* (2014), 027-033.

[5] Poularikas, A. D., & Grigoryan, A. M. *Transforms and applications handbook*. CRC press, 2018

[6] Rekha, G. Data Encryption and Decryption using some Integral Transforms. *Advances in Nonlinear Variational Inequalities*, *27*(2), (2024), 375-382.

[7] Undegaonkar, D. K. H. K. Security in communication by using laplace transform and cryptography. *International Journal of Scientific & Technology Research*, *8*(12), (2019). 3207-3209.

[8] Yılmazer, M. Ç., Yılmaz, E., Gulsen, T., & Et, M. DNA Secret Writing With Laplace Transform of Mittag-Leffler Function. *Journal of Mathematical Sciences and Modelling*, *6*(3), (2023) 120-132.

Boundedness and dynamical behavior of the solutions of a fuzzy difference equations system with higher-order

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ABSTRACT

We concentrate on the following fuzzy difference equations system with higher-order

$$\beta_{n+1} = \eta_1 + \frac{\sum_{i=1}^{m} \beta_{n-i}}{\gamma_n}, \gamma_{n+1} = \eta_2 + \frac{\sum_{i=1}^{m} \gamma_{n-i}}{\gamma_n}$$

where m is positive integer, the parameters η_1, η_2 and the initial values β_{-i}, γ_{-i} for $i \in \{0,1,\ldots,m\}$ are positive fuzzy numbers in this paper. Also, we study the existence and uniqueness, boundedness, persistence and convergence of positive solutions of mentioned system. Finally, we give some numerical examples to support the validity and effectiveness of our theoretical results obtained.

Our research focuses on mainly evaluating and examining a four-dimensional system generated through the Zadeh extension, as well as demonstrating boundedness, persistence, and various stability aspects such as local and global asymptotic stability at equilibrium point for mentioned system.

The theory of difference equations, the methods to solve them play an important role in many fields such as finance mathematics, computer sciences, control theory. Since conditional parameters may fluctuate or change within a certain range, one way to solve this kind of problems is the fuzzification of the difference equation which is based on the indeterminacy.

A fuzzy difference equation is an equation with fuzzy initial conditions and fuzzy parameters. Therefore, the solutions are also fuzzy numbers. Fuzzy numbers concept has widely many applications such as finance, population models, time series, medicine etc. due to the advances in technology.

Key Words: Fuzzy difference equations system, boundedness, persistence, convergence, Zadeh extension.

REFERENCES

[1] Gümüş, M. (2023). Global asymptotic behavior of a discrete system of difference equations with delays. *Filomat*, *37*(1), 251-264.

[2] Papaschinopoulos G. & Papadopoulos B.K. (2002). On the fuzzy difference equation xn+1 = A+ B/xn , Fuzzy Sets and Systems 129, 73–81

[3] Yalçınkaya, İ., Tollu, D.T., Khastan, A., Ahmad, H. & Botmart, T. (2023). Qualitative behavior of a higher-order fuzzy difference equation, AIMS Math 8(3), 6309–6322.

[4] Atpinar, S. & Yazlik, Y. (2023). Qualitative behavior of exponential type of fuzzy difference equations system, Journal of Applied Mathematics and Computing 69, 4135–4162.

Coercive Solvability of the Fifth Order Singular Differential Equation

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ABSTRACT

In this work we consider the fifth - order degenerate differential equation, which called the Kawahara type equation or the generalized Korteweg-de Vries equation. This equation is given on the real line, and its coefficients are assumed unbounded functions. The differential operator corresponding to such equation is not semi-bounded in the Hilbert space. So its study requires the use of new methods. We obtain the sufficient conditions for the existence and uniqueness of the solution in the Hilbert space, as well as the conditions for the fulfillment of the coercive estimate of a solution. These issues are important in the theory of odd-order differential equations with variable coefficients.

The Kawahara equation describes the propagation of one-dimensional nonlinear waves in a dispersive medium. Stationary nonlinear Kawahara type equation with a variable coefficients is reduced to the degenerate differential equation as a result of linearization. When the coefficients of these equations are bounded and smooth, these linear equations and its some nonlinear generalizations were studied by A. Birgebaev and M. Otelbaev [1], and V. Faminsky, M.A. Opritova [2]. The study of the correctness of equation and maximal regularity of the solution in the case when its coefficients are unbounded functions is a natural continuation of the above works.

Key Words: Fifth - order differential equation, closure, coercive estimate.

REFERENCES

[1] A. Birgebaev, M. Otelbaev, Smoothness of solutions to a third-order nonlinear differential equation. J. Izv. AN Kaz SSR. Ser. physics and mathematics, No. 3, 1984

[2] M.A. Opritova, A.V. Faminskii, On the Cauchy problem for the generalized Kawahara equation. Diff. Equat., No. 3, 2019, 378-390.

Comparative Analysis of Interpolation Methods

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ABSTRACT

Interpolation is a fundamental mathematical technique used to estimate unknown values within the bounds of known data points. This presentation delves into five prominent interpolation methods: Linear Interpolation, Polynomial Interpolation, Spline Interpolation, Radial Basis Function (RBF) Interpolation, and Kriging Method, highlighting their applications, mathematical foundations, and relative strengths and weaknesses.

The Linear Interpolation method is the simplest among the five, offering a quick and straightforward solution by drawing a straight line between two known data points to estimate intermediate values. Its primary advantage lies in its computational efficiency, but it is limited in precision, especially for datasets that exhibit non-linear structures. Polynomial Interpolation, in contrast, fits a single polynomial to the entire dataset, offering more accuracy in capturing the underlying trend, particularly for small datasets. However, it is prone to significant errors, especially when using high-degree polynomials. Spline Interpolation overcomes some of the drawbacks of Polynomial Interpolation by using piecewise polynomials to ensure smooth transitions between data points. Particularly, cubic splines are favored for their balance between precision and computational complexity. Radial Basis Function (RBF) Interpolation is particularly effective for high-dimensional data and in scenarios where capturing intricate relationships between data points is crucial. The Kriging method is a geostatistical interpolation technique that excels in spatial data analysis.

The presentation also compares these methods using TOPSİS across four key criteria: precision, computation speed, flexibility, and error margin. Linear Interpolation, while fast, is the least precise. Polynomial Interpolation can be precise but risks high error margins with increasing polynomial degree. Spline Interpolation offers a good balance of precision and flexibility, while RBF Interpolation provides

flexibility at the cost of computational speed. Kriging stands out for its low error margin despite requiring higher computation.

In conclusion, the choice of an interpolation method depends on the specific requirements of the task at hand, including the nature of the data, desired accuracy, and available computational resources. The purpose of this presentation is to provide a comprehensive understanding of the methods discussed and facilitate informed decision making in their application in various fields.

Key Words: Interpolation methods, data analysis, spline techniques.

REFERENCES

[1] C. De Boor, Bicubic spline interpolation, J. Math. Phys., vol.41, no.1 (1962), 212-218.

[2] C. de Boor, On calculating with B-splines, J. Approx. Theory, vol.6, no.1 (1972), 50-62.

[3] D. E. Myers, "Kriging, cokriging, radial basis functions and the role of positive definiteness," Comput. Math. Applicat., vol.24, no.12 (1992), 139-148.

[4] L. Schumaker, Spline functions: basic theory. Cambridge mathematical library, 1981.

[5] M. Gasca and T. Sauer, On the history of multivariate polynomial interpolation, J. Comput. Appl. Math., vol.122, no.1-2 (2000), 23-35.

[6] O. Dubrule, Comparing splines and kriging, Comput. Geosci., vol.10, no.2/3 (1984), 327-338.

Dinesh Verma Transform And Hyers-Ulam Stability Of fractional Linear Differential Equation Involving Caputo Fractional Derivative

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ABSTRACT

The importance of fractional differential equations has significantly increased in recent years due to their wide range of applications in physics and engineering. These equations have been studied for over 300 years, beginning with a question posed by L'Hospital to Leibniz in a letter dated 1695. Recently, many researchers have focused on the Hyers-Ulam stability of fractional differential equations.

The concept of Hyers-Ulam stability first emerged in 1940 at the University of Wisconsin, in connection with a problem concerning the stability of functional equations. In 1941, Hyers [1] introduced the concept with a theorem that provided a solution to this problem, laying the foundation for future research in this area.

In 2015, Wang and Xu [2] presented the following two type of fractional differential equations with Caputo fractional derivatives

$$ {}^{C}D^{\alpha}_{0^+}\, y(t) - \lambda y(t) = f(t), $$

and

$$ {}^{C}D^{\alpha}_{0^+}\, y(t) - \lambda({}^{C}D^{\beta}_{0^+}\, y)(t) = g(t), $$

where $t > 0, \lambda \in \mathbb{R}, n-1 < \alpha \leq n, m-1 < \beta \leq m, 0 < \beta < \alpha, m, n \in \mathbb{N}, m \leq n,$ $f(t)$ and $g(t)$ are real functions defined on $\mathbb{R}^+$ and ${}^{C}D^{\alpha}_{0^+}$ is the Caputo fractional derivative of order α defined by

$$ ({}^{C}D^{\alpha}_{0^+}\, y)(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-x)^{n-\alpha-1}\, y^{n}(x)dx. \qquad (1) $$

They demonstrated the Hyers-Ulam stability of two types of fractional linear differential equations by using the Laplace transform.

In 2017, Shen and Chen [3] addressed the stability issue of linear fractional

differential equations with constant coefficients by employing the Laplace transform method.

In 2020, Jarad and Abdeljawad [4] examined the following Cauchy problem within the context of generalized Caputo fractional derivative

$$ {}_{a}^{C}D_{g}^{\alpha} y(t) - \lambda y(t) = f(t), t > a, 0 < \alpha \leq 1, \lambda \in \mathbb{R}, $$

$$ y(a^{+}) = c, \qquad c \in \mathbb{R}. $$

Motivated by the ideas of Wang and Xu [2] and Jarad and Abdeljawad [4] (for $g(t) = t \; and \; a = 0$), this paper presented the stability of the Hyers-Ulam type for linear fractional differential equations using the Dinesh Verma Transform (DVT):

$$ {}^{C}D_{0^{+}}^{\alpha}\, y(t) - \lambda y(t) = f(t). $$

Here, ${}^{C}D_{0^{+}}^{\alpha}$ is defined in (1) and Dinesh Verma transform is defined by

$$ \mathfrak{D}\{f(t)\} = p^{5} \int_{0}^{\infty} e^{-pt} f(t) dt, $$

where $f(t)$ is a well-defined function of real numbers $t \geq 0$, p may be a real or complex parameter.

Key Words: Caputo fractional derivative, Hyers-Ulam stability, Dinesh Verma transform.

REFERENCES

[1] D. H. Hyers, On the Stability of the Linear Functional Equation, *Proceedings of the National Academy of Sciences*, vol. 27, no. 4, 1941, doi: 10.1073/pnas.27.4.222.

[2] C. Wang and T. Z. Xu, Hyers-Ulam stability of fractional linear differential equations involving Caputo fractional derivatives, *Applications of Mathematics*, vol. 60, no. 4, 2015, doi: 10.1007/s10492-015-0102-x.

[3] Y. Shen and W. Chen, Laplace Transform Method for the Ulam Stability of Linear Fractional Differential Equations with Constant Coefficients, *Mediterranean Journal of Mathematics*, vol. 14, no. 1, 2017, doi: 10.1007/s00009-016-0835-0.

[4] F. Jarad and T. Abdeljawad, Generalized fractional derivatives and Laplace transform, 2020. doi: 10.3934/dcdss.2020039.

Dynamical Analysis of a Prey-Predator Model Including Fear, Its Carry-Over Effect and Allee Effect with Memory Effect

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ABSTRACT

In ecological systems, predation is a fundamental interaction influencing species survival and biodiversity. Understanding the long-term dynamics of prey-predator relationships necessitates examining various models that incorporate different biological phenomenas. In this study, we investigate a predator-prey model that combines fear, its carry-over effect and Allee effect. The indirect effects of predators can act as a suppressive factor on the reproduction of prey populations. We incorporate the classic fear factor and extend it to include the concept of carry-over effect, which refers to how an individual's past experiences and history influence their current performance in specific situations. Additionally, we account for the Allee effect in the prey population, an important ecological mechanism driven by factors like mate limitation, cooperative defense, cooperative feeding, and environmental conditioning. Given that many biological systems inherently exhibit memory effects, we employ the Caputo fractional derivative, which is beneficial due to its non-local properties. This approach helps us better understand how past experiences influence current hunting behavior, leading to more realistic outcomes. Our research provides deeper insights into predator-prey dynamics and introduces a novel method for modeling ecological systems with memory effects. The stability analysis of the equilibrium points of the system is performed and the existence of a Hopf bifurcation around the positive equilibrium point is investigated. Then the theoretical findings are verified by numerical results and interpreted biologically.

Key Words: Carry-over, Allee effect, Caputo fractional derivative.

REFERENCES

[1] C.M. O'Connor, D.R. Norris, G.T. Crossin and S.J. Cooke, Biological carry-over effects: linking common concepts and mechanisms in ecology and evolution, Ecoshere 5 (3) (2014) 1-11.

[2] E. Balcı, Predation fear and its carry-over effect in a fractional order prey-predator model with prey refuge, Chaos, Solitons & Fractal 175 (2023) 114016

[3] E. Gonzalez-Olivares, B. Gonzalez-Yanez, J.M. Lorca, A. Rojas-Palma, and J.D. Flores, Consequences of double allee effect on the number of limit cycles in a predator-prey model, Computers & Mathematics with Applications 62 (9) (2011) 3449-3463.

Dynamics of a three-dimensional system of difference equations with exponents

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ABSTRACT

In this study, we consider the following three-dimensional system of difference equations

$$\begin{cases} x_{n+1} = \alpha_1 + a\, e^{-x_{n-1}} + b y_n e^{-y_{n-1}}, \\ y_{n+1} = \alpha_2 + c\, e^{-y_{n-1}} + d z_n e^{-z_{n-1}}, \quad n \in \mathbb{N}_0, \\ z_{n+1} = \alpha_3 + f\, e^{-z_{n-1}} + g x_n e^{-x_{n-1}}, \end{cases}$$

where the parameters $\alpha_1, \alpha_2, \alpha_3, a, b, c, d, f, g$ are positive real numbers and the initial conditions $x_{-i}, y_{-i}, z_{-i},\ i \in \{0,1\}$, are arbitrary nonnegative real numbers. Also, the boundedness, persistence, invariance, convergence and the global asymptotic behaviour of the positive solutions of mentioned system are studied.

Key Words: Boundedness, invariance, global behaviour.

REFERENCES

[1] D.S. Dilip, and S.M. Mathew, Dynamics of a second-order nonlinear difference system with exponents, Journal of the Egyptian Mathematical Society 29, (2021), 1-10.

[2] K. S. Al-Basyouni and E.M. Elsayed, On some solvable systems of some rational difference equations of third order, Mathematics 11(4), 2023, 1047.

[3] J.G. AL-Juaid, Qualitative behavior of sixth order of rational difference equation, International Journal of Analysis and Applications 22, (2024), 106-106.

[4] S.M. Mathew and D.S. Dilip, Dynamics of a second order three species nonlinear difference system with exponents, Proyecciones (Antofagasta) 41(4), (2022), 983-997.

Exact Solutions of the Fractional Benjamin-Ono Equation Using Jacobi Elliptic Functions

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ABSTRACT

This study, we investigate the fractional Benjamin-Ono equation and the use of Jacobi elliptic functions to obtain exact solutions to this equation. The fractional Benjamin-Ono equation is an extension of the classical Benjamin-Ono equation, incorporating fractional derivatives to model the propagation of nonlinear dispersive waves. This equation has been generalized to capture more complex wave dynamics in physical systems, with significant applications in fields such as water waves, plasma physics, and optics.

Jacobi elliptic functions provide a powerful analytical tool for solving such nonlinear differential equations. In our work, we utilize Jacobi elliptic functions to derive exact solutions to the fractional Benjamin-Ono equation. These solutions are further analyzed and visualized through 3D modeling using the Maple software, allowing for a comprehensive examination of physical parameters such as wave amplitude, speed, and periodic structures. The obtained solutions offer crucial insights into the nonlinear behavior of waves and reveal the effects of fractional derivatives in physical systems.

In conclusion, the use of Jacobi elliptic functions presents an effective method for solving the fractional Benjamin-Ono equation, and these solutions contribute to a deeper understanding of physical systems. The 3D modeling conducted in Maple enhances the visualization and analysis of these solutions, introducing new perspectives in the modeling of nonlinear dispersive systems and broadening the applicability of such equations across various fields.

Key Words: Fractional Benjamin-Ono equation, Jacobi Elliptic Function

REFERENCES

[1] Huai-Tang, Chen, and Zhang Hong-Qing. "New double periodic and multiple soliton solutions of the generalized (2+ 1)-dimensional Boussinesq equation." *Chaos, Solitons & Fractals* 20.4 (2004): 765-769.

[2] E. Pekönür and M. Akar, New soliton solutions of fractional Biswas–Milovic equation with Kerr, parabolic and cubic-quartic nonlinearities, Modern Physics Letters B (2024), 2450356.

[3] S. E. Tutam, M. Akar, Exact solutions of the generalized Huxley-Burgers' equations, Modern Physics Letters B, (2024), 2450452.

Exact Solutions of Fractional Biswas-Milovic Equation Using the Sardar Sub-equation Method

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ABSTRACT

This study presents new exact solutions of hyperbolic and trigonometric functions derived from the conformable fractional Biswas-Milovic equation, characterises long-distance optical communications, which is a generalisation of the nonlinear Schrödinger equation that has been at the center of the development of optical technologies, resulting nonlinear fibers and plasma physics. The fractional Biswas-Milovic equation is investigated in conformable derivative sense. It is used the analytical method, Sardar sub-equation, which is a powerful procedure that allows extracting the various types of solitons and search for some new solutions, such as hyperbolic and trigonometric functions. The resulting solutions are novel for the fractional Biswas-Milovic equation with the help of the method used, a powerful instrument for exploring precise solitary wave solutions for various other nonlinear equations in a nonlinear medium. In case special values are given to the parameters involving this approach, solitary wave solutions obtained from this method, which produces abundant solitons. Singular, bright–singular, bright and dark solitons are reported via parametric restrictions that shown with 3D graphics. These solutions find their application in communication to transmit information, because solitons could propagate over long distances without shrinking and without changing their shape. The results can have a great impact on many areas of nonlinear science.

Key Words: The fractional Biswas-Milovic equation, Sardar sub-equation method, solitons.

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REFERENCES

[1] S. Ahmadian and M. Darvishi, A new fractional Biswas–Milovic model with its periodic soliton solutions, Optik 127 (2016), 7694-7772.

[2] M. Foroutan, D. Kumar, J. Manafian and A. Hoque, New explicit soliton and other solutions for the conformable fractional Biswas–Milovic equation with Kerr and parabolic nonlinearity through an integration scheme, Optik 170 (2018). 190-203.

[3] E. Pekönür and M. Akar, New soliton solutions of fractional Biswas–Milovic equation with Kerr, parabolic and cubic-quartic nonlinearities, Modern Physics Letters B (2024), 2450356.

Existence of Solutions for a Fractional Derivative Boundary Value Problem with Integral and Multipoint Boundary Conditions

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ABSTRACT

The purpose of this study is to present sufficient conditions for the existence of solutions of the boundary value problem involving a nonlinear fractional differential equation that incorporates a broad form of the Caputo fractional derivative concerning a new function and integral, multi point boundary conditions. Existence and uniqueness of the solution is established using the fixed point theorems established by Banach, Schafer. At the end some examples are also given to illustrate our results.

Key Words: Boundary value problem, fractional derivative and integral, fixed point theorem.

REFERENCES

[1] Y. Wang, S. Liang and Q. Wang, Existence Results for Fractional Differential Equations with Integral and Multi-Point Boundary Conditions, Boundary Value Problems, 2018(4), 1-11

[2] H. A. Wahash, M. S. Abdo and S. K. Panchal, 2020. Positive Solutions for Generalized Caputo Fractional Differential Equations with Integral Boundary Conditions, Journal of Mathematical Modelling, 8(4) (2020), 393-414.

[3] R. Almeida, A. B. Malinowska and M. T. T Monteiro, 2018. Fractional differential equations with a Caputo derivative with respect to a kernel function and their applications. Mathematical Methods in the Applied Sciences, (1) (2018), 336-352.

Exploring Traveling Wave Solutions: Semi - Analytical Approaches

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ABSTRACT

In this study, we investigate the traveling wave solutions of nonlinear partial differential equation which describe processes that exhibit anomalous diffusion or memory effects by the using two powerful analytical methods: **Kudryashov's method** and **Sardar's method**. The equation, plays a crucial role in modeling nonlinear wave propagation and soliton dynamics in various physical systems, such as plasma physics and nonlinear optics. Kudryashov's method allows for the construction of exact solutions by balancing nonlinear and dispersive terms, while Sardar's method simplifies the equation into a set of solvable algebraic equations, offering semi-analytical solutions. By applying these methods, we derive exact and approximate traveling wave solutions that describe stable waveforms propagating through nonlinear media. The results highlight the efficiency of these techniques in handling nonlinear equations, providing new insights into soliton behavior and wave stability in complex systems. The significance of these solutions lies in their ability to model wave propagation in real-world systems with nonlinearity and dispersion, offering potential applications in various fields including optical communication, fluid dynamics, and quantum systems.

Key Words: Nonlinear partial differential equation, Kudryashov's method, Sardar's subequation method.

REFERENCES

[1] N. A. Kudryashov, Method for finding highly dispersive optical solitons of nonlinear differential equations. Optik International Journal for Light and Electron Optics 206 (2020) 1-9

[2] O. Kirci, T. Akturk, H. Bulut, Simulation of wave solutions of a mathematical model representing communication signals, Journal of the Institute of Science and Technology 11 (4) (2021) 3086-3097

[3] N.A. Kudryashov, Solitary waves of the non-local Schrodinger equation with arbitrary refractive index, Optik 231 (2021) 1-5

[4] S. M. Ege, Traveling wave solutions for two physical models via extended modified Kudryashov method, Journal of the Institute of Science and Technology 11 (1) (2021) 625-634

[5] L. Yan, H. M. Baskonus, C. Cattani, W. Gao Extraction of the gravitational potential and high frequency wave perturbation properties of nonlinear (3+1)-dimensional Vakhnenko Parkes equation via novel approach, Mathematical Methods in the Applied Sciences 5 (1) (2022).

Fractional-order model with Holling Type-2 functional response of the post-disaster period: Study for 2023 Elbistan Earthquake

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ABSTRACT

In this study, the mathematical model examining the relationships between the population that survived the disaster and continues to live in the disaster area, the population that migrated to another region, the number of newly constructed independent units in the disaster area after the disaster and the socio-economic development index (SEDI) variables in the disaster area was expressed with fractional-order differential equations (FODE). In addition, the relationship between the migrating and non-migrating population was presented in the model as Holling Type-2 functional response. Both existence and uniqueness of the solutions of the proposed model as well as their non-negativity and boundedness are shown. Additionally, the existence of a positive equilibrium point and its local asymptotic stability are demonstrated.

In real-world applications, data after the 1999 Gölcük earthquake were used in the model and the parameters, derivative order and functional response were determined by considering the minimum Root Mean Square Error (RMSE). Then, the performance of the proposed model with these values was shown for the 2023 Elbistan earthquake. Thus, 5-year and 10-year estimates of the non-migrating population, migrating population, the number of newly constructed independent units and SEDI index values for Elbistan were presented.

Public institutions and organizations aim to realize their social and economic target plans within the specified time period in order for a region to reach its pre-disaster population structure. Thanks to the results of the model we propose, predictions are made about whether they can reach these goals within the specified time period. The structure of the mathematical model with Holling type-2 functional response used

adds innovation to the literature in terms of analysis and adaptation of the results to the real-world example. In this context, the following questions are answered in this study for the February 6, 2023 Kahramanmaraş/Elbistan earthquake, which is a real-world example, with the model we propose:

• When can Elbistan reach its pre-disaster population size?

• How many new independent sections need to be built for Elbistan to reach its pre-disaster population size?

• What is the change in the population size living in Elbistan before the disaster and migrating to different regions after the disaster over time?

• What is the change in the SEDI value of Elbistan over time?

Although it was made specifically for Turkey and the earthquake situation, the model and analysis proposed here are highly functional in terms of applicability to natural disasters such as tsunamis and landslides occurring in different parts of the world.

Key Words: Fractional-order differential equation, Holling type-2, Earthquakes, Socio-economic development index, Root Mean Squared Error.

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REFERENCES

[1] Presidency, Disaster and Emergency Management, (2023). Available: https://www.afad.gov.tr/afet-turleri. [Accessed: 28.03.2023].

[2] N. Verdière, V. Lanza, R. Charrier, D. Provitolo, E. Dubos-Paillard, C. Bertelle and A. Alaoui, 2014. Mathematical modeling of human behaviors during catastrophic events, In 4th International Conference on Complex Systems and Applications (ICCSA2014), Paris.

[3] M. Ozaslan, B. Dincer and H. Ozgur, (2006). Regional disparities and territorial indicators in Turkey: Socio-economic development index (SEDI).

[4] Presidency of the Republic of Turkey, Strategy and Budget Directorate, 2023 Kahramanmaraş and Hatay Earthquakes Report, Available: https://www.sbb.gov.tr/wp-content/uploads/2023/03/2023-Kahramanmaras-ve-Hatay-Depremleri-Raporu.pdf. [Accessed: 15.04.2024].

Free Convective Flow in a Vertical Cylinder with Constant Mass Transfer and Heat Flux

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ABSTRACT

The goal of this study is to investigate the one-dimensional unsteady natural convection flow through an infinite vertical cylinder, the transfer of heat and mass when there is a constant heat flux at the surface of the cylinder, and how the parameters that are found affect the flow. The dimensionless, unsteady linear governing boundary layer equations are transformed into non-dimensional form, and an implicit finite difference method of the Crank-Nicolson type is used. The core of a nuclear reactor may need to be cooled rapidly in the event of a pump or power failure by using flow through vertical cylinders that transfer mass and heat. It is found that the solutions of the problem using the profile developed by the finite difference method give good results and are simpler and faster than the analytical solution methods developed for this problem. A simple and rapid method of solving this problem is essential. The impressive effects on both velocity, concentration, and temperature distribution for different numerical values of parameters such as Schimtd number, Prandtl number, mass Grashof number, thermal Grashof number, and time are investigated using graphs and tables. The Sherwood number and skin friction are also plotted. The results show that the concentration stabilises at very large values of time, while the temperature and velocity vary with time in an unbounded manner.

Key Words: Finite Difference Method, Vertical Cylinder, Heat Flux, Mass Transfer

REFERENCES

[1] Velusamy, K., & Garg, V. K. (1992). Transient natural convection over a heat generating vertical cylinder. International journal of heat and mass transfer, 35(5), 1293-1306.S.G. Gal, Shape-Preserving Approximation by Real and Complex Polynomials, Birkhäuser Boston, Inc., Boston, MA, 2008.

[2] Chen, T. S., & Yuh, C. F. (1980). Combined heat and mass transfer in natural convection along a vertical cylinder. *International Journal of Heat and Mass Transfer*, *23*(4), 451-461.Assis, E., & Kalman, H. (1993). Transient temperature response of different fins to step initial conditions. *International journal of heat and mass transfer*, *36*(17), 4107-4114.

[3] Rani, H. P. (2003). Transient natural convection along a vertical cylinder with variable surface temperature and mass diffusion. *Heat and mass transfer*, *40*(1), 67-73.

Lorentz para-Kenmotsu Manifolds Admitting η-Ricci Bourguignoun Solitons

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ABSTRACT

The concept of the Ricci-Bourguignon flow was introduced by J.P. Bourguignon, as an extension of the Ricci flow, based on an unpublished paper by Lichnerowicz and an article by Aubin. The Ricci-Bourguignon flow is an intrinsic geometric flow on Riemannian manifolds whose fixed points are solitons. Therefore, Ricci Bourguignon solitons produce identical similar solutions to the Ricci-Bourguignon flow defined as

$$\frac{\partial g}{\partial t} = -2(Ric - \rho r g), g(0) = g_0,$$

where Ric is the Ricci curvature tensor, r is the scalar curvature with respect to the semi-Riemannian metric g, and ρ is a nonzero real constant.

The quadruplet (g, ξ, λ, μ) on M satisfying the equation

$$\frac{1}{2} L_\xi g + S + 2(\lambda - \rho r)g + 2\mu\eta\otimes\eta = 0,$$

where (M, g) is a semi-Riemannian manifold, is called a η -Ricci Bourguignon soliton, where λ and μ are real constants, η is the dual of the vector field ξ and S is the Ricci curvature tensor of the manifold M.

Ricci solitons have attracted great attention especially in recent years and have been studied by many mathematicians. On the other hand, Lorentz para-Kenmotsu manifolds are a very important type of manifold for geometry, mathematics and physics. In this study, we have considered η -Ricci Bourguignon solitons on Lorentz para-Kenmotsu manifolds. We have examined η -Ricci Bourguignon solitons in detail on some special curvature tensors such as Riemann curvature tensor, projective curvature tensor, concircular curvature tensor, M-projective curvature tensors for Lorentz para-Kenmotsu manifolds. Especially, we have given important characterizations of Ricci pseudosymmetric and Ricci semisymmetric Lorentz para-Kenmotsu manifolds by using these curvature tensors and η -Ricci Bourguignon solitons.

Key Words: η- Ricci Bourguignon soliton, Lorentz para-Kenmotsu Manifold, Ricci Pseudoparallel Manifold.

REFERENCES

[1] G. Perelman, The entropy formula for the Ricci flow and its geometric applications,http://arXiv.org/abs/math/0211159, 2002, 1--39.

[2] G. Perelman, Ricci flow with surgery on three manifolds, http://arXiv.org/abs/math/0303109, 2003, 1-22.

[3] S. R. Ashoka, C. S. Bagewadi and G. Ingalahalli, Certain results on Ricci Solitons in α-Sasakian manifolds, Hindawi Publ. Corporation, Geometry, Vol.(2013), Article ID 573925,4 Pages.

[4] S. R. Ashoka, C. S. Bagewadi and G. Ingalahalli, A geometry on Ricci solitons in (LCS)_{n} manifolds, Diff. Geom.-Dynamical Systems, 16 (2014), 50--62.

[5] C. S. Bagewadi and G. Ingalahalli, Ricci solitons in Lorentzian-Sasakian manifolds, Acta Math. Acad. Paeda. Nyire., 28 (2012), 59-68.

[6] G. Ingalahalli and C. S. Bagewadi, Ricci solitons in α-Sasakian manifolds, ISRN Geometry,Vol.(2012), Article ID 421384, 13 Pages.

[7] C. L. Bejan and M. Crasmareanu, Ricci Solitons in manifolds with quasi-contact curvature, Publ. Math. Debrecen, 78 (2011), 235-243.

[8] A. M. Blaga, η-Ricci solitons on para-kenmotsu manifolds, Balkan J. Geom. Appl., 20 (2015), 1--13.

[9] S. Chandra, S. K. Hui and A. A. Shaikh, Second order parallel tensors and Ricci solitons on (LCS)_{n}-manifolds, Commun. Korean Math. Soc., 30 (2015), 123--130.

[10] B. Y. Chen and S. Deshmukh, Geometry of compact shrinking Ricci solitons, Balkan J. Geom.Appl., 19 (2014), 13--21.

[11] S. Deshmukh, H. Al-Sodais and H. Alodan, A note on Ricci solitons, Balkan J. Geom. Appl.,16 (2011), 48--55.

[12] C. He and M. Zhu, Ricci solitons on Sasakian manifolds, arxiv:1109.4407V2, [Math DG], (2011).

[13] M. Atçeken, T. Mert and P. Uygun, Ricci-Pseudosymmetric (LCS)_{n}-manifolds admitting almost η-Ricci solitons, Asian Journal of Math. and Computer Research, 29(2), 23-32,2022.

[14] H. Nagaraja and C. R. Premalatta, Ricci solitons in Kenmotsu manifolds, J. Math. Analysis, 3(2) (2012), 18--24.

[15] M. M. Tripathi, Ricci solitons in contact metric manifolds, arxiv:0801,4221 V1, [Math DG], (2008).

[16] Cho J.T., Kimura M. Ricci solitons and real hypersurfaces in a complex space form. Tohoku Math. J. 2009:61(2);205-212.

Modeling Epidemic Dynamics: Routh-Hurwitz Stability and Bifurcation Analysis in an Asymptomatic Compartmental Model

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ABSTRACT

Compartmental models are widely used in epidemiology to study the spread of infectious diseases by dividing populations into distinct groups based on disease status. A key extension of these models includes asymptomatic individuals, who do not exhibit symptoms but can still transmit the infection, offering a more realistic approach to understanding disease dynamics. In this work, we further refine this approach by incorporating In this work, we refine the model by introducing logistic growth dynamics to account for more realistic population changes over time.

To analyze the behavior of epidemic models, stability and bifurcation analysis are crucial tools. In this work, we focus on determining the stability of a compartmental model includes asymptomatic individuals by using the Routh-Hurwitz criteria, which provides conditions for the stability of equilibrium points based on the system's characteristic equation. By calculating the eigenvalues of the Jacobian matrix at equilibrium points, we assess stability under varying parameters and identify when bifurcations may occur. This approach allows us to detect shifts in the model's dynamics driven by changes in key parameters.

Our results provide deeper insight into the potential for disease spread and offer a framework for predicting critical thresholds where small changes in parameters lead to significant shifts in infection dynamics.

Key Words: Epidemic modelling, Routh-Hurwitz criteria, stability analysis.

REFERENCES

[1] DeJesus, Edmund X., and Charles Kaufman. "Routh-Hurwitz criterion in the examination of eigenvalues of a system of nonlinear ordinary differential equations." *Physical Review A*35.12 (1987): 5288.

[2] J. Li et al., Stability and bifurcation analysis of an SIR epidemic model with logistic growth and saturated treatment, Chaos, Solitons & Fractals 99 (2017): 63-71.

[3] M. Robinson and S. I. Nikolaos, A model for the emergence of drug resistance in the presence of asymptomatic infections, Mathematical Biosciences 243.2 (2013): 163-177.

Nonlinear Dynamics of a Discrete Leslie Type Predator-Prey System with Prey Harvesting

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ABSTRACT

In this paper, we investigate the nonlinear dynamics of a discrete-time Leslie type predator-prey system with prey harvesting. We use nonstandard finite difference method to discretize a continuous model into a discrete system. We investigate existence of equilibrium points and stability analysis of the discrete system using some analytical techniques. We not only explore the Neimark-Sacker bifurcation of the discrete system but also apply some chaos control techniques to stabilize the chaotic fluctuations and ensure sustainable population levels. We present numerical simulations such as phase portraits, maximum Lyapunov exponents, bifurcation diagrams, and controlled regions to support the theoretical results.

Keywords: Stability Analysis, Neimark-Sacker Bifurcation, Chaos Control, Leslie Type Predator-Prey Model, Nonstandard Finite Difference Method, Prey Harvesting Effect.

REFERENCES

[1] Y. A. Kuznetsov, *Elements of applied bifurcation theory*, Springer, (1998).

[2] S.N. Chow, & K. J. Hale, *Methods of bifurcation theory*, Springer Science & Business Media, (2012).

[3] P. Baydemir, H. Merdan, E. Karaoglu, & G. Sucu, Complex dynamics of a discrete-time prey–predator system with Leslie type: stability, bifurcation analyses and chaos. *International Journal of Bifurcation and Chaos*, *30*(10), (2020), 2050149.

[4] X. Jia, M. Zhao, & K. Huang, Bifurcation analysis and simulations of a modified Leslie–Gower predator–prey model with constant-type prey harvesting. *Mathematical Methods in the Applied Sciences*, *46*(18), (2023), 18789-18814.

[5] M. Bilal Ajaz, U. Saeed, Q. Din, I. Ali, & M. Israr Siddiqui, Bifurcation analysis and chaos control in discrete-time modified Leslie–Gower prey harvesting model. *Advances in Difference Equations*, *2020*, (2020), 1-24.

Numerical Investigations of the Gilson-Pickering Eqaution with Septic B-spline Functions

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ABSTRACT

This study uses the B-spline functions as approximate functions to offer a finite element scheme for numerical solutions of the Gilson-Pickering (G-P) equation. Additionally, an investigation of the algorithm's Von-Neumann stability has been done [1]. Furthermore, the method's practicality and dependability are illustrated through an analysis of a single soliton's behavior. To regulate the efficiency and conservation features of the proposed algorithm, the L2 and L∞ error norms as well as the two lowest invariants, I1 and I2, of the equation have been calculated [2]. The aspects of the problem modelled are easily visualized thanks to the tables and images that accompany the obtained numerical results. The outcomes also show that our approach is effective .

Keywords : Gilson-Pickering Equation; Finite Element Method; Collocation; Septic B-Spline

REFERENCES

[1] Karakoc SBG, Zeybek H. A numerical investigation of the GRLW equation using lumped Galerkin approach with cubic B spline. SpringerPlus 2016;5:1−17.

[2] Karakoc SBG, Saha A, Sucu D. A novel implementation of Petrov-Galerkin method to shallow water solitary wave pattern and superperiodic traveling wave and its multistability generalized Korteweg-de Vries equation. Chin J Phys 2021;68:605−617

On Semi-Analytical Solutions of Variable-Order Nonlinear Volterra and Fredholm Integro-Differential Equations

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ABSTRACT

Researchers have recently focused on the concepts of fractional differentiation and integration. Interest in fractional calculus is increasing day by day. It is frequently used in many areas such as physical processes, natural processes, engineering problems and real world problems. Physical and engineering problems that classical differentiation and integration are insufficient to explain are better explained by fractional calculus increases the interest in this field considerably.

In this study, it is considered nonlinear variable order integro-differential equations in sense of conformable and beta derivative. Two examples of integro-differential equations with variable order derivatives are considered and a modified variational iteration method (MVIM) is proposed and a solution procedure for these equations is established.

The definitions of the derivative in the conformable and beta sense are generalized and different functions are used as the order of the derivative. To demonstrate the effectiveness of the method, it is obtained semi-analytical solutions using various functions.

It is used figures to show the effect of various order functions with respect to the exact solution. It is utilized figures to compare oscillations and fluctuations due to fractional derivatives. Also, the effects of conformable and beta derivatives on the solution of nonlinear variable order integro-differential equations are investigated.

Key Words: modified variational iteration method (MVIM), variable order conformable derivative, variable order beta derivative, variable order integro-differential equation

REFERENCES

[1] F.M. Al-Saar and K.P. Ghadle, Solving nonlinear Fredholm integro-differential equations via modifications of some numerical methods, Adv. Theory Nonlinear Anal. Appl. 5 (2021), 260-276.

[2] S. Abbasbandy and E. Shivanian, Application of the variational iteration method for system of nonlinear Volterra's integro-differential equations, Math. Comput. Appl.14 (2009), 147-158.

[3] P. Darania and A. Ebadian, A method for the numerical solution of the integro-differential equations, Appl. Math. Comput.188 (2007), 657-668.

On the Sensitivity of Schur Stability of Some Discrete-time Mathematical Models

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ABSTRACT

Behavior of solutions of mathematical models can be evaluated without solving them with the help of stability analysis of equilibrium points. The most common method used to analyze the behavior of solutions of systems is the analysis which based on spectral criteria. It is well known that the problem of calculating the eigenvalues of non-symmetric matrices, i.e. the problem of studying spectral properties, is an ill-posed problem [1,2].

The other method used for stability analysis of linear systems is Lyapunov's theorem. According to Lyapunov's theorem, if the system is schur stable, Lyapunov matrix equation (LME) has a positive definite solution [1,4,5,6]. However, the existence of this solution does not give information about the quality of the stability of the system. The stability parameter $\omega(A) = \|H\|$, which depends on the solutions of the (LME), gives an idea about the quality of the system's stability, robustness to external influences and sensitivity of the system. If A is Schur stable then $\omega(A) < \infty$, otherwise it is shown with "$\omega(A) = \infty$" [3]. Moreover, sensitivity analysis is performed with the continuity theorems [1,4]. Thus, stability intervals that preserve Schur stability of matrix families consisting of linear combinations can be obtained. These intervals can be extended with the given methods and algorithms [5,6].

In nonlinear systems, decisions are usually made by linearizing with the help of Jacobian matrix. In this study, we will perform stability analysis on the Nicholson-Bailey model and SIR model by taking into account the studies in the literature, and

we will obtain intervals by performing sensitivity analysis for Schur stability with the help of stability parameter and continuity theorems in linear systems. We will give results on the stability analysis of Nicholson-Bailey model and SIR models. We will test these results with numerical examples [7,8].

Key Words: Schur stability, continuity theorems, stability parameter, mathematical model.

REFERENCES

[1] H. Bulgak, Pseudoeigenvalues, spectral portrait of a matrix and their connections with different criteria of stability, Error Control and Adaptivity in Scientific Computing,(1999) pp. 95-124.

[2] H. Bulgak, Pseudospectrum and Different Quality of Stability Parameters, Continuum Mechanics, Applied Mathematics and Scientific Computing: Godunovs Legacy, Springer, Cham, pp. (2020) 61-67.

[3] A.Ya. Bulgakov, Effectively calculable parameter of stability quality of systems of linear differential equations with constant coefficients, Sibirskii Matematicheskii Zhurnal, vol. 21(3), (1980) pp. 32-41.

[4] A. Duman and K. Aydin, "Sensitivity of Schur stability of systems of linear difference equations with constant coefficients," *Scientific Research and Essays*, vol. 6, no. 28, pp. 5846–5854, Nov. 2011.

[5] G.Topcu and K. Aydın, A new approach to construct and extend the Schur stable matrix families. Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics, Volume 73, Number 2, Pages 538–553 (2024)

[6] G. Topcu, Polinom Ailesinin Kararlılığı ve Kararlılığın Hassasiyeti, Doktora Tezi, Selçuk Üniversitesi Fen Bilimleri Enstitüsü, Konya (2023).

On the spectrum of a finite system of Klein-Gordon s-wave equations

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ABSTRACT

This paper is concerned with the spectral properties of the operator L generated by the finite system of Klein-Gordon s-wave differential expressions in the space $L_2(\mathbb{R}_+, \mathbb{C}^N)$. We investigate the spectrum of L and find the conditions for the potential, under which there exist finite number of eigenvalues and spectral singularities with finite multiplicities. Along with generalizing and adopting the recent results to Klein-Gordon operators, this paper may lay the groundwork for some problems including matrix valued operators, inverse problems and scattering theory.

Key Words: Spectral analysis, eigenvalues, spectral singularities.

REFERENCES

[1] N. Yokus and N. Coskun, Spectral properties of discrete Klein-Gordon s-wave equations with quadratic eigenparameter-dependent boundary condition, Iranian Journal of Science and Technology, Transactions A: Science, 4 (2019), 1951-1955.

[2] E. Kır Arpat and N. Yokus, Uniqueness of the solution to the inverse problem of scattering theory for spectral parameter polynomially dependent Klein-Gordon s-wave equation, Tbilisi Mathematical Journal, 16, (2023), 129–140.

Optimization of Some Special Curves by Gradient Descent Method

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ABSTRACT

This study consists of five chapters. Basic information is included in the introduction section. Information about optimization definition, daily survival and geometry application areas is given. Optimization problems of some special curves are expressed. In the second part, information about resource research and previous studies is given. In the third section, which is the material and method section, the objective function, the optimal point, some special cycloid type curves and the helix curve are expressed. The Gradient descent method to be specifically applied is explained in detail in the third section. The application steps of the gradient descent method are explained in detail and its application on a curve is shown. The advantages of the gradient descent method and the reasons for its preference are expressed. The definitions and equations of the special curves to be optimized with the gradient descent method are expressed. In the fourth part, which is the original part of the study, epicycloid, hypocycloid and helix special curves were optimized by the gradient descent method. With this algorithm, the minimum and maximum values of the curves within the specified range were calculated. These values are expressed in tables and graphs. In the fifth chapter, which is the last part of the study, information is given about the continuation and diversification of this study.

Key Words: Gradient descent algorithm, Helix curve, Optimization, Cycloid type curve

REFERENCES

[1] Bağcıoğlu, E., Optimizasyonda Genetik Algoritma Yaklaşımı, Master's Thesis, Institute of Science., Ankara University, Ankara, 2021.

[2] Boumal, N. Optimization and estimation on manifolds. Institute of Information Communication Technologies, Electronics and Applied Matehematics, 2014.

[3] Gottfried, B.S., Weisman, J. Introduction to Optimization Theory, New Jersey, Prentice Hall Inc, 1973.

[4] Huang, W. Optimization algorithms on Riemannian Manifolds with Applications. Doctoral dissertation, The Florida State University, 2013.

[5] Reeds, J. A. and Shepp, L. A. Optimal Paths For A Car That Goes Both Forwards And Backwards, Pacific Journal Of Mathematics, Vol. 145, No. 2, 367-393, 1990.

[6] Qi, C. Numerical Optimization Methods on Riemannian Manifolds, Phd Dissertation, The Florida State University, America, 2011.

[7] Öğüt, H. Kısıtlı Optimizasyon Problemlerinin Çözümü İçin Düzgünleştirilmiş Ceza Algoritması, Master's Thesis, Institute of Science. Süleyman Demirel University, 2023.

[8] Yang, H. and Wang, G. Optimized Design of Bézier Surface Through Bézier Geodesic Quadrilateral, Journal of Computational and Applied Mathematics, Vol. 273, 264-273, 2015.

Some Identities of the Generalized Higher Order Fibonacci and Lucas Polynomials

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ABSTRACT

Higher order Fibonacci and Lucas polynomials are polynomial sequences formed by generalizing classical Fibonacci and Lucas number sequences. These polynomials have an important place in theoretical and practical mathematical applications. This study aims to generalize higher order Fibonacci and Lucas polynomials. First, existing definitions, theorems and properties in the literature are used to describe the generalization of these higher order polynomials. Moreover, generating functions and Binet formulas for generalized higher order Fibonacci and Lucas polynomials are derived. In addition, several important identities of these generalized higher order polynomials are obtained. The findings are expected to increase the importance of generalized higher order polynomials in the mathematical literature and contribute to the research to be done in this field.

Key Words: Higher-order Fibonacci Polynomials, Higher-order Lucas Polynomials, Generating functions, Binet formulas.

REFERENCES

[1] N. Ait-Amrane, H. Belbachir and E. Tan, On the Generalized Fibonacci and Lucas Hybrid Polynomials, Turkish Journal of Mathematics, 46(6) (2022), 2069-2077.

[2] S. Köme and H. Kirik, On the Generalized Fibonacci and Lucas 2^k-ions, Notes on Number Theory and Discrete Mathematics, 26(4) (2020), 173-186.

[3] C. Kızılateş, W.S. Du and N. Terzioğlu, On Higher-Order Generalized Fibonacci Hybrinomials: New Properties, Recurrence Relations and Matrix Representations, Mathematics, 12(8) (2024), 1156.

Stability and Bifurcation Analysis of Discrete Time Predator-Prey Population Model

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ABSTRACT

In [1], the continuous-time predator –prey population model has been investigated. In this study we have considered a discrete-time type of these system by applying the forward Euler scheme. In [2], existence of the fixed points and stability of the system have been studied. Therefore, we focus on the topological classification of the co-existence fixed point and possible bifurcation types of the system such as Neimark-Sacker and flip bifurcations using both bifurcation theory and center manifold theory. The control method of OGY feedback is investigated for the system. Finally, numerical examples are given to verify theoretical results obtained. These examples not only support our theoretical finding but also show the complex dynamical behaviors of the discrete-time system.

We prove that if the bifurcation parameter exceedes a critical bifurcation value, the stability of the co-existence fixed point of the system goes from stable to unstable and the flip and Neimark-Sacker bifurcations arise from at this critical value.

Analytical findings and numerical simulations show that the integral step size plays an important role in the local stability of the co-existence fixed point, flip, and Neimark-

Sacker bifurcations and dynamics of the discrete-time predator-prey system after the original continuous-time the predator-prey system is discretized.

Key Words: Predator-prey population model, Flip bifurcation, Neimark-Sacker bifurcation, Chaos control.

REFERENCES

[1] İ. Çay A note on Hopf bifurcation and steady state analysis for a predator-prey model. Turkish Journal of Science 2020; 5 (3): 220-225. S.G. Gal, Shape-Preserving Approximation by Real and Complex Polynomials, Birkhäuser Boston, Inc., Boston, MA, 2008.

[2] A. Haskan A. Ayrık zamanlı bir av-avcı popülasyon modeli ve kararlılık analizi. Sivas Cumhuriyet Universitesi Fen Bilimler Enstitüsü, Yüksek Lisans Tezi 2021, Sivas.

Stability and Numerical Analysis of Magnetohydrodynamic (MHD) Flow with Fractional Derivatives in Porous Media

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ABSTRACT

In this study, the dynamics of fluid flow in a porous medium under the influence of magnetohydrodynamics (MHD) have been thoroughly investigated. A time-dependent flow model between parallel plates has been developed, where fractional-order differential equations are employed. The reason for preferring fractional derivatives is that they provide more flexible and realistic modeling results, going beyond classical derivatives. Fractional derivatives allow for a broader examination of the system's dynamics, as they consider both past and present states of the flow, offering a richer solution. In the study, the existence and uniqueness conditions for the solutions of the developed fractional differential equation system were analyzed. This analysis is crucial to ensure the validity of the model and the reliability of the solutions. Additionally, the Hyers-Ulam stability of the solutions was examined, assessing whether the system remains stable under various conditions. Numerical solutions were obtained using the finite difference method. The time-dependent Caputo fractional derivative was discretized using the Grünwald-Letnikov (GL) approach, while the terms related to space were computed with the Crank-Nicolson method. The Thomas algorithm was used for solving the fractional differential equation system. After obtaining the velocity, temperature, and concentration profiles, the effects of flow parameters such as Gr, Gm, Pr, M, Sc, and K on these profiles were thoroughly analyzed. Furthermore, the motion of the plate under constant speed and periodic acceleration was separately evaluated. The effects of varying the fractional-order derivative were illustrated through graphical representations.

Key Words: Fractional derivative, Grünwald Letnikov approach, Hyers Ulam stability.

REFERENCES

[1] Khan, H., Gómez-Aguilar, J. F., Khan, A. And Khan, T. S. (2019). Stability analysis for fractional order advection–reaction diffusion system. *Physica A: Statistical Mechanics and Its Applications*, 521, 737-751.

[2] Inc, M., Yusuf, A., Aliyu, A. I., Baleanu, D. (2018). Investigation of the logarithmic-KdV equation involving Mittag-Leffler type kernel with Atangana–Baleanu derivative. *Physica A: Statistical Mechanics and its Applications*, 506, 520-531.

[3] Khan, H., Li, Y., Chen, W., Baleanu, D., & Khan, A. (2017). Existence theorems and Hyers-Ulam stability for a coupled system of fractional differential equations with p-Laplacian operator. *Boundary Value Problems*, 1-16.

Steady State solution and its stability Analysis of a Reaction-Diffusion Mathematical Model for LDL, Lipoprotein(a), and CRP in Atherosclerosis

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ABSTRACT

In this paper, a mathematical model of reaction-diffusion artificially evaluates the behavior of low-density lipoprotein (LDL), lipoprotein (a), and c-reactive protein (CRP) in the arterial walls, which contribute to processes of diseases such as heart attack and stroke by forming plaques on their interior wall has been proposed. It models these molecules' diffusion and biochemical interactions via partial differential equations to reproduce their functions in oxidation, inflammation, and formation of plaques. Computational simulations via finite-difference methods reveal that C-reactive protein increases the rate of low-density lipoprotein oxidation and promotes plaque deposition, especially in regions of elevated local lipoprotein concentration. Results Pointedly, this study revealed that elevated CRP could efficiently facilitate development by the interaction of LDL and Lp(a), suggesting inflammation-targeted treatment may be a potential strategy in the clinic. It, therefore, presents a robust model to examine the interactive inflammatory effects of lipid metabolism in cardiovascular disease. This study presents a comprehensive stability analysis of a reaction-diffusion model that simulates the interactions between LDL, Lipoprotein(a), and CRP, key players in the progression of atherosclerosis. This mathematical modeling using finite difference methods and eigenvalue estimation provides deep insights into the dynamics of lipid accumulation, inflammation, and their combined effects on cardiovascular health. The main results are as follows.

Key Words: Finite Difference, Reaction-Diffusion, Atherosclerosis

REFERENCES

[1] G. Douglas and K.M. Channon, The pathogenesis of atherosclerosis, Medicine (Baltimore) 42 (2014), pp. 480–4.

[2] A.J. Lusis, Atherosclerosis, Nature 407 (2000), pp. 233–41.

[3] P. Libby and G.K. Hansson, Atherosclerosis: Current pathogenesis and therapeutic options, Journal of Clinical Investigation 120 (2010), pp. 1252–60.

[4] Ridker P.M., Moorthy M.V., Cook N.R., et al., Inflammation, cholesterol, lipoprotein(a), and 30-year cardiovascular outcomes in women, N Engl J Med 390 (2024), pp. 1–11.

The Novel Study for Solving the Some Nonlinear Fractional Partial Differential Equations

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ABSTRACT

In this paper, new numerical solutions for some nonlinear fractional differential equations are obtained. The structure of fractional derivatives is investigated in a conformable sense. Two-dimensional and three-dimensional graphs of the obtained numerical solutions are plotted using the Maple software program. Fractional derivatives, or derivatives of arbitrary (non-integer) order, are incorporated into nonlinear fractional partial differential equations (NFPDEs), which are an extension of classical partial differential equations (PDEs). These equations appear in many domains, including physics, engineering, biology, and finance, because they are superior to fractional partial differential equations in simulating anomalous diffusion, memory effects, and complex dynamical systems. Fractional derivatives, such as the Riemann-Liouville, Caputo, or Grünwald-Letnikov derivatives, are involved in these equations. Each of these derivatives has distinct properties and can be applied to various kinds of situations. The understanding and solution of the problem are influenced by the selection of the derivative.

NFPDEs may have nonlinear terms in the dependent variable, its fractional derivatives, or both as sources of nonlinearity. Due to their nonlinearity, the equations are frequently difficult to understand and solve, necessitating the use of the analytical and numerical techniques used to explain quantum physics, wave propagation in nonlinear, viscoelastic materials, and anomalous diffusion in complex media models of electromagnetic waves in complex media, viscoelastic materials, and porous media flow. Population dynamics in biological tissues: anomalous diffusion and memory effects models that account for genetic traits and memory in financial

markets.

Key Words: Fractional derivative, nonlinear fractional partial differential equations, Maple software.

REFERENCES

[1] J. Liouville, Mémoire sur quelques questions de géométrie et de mécanique, et sur un nouveau genre de calcul pour résoudre ces questions, Ecole polytechnique, 13 (1832), 71-162.

[2] K.S. Miller and B. Ross, An introduction to the fractional calculus and fractional differential equations, Wiley, New York, 1993.

[3] P. Veeresha, D.G. Prakasha and H.M. Baskonus, Novel simulations to the time-fractional Fisher's equation, Mathematical Sciences 13(1) (2019), 33-42.

[4] D. Baleanu, A. Jajarmi and M. Hajipour, On the nonlinear dynamical systems within the generalized fractional derivatives with Mittag–Leffler kernel, Nonlinear Dynamics, 2018(1) (2018), 1–18.

Transient Heat Transmission of Functionally Graded Longitudinal Fin Using Lines Approximation Method

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ABSTRACT

This paper investigated the transient thermal behaviour of a functional graded material (FGM) in a longitudinal fin. A longitudinal fin is a longitudinal structure used to improve heat transfer in various systems, such as heat exchangers, electronic cooling systems, and power generation equipment. The main objective of this study is to analyse the efficiency of the longitudinal fin in terms of heat transfer and temperature distribution. The fin surfaces are subjected to convection and radiation to disperse heat. The exponential change in thermal conductivity with distance of the longitudinal fin is considered. Also the cross-sectional area is assumed to be an exponential function. The partial differential equation resulting from these assumptions is the closed-form solution of the heat transfer equation for the FGM wing and is obtained using the method of lines. The equation is first decomposed using the method of lines, and then its numerical solution is obtained using the matrix method. These solutions allow to check the accuracy of the transient numerical predictions for large times. The thermal distribution along the fin is shown graphically and compared with the literature. The results presented are not only of fundamental interest but can also be used to design a functionally graded fin with the desired stable and transient performance. The exponent of the heat transfer coefficient on the temperature distribution are studied.

Key Words: Method of Lines, Functionally Graded Longitudinal Fin, Numerical Solution, Heat Transfer Coefficient

REFERENCES

[1] S.G. Gal, Shape-Preserving Approximation by Real and Complex Polynomials, Birkhäuser Boston, Inc., Boston, MA, 2008.

[2] Aziz, A., & Na, T. Y. (1981). Periodic heat transfer in fins with variable thermal parameters. *International Journal of Heat and Mass Transfer*, *24*(8), 1397-1404.

[3] Assis, E., & Kalman, H. (1993). Transient temperature response of different fins to step initial conditions. *International journal of heat and mass transfer*, *36*(17), 4107-4114.

[4] Abu-Abdou, K., & Mujahid, A. A. M. (1989). Heat transfer in straight fins of arbitrary profile subjected to periodic base and environment temperatures. *Wärme-und Stoffübertragung*, *24*(6), 353-361.

GEOMETRY AND TOPOLOGY

Agnesi Curves in Lorentz-Minkowski Plane

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ABSTRACT

The Agnesi curve, one of the curve examples in the book Instituzioni Analitiche ad Uso della Gioventù Italiana (1748) written by the 18th century Italian mathematician Maria Gaetana Agnesi, is a curve with wide application areas. The curve, which was examined and defined by Pierre de Fermat in 1630, Guido Grandi in 1703 and Maria Gaetana Agnesi in 1748 as a translation error of the name given in Agnesi's book, and known in the literature as the Agnesi curve or the Witch of Agnesi Curve in common usage, is one of the kinematic, geometric ground-based curves. The Agnesi curve plays an important role in the approximate calculation of the spectral energy distributions of spectral lines. The Agnesi curve is defined as the geometric location of points that satisfy certain geometric conditions around the circle. The Agnesi curve is similar to the probability density function of some probability distributions such as the Cauchy distribution. The similarity between the shapes of the Agnesi curve and the Cauchy distribution curve can help visualize how certain probability distributions behave, especially in terms of central tendencies and tail behavior. The Agnesi curve also has an important relationship with the Lorentzian distribution, especially in the context of physics. The Agnesi curve has an important place both mathematically and historically. Maria Gaetana Agnesi's contributions demonstrate the richness of her mathematical thought, while the curve's geometric and analytical properties combine mathematical elegance and complexity. The curve has an important place both in its time and in modern mathematics. The form of the Agnesi curve in the Lorentz-Minkowski space will yield important results. This study tries to obtain the equations of the Witch of Agnesi curve in the Lorentz plane.

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Key Words: Agnesi curve, Lorentz space, distribution, probability

REFERENCES

[1] Agnesi, D.M.G., 1801. Analytical Institutions. (Translate to English By John Colson) Taylor and Wills, Chancery-Jane, London.

[2] Baydaş, Ş., Karakaş, B., 2019a. The cissoid of Diocles in the Lorentz–Minkowski plane. Turkish Journal of Mathematics, 43: 1232 – 1240.

[3] Madson, M.R., 2012. Golf Ball Dimple Based on Witch og Agnesi Curve. United States Patent Application Publication, US2012/0122613 A1.

[4] Baydaş, Ş., Karakaş, B., 2019b. Defining a curve as a Bezier curve. Journal of Taibah University for Science 13(1): 522-528.

[5] Spencer, R.C., 1940. Properties of the Witch of Agnesi-Application to Fitting the Shapes of Spectral Lines. Journal of the Optical Society of America, 30, 415-419.

[6] Stigler, S.M., 1974. Studies in the History of Probability and Statistics. XXXIII Cauchy and the Witch of Agnesi: An Historical Note on the Cauchy Distribution. Biometrika, 61(2), 375-380.

Agnesi Surfaces

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ABSTRACT

The curve, which was studied and defined by Pierre de Fermat in 1630, Guido Grandi in 1703 and Maria Gaetana Agnesi in 1748 and is commonly known as the Agnesi curve or the Witch of Agnesi Curve in literature, is a kinematic, geometric ground-based curve. The curve, which was studied by Pierre de Fermat, Guido Grandi and Maria Gaetana Agnesi in 1748 and is widely known as the Witch of Agnesi Curve in literature, is one of the kinematic, geometric ground-based curves that play an important role in the approximate calculation of spectral energy distributions of spectral lines. The similarity between the shapes of the Agnesi curve and the Cauchy distribution curve can help visualize how certain probability distributions behave, especially in terms of central tendencies. The Agnesi curve has an important place both mathematically and historically. Maria Gaetana Agnesi's contributions show the richness of her mathematical thought, while the geometric and analytical properties of the curve offer mathematical elegance and complexity together. The curve has an important place both in its time and in modern mathematics. The curve, which is in the form of the probability density function of the Cauchy distribution, has also widespread applications in probability theory. To examine multiple distributions together, the Agnesi surface is needed. The study studies the theory of defining and examining the Agnesi surface and its Matlab application.

*This work is supported by the project number 9953 supported by the Scientific Research Projects Coordination Unit of Van Yüzüncü Yıl University of the Republic of Turkiye.

Key Words: Agnesi curve, distribution, probability

REFERENCES

[1] Al-Ossmi, L.H.M., 2021. Noor's Curve, a New Geometric Form of Agnesi Witch, a Construction Method is Produced. Baghdad Science Journal, 18(2), 1113-1124.
[2] Agnesi, D.M.G., 1801. Analytical Institutions. (Translate to English By John Colson) Taylor and Wills, Chancery-Jane, London.
[3] Baydaş, Ş., Karakaş, B., 2019a. The cissoid of Diocles in the Lorentz–Minkowski plane. Turkish Journal of Mathematics, 43: 1232 – 1240.
[4] Bottema, O., Roth, B., 1990. Theoretical Kinematics, Dover Publications, New York.
[5] Farin G., 2002. Curves and Surfaces for CAGD. Academic Press, Boston.
[6] Madson, M.R., 2012. Golf Ball Dimple Based on Witch og Agnesi Curve. United States Patent Application Publication, US2012/0122613 A1.
[7] Stigler, S.M., 1974. Studies in the History of Probability and Statistics. XXXIII Cauchy and the Witch of Agnesi: An Historical Note on the Cauchy Distribution. Biometrika, 61(2), 375-380.

Computation and Visualization of Involute and Evolute Curves in Euclidean Space Using Python

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ABSTRACT

The geometric properties of involute and evolute curves in Euclidean space play a significant role in differential geometry, particularly in understanding the local and global behaviour of curves. These properties, such as curvature, torsion, and the Frenet frame, can provide deep insights into the nature of the curves and their applications in various fields. In this study, we present a Python-based framework for the calculation and visualization of these properties, with a focus on involute and evolute curves. Using libraries like NumPy, SymPy and Matplotlib, we demonstrate how to compute the tangent, normal, and binormal vectors, as well as the curvature and torsion, for parametric curves. The framework also enables the analysis of involute and evolute relationships, providing a computational tool for deeper exploration of these curves. By facilitating both symbolic and numerical computations, this approach enhances accessibility to complex geometric concepts, bridging the gap between theoretical understanding and practical implementation. Additionally, the framework allows for graphical representation of the curves and their geometric properties, further aiding in visualization and comprehension. Future work will extend these methods to more complex geometries and higher-dimensional spaces.

Key Words: Euclidean space, curvature, torsion, Frenet frame, involute, evolute, Python, differential geometry.

REFERENCES

[1] Müller, W., et al. (2007). "Curves with Constant Curvature Radius and Their Evolutes." Journal of Differential Geometry, 45(3), 567-583.

[2] Fenchel, W. (1951). The Differential Geometry of Curves. The University of Chicago Press.

[3] Do Carmo, M. P. (1976). Differential Geometry of Curves and Surfaces. Prentice-Hall.

[4] Litvin, F. L., & Fuentes, A. (2004). Gear Geometry and Applied Theory. Cambridge University Press.

[5] Guggenheimer, H. W. (2012). Differential Geometry. Dover Publications.

[6] Gray, A., et al. (2006). Modern Differential Geometry of Curves and Surfaces with Mathematica. Chapman & Hall/CRC.

[7] O'Neill, B. (2006). Elementary Differential Geometry. Academic Press.

[8] Izumiya, S., & Takeuchi, N. (2002). "Singularities of Curves and Surfaces with Applications." Journal of Mathematical Sciences, 115(1), 32-46.

[9] Struik, D. J. (2012). Lectures on Classical Differential Geometry. Dover Publications.

[10] Kim, Y. H., et al. (2013). "Curves on the Unit Spheres." Mathematical Proceedings, 24(2), 233-245.

[11] Pavlenko, A., & Litecka, T. (2012). "Involute Gears and Their Mathematical Properties." Applied Mechanics and Materials, 162, 101-110.

[12] Romero-Fuster, M. C., & Sanabria-Codesal, E. (1999). "Generalized Evolutes in Higher Dimensions." Journal of Geometry, 64(4), 301-318.

[13] Borceux, F. (2014). Differential Geometry: From Curves to Manifolds. Springer.

[14] Weatherburn, C. E. (1927). Differential Geometry of Three Dimensions. Cambridge University Press.

[15] VanderPlas, J. (2016). Python Data Science Handbook: Essential Tools for Working with Data. O'Reilly Media.

[16] McKinney, W. (2017). Python for Data Analysis: Data Wrangling with Pandas, NumPy, and Python. O'Reilly Media.

Multiplicative partner curve in E_*^3

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ABSTRACT

The aim of this article is to characterize pairs of curves within multiplicative (non-Newtonian) spaces. Specifically, we examine how notable curve pairs in differential geometry, such as Bertrand and Mannheim partner curves, are transformed through the framework of multiplicative analysis. By utilizing the relationships between multiplicative Frenet vectors, we introduce multiplicative versions of Bertrand and Mannheim curve pairs. Subsequently, these pairs are characterized through multiplicative arguments. To further enhance comprehension of the subject, examples are provided along with multiplicative graphs, which visually illustrate key concepts.

This study seeks to shed light on the behavior and intrinsic properties of these curve pairs within the context of multiplicative geometry. By exploring the intersection of differential geometry and multiplicative analysis, we aim to offer a novel perspective that enriches the understanding of both fields. The examples and graphs presented in the article serve as crucial tools for demonstrating the practical implications of this innovative approach. Ultimately, the analysis undertaken herein contributes to a deeper understanding of how multiplicative structures influence well-known geometric entities.

Key Words: Multiplicative Frenet frame, multiplicative Euclidean space, non-Newtonian calculus, multiplivative Bertrand curve, multiplivative Mannheim curve .

REFERENCES

[1] M.E. Aydin, A. Has and B. Yılmaz, A non-Newtonian approach in differential geometry of curves: multiplicative rectifying curves, Bulletin of the Korean Mathematical Society 61(3) (2024), 849-866.

[2] S.G. Georgiev, Multiplicative Differential Geometry (1st ed.), Chapman and Hall/CRC, New York 2022.

[3] M.P.D. Carmo, Differential Geometry of Curves and Surfaces, Prentice Hall Inc., New Jersey 1976.

[4] M. Grossman and R. Katz, Non-Newtonian Calculus, 1st ed. Lee Press, Pigeon Cove Massachussets 1972.

The Application of the Principle of Least Action to the Some Special Curve Types Under the Inverse Motion

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ABSTRACT

This study explores the application of the principle of least action to epicycloid and cardioid cycloid curves under closed planar reverse motion. Initially, a summary of relevant literature is provided, along with the foundational concepts necessary for understanding the research. The study delves into the mathematical definitions and equations of the epicycloid and cardioid curves, explaining how the principle of least action can be applied to these specific geometric shapes.

A key aspect of the research involves calculating the energies of points on a moving plane under reverse motion to identify the least action points on these curves. The study compares the characteristic points of the epicycloid and cardioid curves, highlighting the differences and similarities between them.

Finally, the potential for expanding and applying this research in various fields is discussed, emphasizing its broader implications and future directions.

Key Words: Inverse motion; Cycloid type curve; Minimal action; Kinematics

REFERENCES

[1] Muller, H. R, Egesoy, E. and Oruc, M, *Kinematik Lessons,* Ankara University, 1963.

[2] Whitman, E. A, Some Historical Notes on the Cycloid. The American Mathematical Monthly. 50(5), 30-31, 1943.

[3] Blaschke, W, Muller, H. R, *Ebene Kinematik,* Munchen, 1956.

[4] Dathe, H, Gezzi R, Kubein-Meesenburd D, and Nagerl, H, Characteristic Point and Cycles in Planar Kinematics with Applcations to Human Gait. *Acta Bioengineering and Biomechanics.* 17(1), 75-86, 2015.

THE KINEMATICS OF SOME CURVES IN GALILEAN GEOMETRY

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ABSTRACT

This study explores the application of the principle of least action to special cycloid-type curves within the Galilean plane. The research begins with a review of relevant literature and introduces key concepts, including the definitions of these curves and the general Galilean transformations. The study focuses on the transformations of these special curves under planar motion, calculating the energies of points on the moving plane to determine the minimal action points. Additionally, the pole points of these curves are examined under general Galilean motion, encompassing shear, rotation, and displacement. The findings offer insights into the behavior of these curves within the framework of Galilean geometry.

Key Words: Planar Motion, Cycloid-Type Curves, Principle of Least Action, Galilean Plane

REFERENCES

[1] Müller, H. R., Egesoy, E. ve Oruç, M., Kinematik dersleri. Ankara Üniversitesi, 1963.

[2] Yaglom, I.M., A Simple Non-Euclidian Geometry and Its Physical Basis, Springer- Verlag Inc.,New-York, 1979

[3] Blaschke, W., & Müller, H. R., Ebene Kinematik, München, 1956.

[4] Hacısalihoglu, H. H., Dönü¸sümler ve Geometriler, Ankara Üniversitesi, Fen Fakültesi, Matematik Bölümü, 1998.

Umbrella Matrices in Galilean Space

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ABSTRACT

In this study, we examine Umbrella matrices in Galilean space and the Lie group formed by these matrices. Additionally, we will provide examples of the orbit surfaces of curves under Umbrella motions.

Key Words: Umbrella matrix, Galilean space, Lie Group.

REFERENCES

[1] G. Yüca and Y. Yaylı. Dual transformations in galilean spaces, Inte. Elec. J. Geo. 13 (2) (2020), 52-61. https://doi.org/10.36890/iejg.683738

[2] M. Çarboğa and Y. Yaylı, Geometric applications and kinematics of umbrella matrices, Korean J. Math. 31 (3) (2023), 295-303. https://doi.org/10.11568/kjm.2023.31.3.295

[3] M. Çarboğa and Y. Yaylı, The Lie Group of Umbrella Matrices and Pairwise Comparisons Matrices, Acta Uni. Apu., 74 (2023) 1-10. https://doi.org/10.17114/j.aua.2023.74.01

[4] M. Çarboğa and Y. Yaylı, Geometry Of Dual Umbrella Matrices, Advanced Stud. Euro-Tbilisi Math. J. 16 supp. issue (3) (2023), 77–87. https://doi.org/10.32513/asetmj/1932200823306

[5] O. Bottema and B. Roth, Theoretical kinematics, Vol. 24, Courier Corporation, ABD, 1990.

[6] M. Çarboğa and Y. Yaylı, Differential Geometry of Umbrella Matrices, Int. Geo. Sym. in Memory of Prof. Erdogan ESIN's Proc. Book, Turkey, 2023. https://igsm-erdoganesin.gazi.edu.tr/view/page/294511/ proceeding-book

[7] E. Esin, Umbrella matrices and higher curvatures of a curve, Commu. Facul. Scie. Uni. Ankara Series A1 Math. and Stat. 35 (01.02) (1986), 28–34. https://doi.org/10.1501/Commua1_0000000251

[8] N. Kuruoglu, On the lie group of umbrella matrices, Commu. Facul. Scie. Uni. Ankara Series A1 Math. and Stat. 32 (1) (1983) 132–144. https://doi.org/10.1501/Commua1_0000000231

Closed and Strongly Closed Quantale-valued Reflexive Spaces

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ABSTRACT

Baran [1, 2] introduced the notions of "closedness" and "strong closedness" in arbitrary set-based topological categories, and he used these notions to generalize some fundamental topological concepts to topological categories. Moreover, it is shown that they form a suitable closure operator in the sense of Dikranjan and Giuli [3] in some well-known topological categories.

In recent years, distinct mathematical frameworks have been studied with lattice structures, including lattice-valued topology [4], quantale-valued metric space [5], quantale-valued approach space [6, 7], and lattice-valued preordered space [4]. This leads us to study quantale-valued reflexive spaces, which is a generalization of quantale-valued preordered spaces. In this paper, we characterize a closed point, closed and strongly closed subsets in Q-RRel [8], the topological category of quantale-valued reflexive spaces, and Q-monotone mappings.

Key Words: Quantale-valued reflexive space, Closedness, Strong closedness, Topological category.

REFERENCES

[1] M. Baran, Separation properties, Indian J. Pure Appl. Math., 23 (1991), 333–341.

[2] M. Baran, The notion of closedness in topological categories, Comment. Math. Univ. Carolin. 34 (1993), 383–395.

[3] D. Dikranjan, E. Giuli, Closure operators I, Topology Appl. 27 (1987), 129–143.

[4] J.T. Denniston, A. Melton, S.E. Rodabaugh, S.A. Solovyov, Lattice-valued preordered sets as lattice-valued topological systems, Fuzzy Sets Syst. 259 (2015), 89–110.

[5] G. Jäger, The Wijsman structure of a quantale-valued metric space, Iranian J. Fuzzy Syst. 17 (2020), 171–184.

[6] G. Jäger, W. Yao, Quantale-valued gauge spaces, Iranian J. Fuzzy Syst. 15 (2018), 103–122.

[7] M. Qasim, S. Özkan, The notions of closedness and D-connectedness in quantale-valued approach spaces, Categories Gen. Algebraic Struct. Appl. 12 (2020), 149–173.

[8] S. Özkan, Local separation, closedness and zero-dimensionality in quantale-valued reflexive spaces, Filomat 37:12 (2023), 3891–3905.

Exploration of Misconceptions of Mathematics Major Students about the Topological Space Concept

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ABSTRACT

This study aims to explore the misconceptions held by mathematics major students regarding the concept of topological space. The topological space concept is a primary concept in topology which is regarded as a difficult subfield of mathematics. The participants in this study were third-year students in the mathematics department enrolled in courses on topology and geometric topology. The research utilized a qualitative method to determine misconceptions about the topological space concept, particularly in point set topology and geometric topology. The qualitative method was employed within a case study framework, guiding the design and implementation of the study. The research interviews focused on the participants' conceptual understanding and their definition of topological concepts, such as topological space. The primary sources of data for this study were video-recorded interviews, in which students provided explanations and insight into their cognitive processes. The interviews utilized specific techniques, namely, the think-aloud protocol and interview-about-events. Additionally, artifacts created by the students during the interviews were also considered as part of the data sources. The methodology for data analysis included the case study approach and the framework proposed by Miles and Huberman (1984), and Creswell (2013), involving three stages: data reduction, data display, and conclusion or verification. The findings of this research revealed several misconceptions about the concept of topological space (Şahin, 2024).

Key Words: Topological space, misconceptions in topology.

REFERENCES

[1] J. W. Creswell, Qualitative Inquiry and Research Design: Choosing Among Five Approaches, Sage Publications, 2013.

[2] M. B. Miles and A. M. Huberman, Qualitative Data Analysis: A Sourcebook of New Methods, Sage, 1984.Ş. Şahin, Investigation of Misconceptions and Mental Models of Mathematics

[3] Major Students About Topological Concepts, (Unpublished PhD thesis), Middle East Technical University, Turkey, 2024.

New Kinds of Soft Covering Based Rough Sets via the Concept of Soft Maximal Description

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ABSTRACT

The present paper aims to use the soft maximal description of the objects, which give rise to new kinds of soft covering based rough sets. We examine the relationships between them. Also, we present its basic, topological properties, and give some illustrative examples.

Key Words: Rough set, soft set, soft covering based rough set.

REFERENCES

[1] Z. Pawlak, Rough sets, International Journal of Computer and Information Sciences, 11 (5) (1982), 341-356.

[2] D. Molodtsov, Soft set theory-first results, Computers and Mathematics with Applications, 37 (1999), 19-31.

[3] F. Feng, X. Liu, V. Leoreanu-Fotea, and Y. B. Jun, Soft sets and soft rough sets, Information Sciences, 181 (2011), 1125-1137.

[4] Ş. Yüksel, Z. Güzel Ergül and N. Tozlu, Soft covering based rough sets and their application, The Scientific World Journal, (2014), Article ID 970893, 9 pages.

[5] Ş. Yüksel, N. Tozlu and T. Dizman, An application of multicriteria group decision making by soft covering based rough sets, Filomat, 29 (2015), 209-219.

[6] Z. Güzel Ergül and Ş. Yüksel, A new type of soft covering based rough sets applied to multicriteria group decision making for medical diagnosis, Mathematical Sciences and Applications E-Notes, 7 (1) (2019), 28-38.

Separation Properties at p for Interval Spaces

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ABSTRACT

The interval operator, a natural generalization of intervals, was introduced by Calder [1] in 1971. It also offers a frequent and natural way to construct convex structures [2]. Convexity is a fundamental property in many areas of mathematics with numerous applications, such as lattice theory, graph theory, and topology.

In 1991, Baran [3] extended the classical separation properties of topology to set-based topological categories [4] concerning discreteness, initial and final structures. He defined these axioms first at a point p, that is, locally, and then point-free. In this work, we characterize local separation properties such as T_0 at p, T_1 at p, pre-Hausdorff at p, and Hausdorff at p for the topological category of interval spaces. Furthermore, we investigate the relationships among them and examine their some invariance properties, such as hereditary and productive.

Key Words: Topological category, Interval spaces, Local separation properties.

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REFERENCES

[1] J. Calder, Some elementary properties of interval convexities, J . London Math Soc., 2 (1971), 422–428.

[2] M.J.L. Van De Vel, Theory of Convex Structures, North Holland, Amsterdam, 1993.

[3] M. Baran, Separation properties, Indian J. Pure Appl. Math., 23 (1991), 333–341.

[4] J. Adamek, H. Herrlich, and G.E. Strecker, Abstract and Concrete Categories: The Joy of Cats, John Wiley & Sons, New York, 1990.

Some Set Operators in terms of Local Closure Function and The Operator Ψ_Γ

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ABSTRACT

The concept of the ideal [2,4] forms the basis of many studies in general topology. Researchers introduced numerous topological operators by means of ideal. The local closure function [1] and the operator Ψ_Γ [1] which are defined by Al-Omari and Noiri are among these operators. Afterward, Γ-boundary operator [8] was presented by Tunç and Özen Yıldırım via local closure function. In addition, Tunç and Özen Yıldırım produced the operators $\wedge_\Gamma$, $\underline{\vee}_\Gamma$ and $\overline{\wedge}_\Gamma$ using local closure function and the operator Ψ_Γ in [7]. They analyzed the behaviors of these operators for some special sets. In [1], Al-Omari and Noiri obtained equivalent conditions of the case $\mathrm{cl}(\tau)\cap\mathfrak{I} = \{\emptyset\}$ and they defined the closure compatibility in ideal topological spaces. Furthermore, different equivalent conditions for the case $\mathrm{cl}(\tau)\cap\mathfrak{I} = \{\emptyset\}$ and closure compatibility were obtained using the operators $\wedge_\Gamma$, $\underline{\vee}_\Gamma$ and $\overline{\wedge}_\Gamma$ in [7]. In this study, we introduce new operators ℓ_1, ℓ_2 and ℓ_3 by using the operators $\wedge_\Gamma$, $\underline{\vee}_\Gamma$ and $\overline{\wedge}_\Gamma$. We also investigate important properties of these new operators. Besides, we acquire some new characterizations of the case $\mathrm{cl}(\tau)\cap\mathfrak{I} = \{\emptyset\}$ by means of these new operators as a practice of these main properties. The attitudes of these operators are researched in ideal topological spaces which are familiar.

Key Words: Ideal topological space, local closure function, operator Ψ_Γ, operator ℓ_i $(i = 1, 2, 3)$.

REFERENCES

[1] A. Al-Omari and T. Noiri, Local closure functions in ideal topological spaces, Novi Sad J. Math. 43(2) (2013), 139-149.

[2] K. Kuratowski, Topology, Vol. I, Academic Press, New York, 1966.

[3] N. Goyal and N. S. Noorie, θ-closure and $T_{2\frac{1}{2}}$ spaces via ideals, Italian Journal of Pure and Applied Mathematics 41 (2019), 571-583.

[4] R. Vaidyanathaswamy, The localisation theory in set-topology, Proc. Indian Acad. Sci., Sect. A. 20 (1944), 51–61.

[5] N. V. Veličko, H-closed topological spaces, Amer. Math. Soc. Transl. 78(2) (1968), 102-118.

[6] A. N. Tunç and S. Özen Yıldırım, A Study on further properties of local closure functions, 7th International Conference on Recent Advances in Pure and Applied Mathematics (ICRAPAM 2020) (2020) 123-123 Muğla, Türkiye.

[7] A. N. Tunç and S. Özen Yıldırım, Characterization of two specific cases with new operators in ideal topological spaces, Journal of New Theory 46 (2024), 51–70.

[8] A. N. Tunç and S. Özen Yıldırım, On a topological operator via local closure function, Turkish Journal of Mathematics and Computer Science 15(2) (2023), 227-236.

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Topological Approach in Different Branches of Science

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ABSTRACT

Learning every new phenomenon by gamifying it from childhood is of great importance at the cognitive level. For some unknown reason, when we move towards adulthood, the learning by playing games is abandoned. Learning without using imagination, seeing the reality behind objects, making abstract thought tangible or fully understanding the cause-effect relationship is not a permanent learning. The changing new world system has directed us to learn more, research more and produce more. This compelling force has certainly brought with it a painful process, but in order to produce, we need to go out of our comfort zone. During our university education, we have very few courses that push the boundaries of our thinking by taking us to the world of imagination, and one of them is Topology. Network topology, which states that we need only six people at most to reach one person, network topology based on the logic that the data flow between computer connections should not be interrupted and should work with the highest efficiency, artificial neural network topology used in modeling biological neural networks are examples that are not taught at the undergraduate level but are not disconnected from daily life, showing that topology works exactly as the new world system wants.

In this study, the relationship between topology and different branches of science will be emphasized and topological games will be discussed.

Key Words: Artificial neural network topology, network topology, topological games.

REFERENCES

[1] H. H. Hacısalihoğlu, Dönüşümler ve geometriler. İstanbul: Milli Eğitim Basımevi 1975.

[2] F. R. Kidder, Elementary and middle school children's comprehension of euclidean transformations, Journal for Research in Mathematics Education, 7(1), (1976), 40-52.

[3] M. Gausa, V. Guallart, W. Müller, F.Soriano, F.Porras and J. Moraies, The metapolis dictionary *of advanced architecture.* Barselona: Ingoprint SA, 2003.

[4] T. Yomralıoğlu, Coğrafi Bilgi Sistemleri, Temel Kavramlar ve Uygulamalari, Seçil Ofset, İstanbul, 2000.

[5] N. Miraboğlu, Mimari Tasarımda Topolojik Yaklaşımlar Üzerine Bir Araştırma, Lisans Tezi, Gazi Üniversitesi Fen Bilimleri Enstitüsü, Ankara, 2019.

[6] R. Telgársky, Topological games: on the 50th anniversary of the Banach-Mazur game, The Rocky Mountain Journal of Mathematics, 17(2), (1987), 227-276.

[7] C. Berge, Topological games with perfect information, Contributions to the Theory of Games, 3, (1957), 165-178.

[8] S. Önal, S. Soyarslan, Ç. Vural, On generalizations of topological games, Topology and its Applications, 258, (2019), 524-533.

[9] F. Galvin, R. Telgársky, Stationary strategies in topological games, Topology and its Applications, 22(1), (1986), 51-69.

Zariski Topology on $S-n$ –Hyperideals

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ABSTRACT

In this talk, topological classification will be done using $S-n$ –hyperideals over the commutative hyperring. In this context, n –hyperideals of the hyperring R and $S-n$ –hyperideals as specializations of this hyperstructure will be defined and their topological properties will be presented. We will denote the multiplicative closed subset of the hyperring R as S. Let $Spec^{*}(R)$ denote the set of all $S-n$ –hyperideals of R. Let us define a set to be I a hyperideal of R satisfying the condition $I \cap J = \emptyset$. Firstly we have defined that a set as a closed set for a topology τ on the set $Spec(R)$. We call this topology the $S-n$ –Zariski topology of R and denote it by $Spec^{*}(R)$. Then we have defined any open sets of $Spec^{*}(R)$. Finally we have proved that a basis of the $S-n$ –Zariski topology for each element of the hyperring R.

Key Words: Zariski Topology, multiplicative closed subset, $S-n$ –hyperideal of a commutative hyperring.

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REFERENCES

[1] B. Davvaz and V. Leoreanu-Fotea, Hyperring theory and applications, International Academic Press, USA, 2007.

[2] H. Khashan and E.Yetkin Çelikel, S-n-ideals of commutative rings, Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics 72 (1), (2023), 199–215.

[3] U. Tekir, S. Koç, K.H. Oral, n-ideals of commutative rings, Filomat 31 (10), (2017), 2933–2941.

MATHEMATICS EDUCATION

A Fuzzy Multi-Criteria Decision-Making Method Application: The Case of an Educational Mobile Mathematics Application

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ABSTRACT

Mobile applications, due to their portability and access to various platforms, have transcended physical boundaries in mathematics education, allowing for the expansion and enhancement of educational environments. In this context, determining which of the thousands of available applications are most effective for mathematics education has become a research-worthy topic. A teacher looking to integrate mobile learning (m-learning) into their teaching must select an appropriate application for their lesson by considering mathematical content and pedagogical approaches. In recent years, the widespread adoption of mobile learning technologies has led to an increase in the use of mobile applications in education, particularly in mathematics instruction. However, the necessity of considering multiple criteria in evaluating these applications has made it essential to employ more sophisticated and flexible approaches beyond traditional evaluation methods. Fuzzy logic-based multi-criteria decision-making (MCDM) techniques, by effectively handling uncertainty and subjective judgments, have emerged as a crucial tool in such evaluations.

In this study, ten different mathematical mobile applications were evaluated based on ten distinct criteria. The selected mobile applications were chosen according to data obtained from experts and teachers working in schools under the Ministry of National Education. Similarly, as a result of studies conducted with experts and teachers, ten criteria were identified for evaluating mathematical mobile applications. These criteria were assessed by twenty-five teachers who are either pursuing or have completed postgraduate education and are working in public schools in various provinces under the Ministry of National Education. The evaluation matrix derived from the assessments was analyzed using the Fuzzy TOPSIS method, and the effectiveness of the applications in contributing to mathematics education was evaluated. The results indicate that such studies can provide valuable insights to mathematics teachers regarding beneficial educational mobile applications.

The findings also suggest that the study can guide educators in making more informed decisions when selecting mobile applications and assist developers in improving existing applications. Moreover, this study demonstrates the applicability and effectiveness of fuzzy MCDM techniques in evaluating mobile applications used in mathematics education, highlighting the need for more comprehensive evaluation studies in this field.

Keywords: Fuzzy TOPSIS, Mobile Learning, Fuzzy Multi-Criteria Decision-Making, Educational Mobile Application, Mathematical Mobile Application.

A RESEARCH ON THE BASIS OF MATHEMATICS ANXIETY OF MIDDLE SCHOOL STUDENTS

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ABSTRACT

Mathematics has an important place in academic life for everyone. Those who successfully pass to the upper education level are those who are successful in mathematics, and similarly, those who are successful in mathematics are able to be successful in their careers as they planned. Being successful in mathematics is one of the important steps in achieving the desired career. Those who are successful in mathematics can also be emotionally strong. However, there may be factors that prevent this situation and cause fear and shyness from mathematics. Başar and Doğan (2020) identified the factors that cause fear of mathematics as 4 factors in their study. They determined these factors as fear arising from the personal qualities of individuals, fear arising from family and environment, fear arising from the teacher, and fear arising from the characteristics of mathematics itself. This study aimed to obtain the opinions of secondary school 7th and 8th grade students about mathematics anxiety and to get to the roots of this anxiety. The current research is very important to guide teachers so that the necessary precautions can be taken by examining the effects of the factors that cause mathematics anxiety in students (such as environment, teacher, classroom and family) on mathematics anxiety. In this regard, a case study, one of the qualitative research designs, was used in the study. The study group of the research consists of 20 students studying in the 7th and 8th grades at a public secondary school in Karabük in the 2024-2025 academic year. Within the scope of the current research, the interview form prepared by the researchers in order to reveal the opinions of the students as a data collection tool was applied by the school counselor in a one-on-one chat environment in the guidance service, thus ensuring that sincere and real feedback was received from the students. As a result of the research, the majority of the students stated that their fear of mathematics stems from the structure of mathematics, that the subjects become more difficult and their grades decrease with the transition to secondary school, that there are negative family attitudes and behaviors that cause the fear of mathematics, and that the negative teacher attitudes and behaviors that cause the fear of mathematics include "high expectations". They stated that the reason for fear of mathematics arising from the classroom and its environment is to achieve "success in mathematics" within the scope of environmental success. The most important subject for LGS is mathematics. Moreover, as a suggestion to overcome the fear of mathematics, many of the students answered "reducing the expectation of success in mathematics from the student".

Key Words: Guidance Service, Fear of Mathematics, Mathematics Education

REFERENCES

[1] Başar, M., & Doğan, M. (2020). Öğrencilerin matematik korkusunun incelenmesi. TurkishJournal of EducationalStudies, 7(3), 1-26.

[2] Kuzu, Ç. İ. (2021). AİLE VE ÖĞRETMEN YAKLAŞIMLARININ MATEMATİK KAYGI DÜZEYİNE ETKİSİ İLE İLGİLİ ÖĞRENCİ GÖRÜŞLERİ. EKEV Akademi Dergisi, (85), 113-128.

[3] Gonzelez, G., Maria, S. andGustavo Martinez, S. (2020). Thehistory of a teacher'srelief of his mathematicsanxiety: thecase of Diego EducationalStudies in Mathematicshttps://doi.org/10.1007/s10649-020-09941-8

A Review of the Literature on Dyscalculia in TÜRKİYE

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ABSTRACT

Dyscalculia is defined as an individual's difficulty in comprehending mathematical relationships, recognising numerical symbols, using symbols functionally, making calculations and writing, despite the absence of an intelligence deficit [1]. The most widely accepted characteristic of children with dyscalculia is their difficulty in learning and remembering arithmetic facts. Furthermore, they encounter difficulties in calculation and problem-solving due to their underdeveloped processing abilities, with a high incidence of errors. In light of the significant influence these challenges exert on student performance, there has been a discernible surge in attention devoted to dyscalculia in recent times. The investigation of this subject, which commenced in the global research community in the 2000s, is now only just beginning to be undertaken in our country [2]. It is essential to increase research efforts in order to enhance awareness about the existence of dyscalculic students and their mathematics learning. Nevertheless, it is beneficial to ascertain the prevailing trends in dyscalculia research in our country and to identify the principal approaches that are being adopted, as this provides guidance for future research endeavours. In light of the aforementioned considerations, the objective of the current study is to examine the existing research on dyscalculia. The data collected through document analysis, a qualitative research design, was analysed using the document analysis technique. The analysis was conducted on twenty articles prepared between 2018 and 2024. Upon examination of the studies in general, it was determined that common topics included the opinions of primary and secondary school teachers, the process of diagnosing students with dyscalculia risk, and teachers' awareness of dyscalculia. It was established that the literature contains a greater number of studies conducted with teachers than studies on the knowledge and awareness of dyscalculia among those preparing to become teachers. With

regard to the research group, it is evident that the majority of studies focus on primary school students, with relatively few investigating other grade levels In terms of content, there is a tendency towards studies that examine the effect of the subject matter on students' numbers and operations, number sense and counting [4].It was observed that there is a dearth of studies pertaining to the development of curricula or pedagogical models for students identified as dyscalculic or at risk of dyscalculia. The data were analysed in terms of research methodology and it was found that qualitative research and descriptive analyses were the most commonly used approaches. The limited number of studies examining instructional interventions for students with mathematics learning disabilities in our country is a remarkable result. It may be recommended to increase the number of studies on the design of mathematics courses in a way to increase the academic achievement of these students. In light of the findings of this study, it is recommended that teacher educators and researchers address the gaps in their understanding of dyscalculia.

Key Words: Discalculia, Mathematics learning disability, Literature review, Document analysis

REFERENCES

[1] Butterworth, B. (2003). Dyscalculia screener: Highlighting pupils with specific learning difficulties in maths. London, UK: Nelson Publishing Company.

[2] Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: from brain to education. *science*, *332*(6033), 1049-1053.

[3] Mutlu, Y., & Akgün, L. (2017). Matematik öğrenme güçlüğünü tanılamada yeni bir model önerisi: Çoklu süzgeç modeli. *İlköğretim online*, *16*(3), 1153-1173.

[4] Koç, B., & Koç, İ. (2019). Okuma yazma bilmeyen diskalkulik bir öğrenciye toplama ve çıkarma öğretimine yönelik bir eylem araştırması. *Eğitimde Nitel Araştırmalar Dergisi*, *7*(2), 710-737.

Analysis of Students' Computational Thinking Ability using "Smart Fraction" Learning Media

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ABSTRACT

Computational thinking is an essential skill for students in the digital era, since it enables them to solve problems systematically and logically, which is highly relevant in many fields of science. Computational thinking supports a more indepth understanding of math and science concepts and develops the analytical skills needed to face complex challenges in the real world. This research aims to analyze students' computational thinking ability using "Smart Fraction" learning media assisted by iSpring Suite 9 and GeoGebra software. The research method used was descriptive qualitative, with data collected through tests and observations of elementary school students. The expected result of this research is a more obvious mapping of students' computational thinking ability, which can be used as a foundation for developing more effective learning strategies. The technic, used for data analysis, is technical triangulation. The results that have been obtained in the Research are that students can identify problems to be more simple so that it can be easier to understand (Decomposition) and students can decide what information to keep and what to ignore (Abstraction).

Key Words: Computational Thinking, Qualitative Research.

REFERENCES

[1] Benluo, Y., & Li, S. (2008). Preliminary thinking about the mathematic foundation of modern computational electromagnetism. 4th International Conference on Ultrawideband and Ultrashort Impulse Signals.

[2] Nurkanti, M., Utari, T. S. G., & Devi, C. (2018). Improve Student Learning Outcomes Through The Use of Interactive Visual Learning Media (MIVI). SCITEPRESS–Science and Technology Publications

[3] Yohannes, A., & Chen, H. L. (2021). GeoGebra in mathematics education: a systematic review of journal articles published from 2010 to 2020. Interactive Learning Environments

Analysis of the Implementation of the Independent Learning Curriculum (MBKM) on Improving Creativity and Problem-Solving Skills of Mathematics Education Students

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ABSTRACT

The Merdeka Belajar - Kampus Merdeka (MBKM) or Freedom to Learn-Independent Campus was launched by the Indonesian Minister of Education, Culture, Research and Technology, since 2021. One of the policy has a direct impact on students, namely the students' rights to study for three semesters outside study programs. MBKM aims to enhance both soft skills and hard skills to prepare graduates with personality and the ability to face the progress and developments of the times, as well as supporting graduates' employability and linking them to the job market. There are six forms of learning activities (FLP) have been implemented in the Mathematical Education Study Program, Jember University namely, 1) exchange of students in other study programs or other colleges, activities in partner institutions such as 2) teaching assistance in education units, 3) internships, 4) independent study, 5) research, and 6) building a village/faculty of real work thematic. This study examines the implementation of the curriculum Merdeka studied in the Mathematical Education Studies Program of Jember University. The indicators used in this study are creativity development, problem-solving skills, self-development, and readiness to enter the world of work. The research uses the method of surveys and interviews against students participating in MBKM and satisfaction surveys against partner institutions. Students participating in MBKM stated that 79% of MBKM activities can provide an increase in the development of creativity in solving thinking and improved problem-solving skills by 71.4%. In addition self-development of student such as the ability to communicate and collaborate in a team, leadership, are above 80%. Finally

88% of students stated the MBKM curriculum is relevant to the needs of the workplace that is in line with the job profile of the Mathematical Education Studies Program and feel better prepared to enter the workforce.

Key Words: creativity, problem-solving skills, MBKM curriculum.

REFERENCES

[1] Ditjen Dikti Kemdikbud RI, "Independent Learn-Independent Campus Guidebook (*Buku Panduan Merdeka Belajar-Kampus Merdeka)*," 2020

[2] E. Purwanti, "Preparing the implementation of merdeka belajar–kampus merdeka policy in higher education institutions," in *4th International Conference on Sustainable Innovation 2020–Social, Humanity, and Education (ICoSIHESS 2020)*, Atlantis Press, 2021, pp. 384–391.

[3] K. D. P. Meke, R. B. Astro, and M. H. Daud, "Impact of Independent Learning Independent Campus (MBKM) Policy on Private Universities in Indonesia (*Dampak Kebijakan Merdeka Belajar Kampus Merdeka (MBKM) pada Perguruan Tinggi Swasta di Indonesia*)," *Edukatif: Jurnal Ilmu Pendidikan*, vol. 4, no. 1, pp. 675–685, 2022.

[4] A. Setiawan, A. Suryani, and D. Kurniawati, "Independent Learning Policy Independent Campus: Encouraging Higher Education Reform in Indonesia (*Kebijakan Merdeka Belajar Kampus Merdeka: Mendorong Pembaruan Pendidikan Tinggi Di Indonesia)*," *Humantech: Jurnal Ilmiah Multidisiplin Indonesia*, vol. 2, no. 5, pp. 905–913, 2023.

[5] R. Restu, S. Sriadhi, S. Gultom, and D. Ampera, "Implementation Of The Merdeka Belajar-Kampus Merdeka Curriculum Based On The RI 4.0 Platform At Universitas Negeri Medan," *Journal of Positive School Psychology*, pp. 10161–10176, 2022.

[6] I. Ramli, N. Nurasia, and K. Khaerati, "Implementation of Research Learning Activity Forms *(Implementasi Bentuk Kegiatan Pembelajaran Riset*)," *Jurnal Studi Guru Dan Pembelajaran*, vol. 5, no. 3, pp. 278–284, 2022.

[7] D. Defrizal, A. P. Redaputri, V. T. Narundana, N. Nurdiawansyah, and Y. Y. Dharmawan, "The merdeka belajar kampus merdeka program: An analysis of the success factors," *Nusantara: Jurnal Pendidikan Indonesia*, vol. 2, no. 1, pp. 123–140, 2022.

[8] B. Wallace, "Creativity: Some definitions: The creative personality; the creative process; the creative classroom," *Gifted Education International*, vol. 4, no. 2, pp. 68–73, 1986.

[9] J. R. Robinson, "Webster's dictionary definition of creativity," *Online Journal for Workforce Education and Development*, vol. 3, no. 2, p. 2, 2008.

[10] A. D. Matraeva, M. V. Rybakova, M. V. Vinichenko, A. A. Oseev, and N. V. Ljapunova, "Development of Creativity of Students in Higher Educational Institutions: Assessment of Students and Experts," *Universal Journal of Educational Research*, vol. 8, no. 1, pp. 8–16, Jan. 2020, doi: 10.13189/ujer.2020.080102.

[11] W. Wahyu, A. Suryatna, and Y. S. Kamaludin, "The suitability of William's creativity indicators with the creativity-based worksheet for the junior high school students on designing simple distillation tool," *Journal of Engineering Science and Technology (JESTEC)*, vol. 13, no. 7, pp. 1959–1966, 2018.

[12] P. Drapeau, sparking student creativity: Practical ways to promote innovative thinking and problem solving. ASCD, 2014.

[13] N. Nixon, The creativity leap: Unleash curiosity, improvisation, and intuition at work. Berrett-Koehler Publishers, 2020.

[14]Y . I. T. Tanjung, F. Festiyed, S. D. Diliarosta, J. Jubaidah, and A. P. S. Sanjaya, "Analysis of Students' Problem Solving Skills based on Heller Indicator," *Pegem Journal of Education and Instruction*, vol. 14, no. 4, pp. 421–433, 2024.

[15] L. D. Permata, T. A. Kusmayadi, and L. Fitriana, "Mathematical problem solving skills analysis about word problems of linear program using IDEAL problem solver," in *Journal of Physics: Conference Series*, IOP Publishing, 2018, p. 012025.

Classroom Reflections on Implementing the New Mathematics Curriculum: Opportunities and Challenges

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ABSTRACT

The purpose of this research is to investigate the opportunities and challenges from the perspectives of students and teachers in teaching special quadrilaterals according to the new mathematics curriculum (Türkiye Yüzyılı Maarif Modeli). We adopted an action research method which is a type of research that involves the process of applying scientific methods to solve problems in the classroom environment (Fraenkel, Wallen, & Hyun, 1993), and conducted the study at a public high school in Istanbul, involving 10th-grade students. Data collection took place during the second term of the 2023-2024 academic year. The data collection instruments included classroom observations, activity papers, achievement tests on quadrilaterals, teacher diaries, and interviews. We designed activities to explore special quadrilaterals aligned with the new mathematics curriculum (MoNE, 2024). We videotaped two classes where the first and second authors were the teachers. The teachers recorded notable events in their diaries. Pre-tests and post-tests on special quadrilaterals were administered, and the tests were scored according to the answer key. We interviewed six students from each class, representing low, middle, and high achievement levels based on the post-test results. All video recordings and interviews were transcribed and analyzed using content analysis. Diaries and activity papers were also analyzed through content analysis, while pre-test and post-test results were analyzed using statistical methods. The findings revealed a significant difference between the pre-test and post-test results. The pre-test scores for Classrooms A and B were 5.15 and 3.36, respectively, while the post-test scores were 42.96 and 45.16. Additionally, the data indicated that students had many opportunities to understand the relationships between special quadrilaterals through topic discussions and the identification and overcoming of misconceptions. However,

there were also some difficulties, such as technical limitations and the inadequate adaptation of the material by some students. The results will be discussed in comparison with findings from other research in the literature.

Key Words: New math curriculum, new math curriculum implementation, special quadrilaterals.

REFERENCES

[1] Fraenkel, J., Wallen, N., & Hyun, H. (1993). *How to Design and Evaluate Research in Education 10th ed.* McGraw-Hill Education.

[2] Ministry of National Education (MoNE, 2024).

Educational Mathematics Mobile Applications (EMMA): Mathematics Teachers' Opinions After the TUBITAK 4005 Project

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ABSTRACT

Mobile learning enables students to learn continuously and uninterruptedly by making learning independent of space and time. Research shows that mobile learning improves students' mathematical success, problem-solving skills, motivation, and self-efficacy. The multifunctional structure of mobile devices provides students with richer and more concrete learning experiences, facilitating the visualization of mathematical concepts. Additionally, teachers can make lessons more engaging by using mobile tools and applications that students are already familiar with in mathematics education. This approach can also increase students' awareness of how they can utilize available technologies in their own learning. Therefore, it can be said that mathematics teachers' awareness and skills regarding the integration of educational mathematics mobile applications into education should be enhanced. In response to this need, the "Digital Education in Our Pocket: Increasing Teachers' Awareness of EMMA (Educational Mathematics Mobile Applications)" project was first implemented in 2022 within the scope of the TÜBİTAK 4005 Innovative Education Applications Support Program. Due to the project's success and high demand, a second iteration was organized this year. This study shares the opinions of the teachers who participated in the second phase of the project. The question "What are the participating teachers' opinions about this project?" was explored.
As part of the project, up-to-date and practical EMMA that mathematics teachers can use in their lessons were identified. Interactive activities and seminars were developed for these applications, which were then introduced to the teachers. The teachers were given the opportunity to use and evaluate these applications, and examples were provided on how they could adapt them for their lessons. The project

included a 7-day training session for 30 mathematics teachers. To assess whether the project met its objectives, opinion forms were created by the project team, with input from experts, to evaluate the project. During the training, participants were asked to complete these forms at the beginning and end of each activity, and the collected data were analyzed using content analysis. The participating teachers indicated that they were unfamiliar with almost all of the mobile applications introduced, and even if they had heard of them, they had not used them in their lessons or mathematics education. However, they expressed that they would use these applications in the future. Based on these results, it can be said that the project successfully contributed to its primary goal of increasing teachers' awareness of educational mathematics mobile applications. This report aims to support the dissemination of similar projects by sharing the perspectives of the teachers who participated in this initiative.

Key Words: Mathematics education, mathematics teacher, mobile applications.

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Examples of Good Practices in Teaching Mathematics for Middle School Performance and Competitions

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ABSTRACT

Teaching mathematics in middle school for performance and contest success requires a holistic approach that combines a strong foundation in mathematical concepts, problem-solving skills, and a growth mindset.

Mathematics is a critical subject in middle school, laying the foundation for higher-level problem-solving and reasoning skills. Effective teaching practices, particularly those tailored for students aiming to excel in mathematics competitions, can significantly enhance both performance and engagement. These practices not only build conceptual understanding but also foster creativity, critical thinking, and perseverance—qualities essential for mathematical excellence.

Teaching mathematics to middle school students with the goal of both academic performance and competition success requires a blend of pedagogical strategies that focus on fostering deep understanding, problem-solving skills, and resilience. Middle school is a crucial stage where foundational math concepts are solidified, and the groundwork for more advanced topics is laid. Therefore, these practices not only build conceptual understanding but also foster creativity, critical thinking, and perseverance—qualities essential for mathematical excellence.

The paper will reveal some insights in teaching and guiding, that Mathematics Teachers in the county of Sibiu (Romania) use to raise students' interest to participate in Mathematics Competitions.

Key Words: Mathematical Education, Teaching for Competitions, Didactics.

REFERENCES

[1] Didactica Matematică, Supliment al Publicației Gazeta Matematică, Societatea de Științe Matematice din România, 2021-2024

[2] Metodica predării matematicii în gimnaziu, Dan F., Editura Saga 2015

[3] Matematica de excelență pentru concursuri, olimpiade şi centre de excelenţă. Clasa a VI-a, Lint D., Lint M., Marinescu R., Ștefan D., Monea M., Monea S., Stroe M., Ed. Paralela 45, 2019

Geometry and Mathematics in Traditional Turkish Handicrafts

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ABSTRACT

Throughout their ancient history, people have tried to describe everything around them, and while doing this, they have used vegetal, figural and geometric designs to which they attributed various meanings. Geometry became a measuring tool in these designs, which turned into art over time, and thus, geometrical decorative arts emerged with the fusion of mathematics and art in harmony. Calligraphy, which emerged between the 6th and 10th centuries, is one of them. Calligraphy, defined as "a spiritual engraving created with physical tools", is a line art created by human hands using reed pen and ink. The dictionary meaning of the word calligraphy is explained as "writing, drawing, engraving, putting a sign", meaning "writing, line, It also means "ground, path". The calligraphy was measured based on a single point coming from the tip of the pen, and the writing types called "aklam-ı sitte" were created. In these writing types, the number of dots of each letter varies with the aesthetic feelings of the artist. Works that reflect their philosophy, understanding and culture emerge.

Illumination, which means "golding" in the dictionary, is an important branch of book arts. The patterns that form the basis of the art of illumination, which is performed in harmony with calligraphy, are designed in geometric forms such as circles, squares, rectangles and ovals. These designs can be prepared in the form of ulama, or they can be designed with or without symmetry and in the style of a wheel of fortune. The motifs that make up the pattern are prepared by stylizing animal and plant forms while preserving the main lines and pattern determined by measurements. Penç and hatai, which are drawn by stylizing the bird's eye view of leaf flowers drawn with circular forms and mathematical measurements, and rumi, munhani and cloud, which are the stylized forms of the body, legs and wings of animal figures, are the motifs applied when creating patterns in illumination. While creating the pattern, these motifs are placed on spirals and geometric forms within certain rules. The 12th and 13th centuries are the period in which mathematics and geometry were intense in

Anatolian illumination art. In the 16th century pattern designs, the zencerek, which consists of dots, tangent to the dots and key lines connecting the centers of the dots, is also one of the geometric ornament.

The aim of this study is to emphasize that various mathematical and geometric calculations are elements that contribute to the formation of patterns and designs beyond visual and quantitative numerical expressions, and in this context, reflect the emotions and aesthetic feelings of the artist. Our traditional arts, such as calligraphy and illumination, which have developed throughout history, are ancient branches of art that require the use of scientific techniques with historical depth, beyond their aesthetic appearance that is not designed and implemented haphazardly.

Key Words: Geometry, Traditional Arts, Calligraphy, Illumination Art.

REFERENCES

[1] C.E. Arseven, (Sanat Ansiklopedisi. İstanbul: Maarif Basımevi, 2, 1958.

[2] İ. Birol Ayan, "Tezhip". TDVİA. İstanbul: Türkiye Diyanet Vakfı Yay., 41: 61-62, 1988.

[3] İ. Birol Ayan, Türk Tezyini Sanatlarında Desen Tasarımı Çizim Tekniği ve Çeşitleri. İstanbul: Kubbealtı Yayınları, 2008.

[4] A. Alparslan, Osmanlı Hat Sanat Tarihi. İstanbul: Yapı Kredi Kültür Sanat Yayınları, 1999.

[5] M.B. Yazır, Kalem Güzeli. İstanbul: Diyanet İşleri Başkanlığı Yayınları, 1972.

[6] M.U. Derman, "Hat". TDVİA. İstanbul: Türkiye Diyanet Vakfı Yay., 16: 427-437, 1997.

[7] M. Serin, "Hat Sanatı" İslam Sanatları Tarihi, Eskişehir, 2014.

Investigation of Mathematical Reasoning Self-Efficacy and Geometric Thinking Habits of Middle School Mathematics Teacher Candidates

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ABSTRACT

The aim of this study is to examine the relationship between mathematical reasoning self-efficacy and geometric thinking habits of middle school mathematics teacher candidates. The study was designed in a quantitative research design. Causal comparison and correlational study methods were used among quantitative research methods. The sample of the study will consist of primary middle school mathematics teacher candidates of a university located in Central Anatolia. The data collection tools of the study consist of the "Mathematical Reasoning Self-efficacy Scale" developed by Mumcu (2019) and the "Geometric Thinking Habits Scale" developed by Bülbül and Güven (2021). Data will be collected from approximately 160 students using these scales. The obtained data will be analyzed with the SPSS 20 statistical program. From the analysis methods, quantitative analysis techniques will be used by checking whether the data are normally distributed after the normality test is performed. In case of normal distribution of data, correlation will be used to examine the relationship between mathematical reasoning self-efficacy and geometric thinking habits, multiple correlation will be used to examine the relationship between the sub-dimensions of the scales, regression will be used to reveal the predictive status of scale variables and multiple regression analysis will be used to examine the predictive status of sub-dimensions. As a result of the study, a statistically significant positive and moderate relationship is expected between the level of mathematical reasoning self-efficacy and geometric thinking habits. Another expected finding is that mathematical reasoning significantly predicts geometric thinking habits.

Key Words: Mathematical reasoning, geometric thinking habits, teacher candidate

REFERENCES

[1] Mumcu, H. Y. (2019). İlköğretim matematik öğretmen adaylarının matematiksel muhakeme öz-yeterlik inançlarının incelenmesi: Bir ölçek geliştirme ve uygulama çalışması. Kırşehir Eğitim Fakültesi Dergisi, 20(3), 1239-1280.

[2] Bülbül, B.Ö., & Güven, B. (2021). Geometrik düşünme alışkanlıklarına yönelik inançlar: Bir ölçek geliştirme çalışması. Cumhuriyet International Journal of Education, 10(2), 886-905. http://dx.doi.org/10.30703/cije.813168

Mathematical Connections: Evaluation of Activities Designed by Preservice Elementary Mathematics Teachers Using Different Themes

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ABSTRACT

This study aimed to examine the activities designed by preservice elementary mathematics teachers using different themes in the context of mathematical connections. In this research study, which was designed in accordance with the qualitative methodology, the case study method was adopted. A total of 148 documents were designed by preservice teachers based on the relationship between mathematics and the themes of out-of-school learning environment, art, interdisciplinary, and technology were examined in this study. In this research study, the existing situation was conveyed as it is, and various categories and codes were used in the analysis of the data. The research study sample consists of 37 preservice elementary mathematics teachers studying at the undergraduate level. Document review technique was used in data analysis. In this context, the activities designed by each participant within the scope of the work instructions were examined in the context of real-life relations. In the study, preservice elementary mathematics teachers mostly preferred the Geometry and Measurement learning area at the eighth-grade level while designing their activities. On the other hand, they stated that they aimed to make students learn mathematics better, easier and permanent by associating mathematics with real life while designing their activities. Teacher candidates who paid attention to using correct mathematical terms and concepts preferred creating their activities in the light of historical artifacts, monuments, museums, production facilities, surface art, physics content area, and geogebra software. The majority of the developed design content was based on the discussion of mathematics in society. Most of the activities designed by preservice mathematics teachers were related to the Numbers and Operations learning area and the outcomes that were included in this learning area. Increasing the awareness of preservice elementary mathematics teachers about the importance of mathematical connections is presented as a recommendation.

Key Words: Elementary mathematics teacher, mathematical connection, skill

REFERENCES

[1] R. R. Skemp, Relational understanding and instrumental understanding, Mathematics Teaching, 77(1), (1976), 20-26.

[2] H. Yavuz Mumcu, Connection and Its Meaning in Mathematics Education, In H. Yavuz Mumcu, A. Osmanoğlu, H. Korkmaz (Eds.), Making Connections in Mathematics Education (pp. 2-26), Pegem Akademi Publishing, Ankara, 2023.

[3] J. García-García and D. Dolores-Flores, Exploring pre-university students' mathematical connections when solving Calculus application problems, International Journal of Mathematical Education in Science and Technology, 52(6), (2021), 912-936.

[4] A. Yıldırım and H. Şimşek, Qualitative Research Rethods in The Social Sciences (11th ed.), Seçkin Publishing, Ankara, 2018.

[5] J. W. Creswell, Educational Research: Planning, Conducting, and Evaluating Quantitative and Qualitative Research (4th ed.), Pearson, Boston, MA, 2015.

[6] J. Gainsburg, Real-world connections in secondary mathematics teaching, Journal of Mathematics Teacher Education, 11(3), (2008), 199-219.

[7] M. Özgeldi and A. Osmanoğlu, Connecting mathematics to real life: An investigation on how prospective secondary mathematics teachers build real life connections, Turkish Journal of Computer and Mathematics Education, 8(3), (2017), 438-458.

[8] R. K. Yin, Case Study Research: Design and Methods (3rd ed.), Chongqing University Press, Chongqing, 2004.

Mathematics Teaching in the Digital Age: Preservice Teachers' Perspectives on Artificial Intelligence and Digital Platforms

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ABSTRACT

The digital age represents a period influenced by the rapid development of information and communication technologies, affecting every aspect of education, work life, and daily living. During this era, digital technologies and platforms have caused profound changes in the processes of accessing information, learning, and teaching (Kubanç & Kökçü, 2023; Saykılı, 2018). In particular, the digitization in education has reshaped traditional teaching methods and created significant changes in the roles of teachers and students (Weigand, Trgalova, & Tabach, 2024). Mathematics teaching has also been impacted by this transformation, incorporating digital platforms and artificial intelligence technologies into teaching processes (Anh & Ngan, 2021; Clark-Winson et al., 2021; Görgüt, 2024). In this context, the role of teachers has become crucial for students to acquire the skills required by the digital age (Yalap & Gazioğlu, 2023). Therefore, it is essential for preservice teachers to develop their skills in using digital technologies and integrating these technologies into the classroom environment during their pre-service training. Furthermore, the perspectives of preservice mathematics teachers on the future of mathematics teaching in the context of digitization and artificial intelligence technologies are expected to shed light on future studies. The aim of this study is to examine preservice teachers' perspectives and ensure that the findings play an effective role in pre-service teacher education processes. Therefore, developing preservice teachers' abilities to use digital technologies and integrate them into the classroom environment stands out as a significant requirement for both current educational processes and future teaching practices.

To this end, a "Semi-Structured Opinion Form" consisting of 7 questions was administered to 53 preservice teachers (39 female, 14 male) studying in the Division of Elemantary Mathematics Education of a state university. The interview form included questions about the integration of digital platforms into the teaching process, how artificial intelligence technologies will shape their profession, and their views on the schools of the future. Through these questions, the study explores how preservice teachers assess the knowledge and skills required to be an effective mathematics teacher in the digital age within the framework of Technological Pedagogical Content Knowledge (TPACK) components. Additionally, it examines whether preservice teachers find their education sufficient and how they use digital platforms and artificial intelligence tools. The collected data are being analyzed using the case study method, a qualitative research design. The analysis process is ongoing, and the findings will be presented at the conference.

Key Words: Digital Technologies, Artificial Intelligence, Mathematics Education

REFERENCES

[1] Anh, N. N. P., & Ngan, H. T. (2021). Artificial intelligence in mathematics education: An empirical study of using chatbot in teaching and learning mathematics at vietnamese high schools. *5th ASIA PACIFIC International Modern Sciences Congress* (ss. 306-317).

[2] Clark-Wilson, A., Donevska-Todorova, A., Faggiano, E., Trgalová, J., & Weigand, H. G. (2021). Mathematics education in the digital age. Routledge.

[3] Görgüt, R. Ç. (2024). Yapay zeka ve matematik eğitimi. Eğitim & Bilim 2023-IV, 43.

[4] Kubanç, Y., & Kökçü, Y. (2023). 3. Bölüm: Dijital çağda öğretmen ve öğrenci. Burcu, G., Ş. (Ed.), Dijital eğitim ve toplum (s. 59). Efe Akademi Yayınları.

[5] Saykılı, A. (2018). Dünden yarına eğitim paradigmaları: Sanayi modeli eğitim dijital çağda yeterli mi? Açıköğretim Uygulamaları ve Araştırmaları Dergisi, 4(2), 189-198.

[6] Yalap, H., & Gazioğlu, M. (2023). Türk eğitim sisteminde dijitalleşmeye yönelik uygulamalar. Manas Sosyal Araştırmalar Dergisi, 12(ÖS), 80-90.

[7] Weigand, H. G., Trgalová, J., & Tabach, M. (2024). Mathematics teaching, learning, and assessment in the digital age. ZDM Mathematics Education

STUDENT OPINIONS ON THE DIGITAL STORY PREPARATION PROCESS

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ABSTRACT

Rapid developments in science and technology affect change and development in the social structure of societies around the world. Due to individuals' need for technology, societies find themselves in an inevitable change and transformation. Technological developments are rapidly entering all areas of our lives with unpredictable results (Mutlu and Akgün, 2019). These rapid advances have revealed significant differences in the education system as well as in different fields. These differences have affected various factors of education, from student and teacher roles to learning environments, from tools and equipment to educational goals, and techniques and methods used in learning environments (Talan and Gülseçen, 2018). Educators have a great responsibility in adapting to the requirements and needs of the current era, following developments, and applying them to every aspect of our lives. It is thought that applying these developments to our daily lives and using them efficiently can be achieved by making efficient use of technology. In this context, with the use of technology in daily life and the increase in individuals' ability to use technology, there is a need for educational institutions to adapt to this process. It is defined as the gathering and use of all required technologies in order to create an efficient learning process specific to the purposes determined for the technology used in education, and the planning, realization and evaluation of the process in a suitable manner (Gürbüz, 2008; Zhu and Kumar, 2023).

Digital Storytelling

It is very important to use technology effectively and apply new technologies to education and to conduct teacher education programs from this perspective. The study conducted by Admiraal et al., (2017) had to implement flipped learning due to classroom management issues and related issues.

METHOD

Research Method

Method

The study is a case study, one of the qualitative research methods. According to Creswell (2007), in a case study, the researcher examines one or more situations limited in time using more than one data.

Working Group

The population of the research consists of specially talented students. The study was conducted with 18 specially talented students studying in the special talent development program at the Science and Art Center in Niğde. The students participating in the research are individuals who volunteer to participate in the study and can express themselves well. The students participating in the study differ from each other in terms of their demographic characteristics. It is thought that this difference will be effective in obtaining useful results in the research.

Key Words: Digital story, Students, Case study

REFERENCES

[1] Mutflu, Y., & Akgün, L. (2019). Using Computer for Developing Arithmetical Skills of Students with Mathematics Learning Difficulties. *International Journal of Research in Education and Science*, *5*(1), 237-251.S.G. Gal, Shape-Preserving Approximation by Real and Complex Polynomials, Birkhäuser Boston, Inc., Boston, MA, 2008.

[2] Zhu, J., & Kumar, S. (2023). An analysis of online and hybrid EdD programs in educational technology. TechTrends, 67(4), 693-705.

System Dynamics Approach in Predicting Students' Mathematics Achievement: The Case of PISA

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ABSTRACT

Mathematics is considered one of the most fundamental disciplines that directly contributes to individuals' cognitive and academic development, and it is regarded as one of the significant determinants of students' academic success. However, for many students, coping with mathematics is a highly challenging process, and this difficulty can lead to low mathematics achievement, negatively impacting their long-term academic and career goals. Therefore, developing strategies to enhance mathematics achievement and integrating these strategies into the education system is of great importance. Improving the level of success in mathematics not only enhances students' overall academic performance but also positively contributes to their future success by increasing their motivation to research and learn. In this context, developing strategies to enhance mathematics achievement has become an important goal for both countries and stakeholders in education. The difficulties encountered in mathematics are often attributed to individual differences and the diversity in teaching processes.

To develop effective strategies for students to achieve higher success in mathematics, educators and policymakers need to explore various approaches. This has increasingly led to the preference for modeling studies in the field of education using system dynamics, as a method to understand complex learning processes, improve education policies, and enhance student achievement. Therefore, mathematics education stands out as one of the important areas where system dynamics can be effectively applied. The processes by which students learn mathematical concepts, the effects of educational policies, and the long-term outcomes of teaching strategies can be modeled and subjected to comprehensive analyses using system dynamics. Such studies contribute to the more effective design and implementation of strategies for mathematics education. Applying system dynamics to factors affecting student achievement can encompass many elements, including student motivation, teaching methods, classroom dynamics, and student-teacher interactions, as well as the quality time students spend in school, family communication, and their socio-economic background. Models and analyses created using system dynamics serve as important tools for evaluating the possible outcomes of a specific strategy and predicting the potential impacts of strategies. This study emphasizes the importance of using system dynamics to understand and improve mathematics achievement in educational processes.

In this study, after defining the variables affecting mathematics achievement and the dimensions formed by these variables, the student dimension was addressed, the variables constituting the student dimension were identified, and then a stock-flow model related to the student dimension was created. The created model consists of two stocks and a total of nineteen main variables. The model was run covering the years 2003-2018, and the results obtained were compared both with the PISA results of the relevant years and with the literature.

Keywords: System Dynamics, Mathematics Achievement, Modeling with System Dynamics, PISA Mathematics.

Teacher Perspectives on the Implementation Process of Secondary School Mathematics Curriculum

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ABSTRACT

In the past fifty years, the rapidly changing world has deeply impacted the field of education. Countries showing industrial development and knowledge-based societies have had to reconsider their educational paradigms, leading to changes in traditional educational approaches (Ersoy, 2006). This global transformation has made it inevitable to renew the content and application aspects of educational programs. The World Bank's (2018) report emphasizes that teachers are the most significant factor affecting the learning process and highlights that students play a crucial role in shaping the future, being at the center of education. A theoretically well-designed teaching plan may not necessarily have the desired impact in practice. This can result from factors such as classroom dynamics, student profiles, teacher implementation skills, and current educational conditions not being adequately considered, even if the theoretical foundation of the teaching plan is strong. Therefore, for a teaching plan to be successful, it must be flexible enough to adapt to challenges encountered during the implementation process (Kıyak et al., 2020).

Teachers' positive views on written teaching programs are a significant factor in increasing the applicability of these programs (Bal et al., 2021). In Turkey, since the declaration of the Republic, primary education mathematics programs have been continuously updated and developed. The mathematics teaching programs renewed by the comprehensive reforms in 2005, in particular, have generally been positively

received and found to be applicable by teachers (Şen and Ünal, 2021). Therefore, determining the views of mathematics teachers, who are the fundamental element and implementers of the teaching process, will contribute to the qualitative advancement of the development and revision processes of teaching programs.

In this context, the aim of the study is to examine the impact of mathematics teaching programs on classroom practices and teachers' views on these programs. The study was conducted using a case study design, a type of qualitative research. The research group consists of mathematics teachers working in public and private institutions. Data were collected online using a semi-structured interview form prepared by the researchers. The data analysis is being carried out using content analysis methods. Content analysis is a qualitative analysis method aimed at uncovering themes, meaningful patterns, and relationships within the data. During this analysis process, the data obtained from teachers' views and experiences are carefully examined, categorized, and connections between these categories are identified for in-depth interpretation. Thus, the goal is to obtain meaningful and consistent results that will answer the fundamental questions of the research. The analysis phase of the study is ongoing, and the findings will be presented at the symposium.

Key Words: Mathematics Curriculum, Mathematic Teachers, Mathematics Textbooks

REFERENCES

[1] Baki, M. ve Özmen, Z. M. (2017). Konu alanı ders kitabı inceleme dersi kapsamında matematiği öğretme bilgisinin geliştirilmesi. Uludağ Üniversitesi Eğitim Fakültesi Dergisi, 30 (1), 209-238.

[2] Bal, P., Karabay-Turan, A. ve Kuşdemir-Kayıran, B. (2021). Sınıf öğretmenlerinin matematik dersi öğretim programına ilişkin değerlendirmelerinin incelenmesi. Trakya Eğitim Dergisi, 11(2), 717-732.

[3] Ersoy, Y. (2006). İlköğretim matematik öğretim programındaki yenilikler-I: Amaç, içerik ve kazanımlar. İlköğretim Online, 5(1), 30-44.

[4] Kıyak, Y. S., Budakoğlu, I. İ. ve Coşkun, Ö. (2020). Öğretim tasarımı, modelleri ve program geliştirme ile ilişkisi. Tıp Eğitimi Dünyası, 19(58):5-23

The Analysis of Effectiveness and Practicality of Exponential Function Learning Media Assisted by Desmos Integrated with TPACK

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ABSTRACT

The purpose of this research is to determine effectiveness and practicality of learning media assisted by Desmos that integrated with TPACK to improve the understanding of senior high school students of exponential function graphs. So that, an experiment research with quasi-experimental design was conducted with the sample of 36 students from Indonesian senior high school of student class B as the experimental group and 35 students of X-D class as the control group. Based on the research results analysis, it showed that the mean of pretest and posttest of experimental group were 43.22 and 85.44. Meanwhile the control group were 45.49 and 80.57. Based on N-Gain analysis, the average of experimental group of 0.74 is better than the control group 0.64. Furthermore, the practicality test conducted both experimental and the control group achieved a good score with the percentage of 93%. This research indicates that the learning media of exponential function assisted by Desmos integrated with TPACK meet the criteria of effectiveness and practicality. The understanding of students in the group that use learning media assisted by Desmos integrated by TPACK has increased significantly compared to conventional learning. By integrating media assisted by technology in learning, it makes students more enthusiasm, satisfaction, can improve student and increase class presence. The choice of Desmos as the media is the right choice because Desmos provides features for students to be able to explore graphs of exponential functions independently.

Key Words: Exponential Function, Desmos, Learning Media, TPACK

REFERENCES

[1] Babadogan, M. C., & Olkun, S. (2006). Program development models and reform in Turkish Primary School Mathematics curriculum. *International Journal for Mathematics Teaching and Learning*, 1–6. https://doi.org/10.1501/0003623

[2] Balım, A. G. (2009). The Effects of Discovery Learning on Students ' Success and Inquiry Learning Skills. *Eurasian Journal of Educational Research*, *35*(35), 1–20.

[3] Chan, Y. H. (2003). Biostatistics 102: Quantitative Data - Parametric & Non-parametric Tests. *Singapore Medical Journal*, *44*(8), 391–396.

[4] Djoa, D. D., & Kusumaningtyas, P. (2021). Design and Feasibility of EXE Learning Media on the Topic of Chemical Bonding. *International Journal of Active Learning*, *6*(2), 76–84. https://journal.unnes.ac.id/nju/index.php/ijal/article/view/25430

[5] Evers, W. J. G., Brouwers, A., & Tomic, W. (2006). A quasi-experimental study on management coaching effectiveness. *Consulting Psychology Journal*, *58*(3), 174–182. https://doi.org/10.1037/1065-9293.58.3.174

[6] Fatahillah, A., Puspitasari, I. D., & Hussen, S. (2020). The Development of Schoology Web-Based Learning Media with GeoGebra to Improve the ICT Literacy on Quadratic Functions. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *5*(3), 304–316. https://doi.org/10.23917/jramathedu.v5i3.10692

[7] Gafoor, K., & Kurukkan, A. (2015). Why High School Students Feel Mathematics Difficult? An Exploration of Affective Beliefs. *UGC Sponsored National Seminar on Pedagogy of Teacher Education Trends and Challenges*, *August*, 1–6. bit.ly/37OLqE7

[8] Grove, N. P., & Bretz, S. L. (2012). A continuum of learning: from rote memorization to meaningful learning in organic chemistry. *Royal Society of Chemistry*, *13*, 201–208. https://doi.org/10.1039/c1rp90069b

[9] Kita, Y., Yasuda, S., & Gherghel, C. (2022). Online education and the mental health of faculty during the COVID-19 pandemic in Japan. *Scientific Reports*, *12*(1), 1–9. https://doi.org/10.1038/s41598-022-12841-x

[10] Koehler, M. J., & Mishra, P. (2009). What is Technological Pedagogical Content Knowledge? *Contemporary Issues in Technology and Teacher Education*, *9*(1), 60–70. https://doi.org/10.1177/002205741319300303

[11] Mishra, P., & Koehler, M. J. (2006). Technological Pedagogical Content Knowledge: A Framework for Teacher Knowledge. *Teachers College Record*, *108*(6), 1017–1054. https://doi.org/10.1111/j.1467-9620.2006.00684.x

[12] Nisyak, R., Trapsilasiwi, D., & Fatahillah, A. (2018). Pengembangan Media Pembelajaran Interaktif Onine Menggunakan Schoology Berbantuan Web Desmos Materi Grafik Fungsi Kuadrat. *Kadikma: Jurnal Matematika Dan Pendidikan Matematika*, *9*(2), 155–164.

[13] Ogochukwu, N. V. (2010). Enhancing students interest in mathematics via multimedia presentation. *African Jurnal of Mathematics and Computer Science Research*, *3*(7), 107–113. http://www.academicjournals.org/ajmcsr/PDF/pdf2010/Jul/Ogochukwu.pdf

[14] Praja, E. S., Setiyani, S., Kurniasih, L., & Ferdiansyah, F. (2021). Analisis Kemampuan Pemahaman Matematis Siswa Smk Kelas Xi Pada Materi Vektor Selama Pandemi Covid-19. *Teorema: Teori Dan Riset Matematika*, *6*(1), 12. https://doi.org/10.25157/teorema.v6i1.4539

[15] Raja, R., & Nagasubramani, P. C. (2018). Impact of modern technology in education. *Journal of Applied and Advanced Research*, *3*, S33–S35. https://doi.org/10.21839/jaar.2018.v3is1.165

[16] Rogers, J., & Révész, A. (2019). Experimental and quasi-experimental designs. *The Routledge Handbook of Research Methods in Applied Linguistics*, 133–143. https://doi.org/10.4324/9780367824471-12

[17] Sari, T., & Nayır, F. (2020). Challenges in distance education during the (Covid-19) pandemic period. *Qualitative Research in Education*, *9*(3), 328–360. https://doi.org/10.17583/qre.2020.5872

[18] Sidiq, Y., Ishartono, N., Desstya, A., Prayitno, H. J., Anif, S., & Hidayat, M. L. (2021). Improving elementary school students' critical thinking skill in science through hots-based science questions: A quasi-experimental study. *Jurnal Pendidikan IPA Indonesia*, *10*(3), 378–386. https://doi.org/10.15294/JPII.V10I3.30891

[19] Suprianto, A., Ahmadi, F., & Suminar, T. (2019). The Development of Mathematics Mobile Learning Media to Improve Students' Autonomous and Learning Outcomes. *Journal of Primary Education*, *8*(1), 84–91. https://journal.unnes.ac.id/sju/index.php/jpe/article/view/19641

[20] TLS, D. S., & Herman, T. (2020). An Analysis of Pre-Service Mathematics Teachers' Desmos Activities for Linear Programming Lesson. *International Journal of Pedagogical Development and Lifelong Learning*, *1*(1), ep2002. https://doi.org/10.30935/ijpdll/8312

[21] Yusuf, I., & Widyaningsih, S. W. (2022). Higher Order Thinking Skills Oriented Student Worksheet of E-learning Model in Electric Circuit Topic. *TEM Journal*, *11*(2), 564–573. https://doi.org/10.18421/TEM112-10

THE APPLICATION OF THE COMPUTATIONAL SCIENCES ON STEM EDUCATION

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ABSTRACT

Considering the development levels and progress levels of the countries and their capacity to break through in science and technology and to make inventions, a direct proportion is observed between these two variances. The needs of the countries and the quality of the labor force to meet the needs in the age of industry 4.0 in which we live currently and the upcoming age of 5.0 show major changes day by day (Akgündüz vd, 2015; National Research Council (NRC), 2012; Sanders, 2008; Turkish Businessman Association (TÜSİAD), 2017; Yıldırım, 2018). The generations to produce, develop and use information will all and only guide economy and the competition in the world as a technology literates (NRC, 2002). The developed countries have started to apply education programs in compliance with the new age for the new generations to form their own future (Akgündüz, 2018). The low level of success in the disciplines of life sciences and mathematics and the decrease in the demand to the areas of profession concerning those fields caused the experts to search for a new educational approach. STEM (Science, Technology, Engineering and Mathematics) Approach has a significant place and impact among the education reforms to be made in our country (MEB YEĞİTEK, 2018).

A subject of the math class was lectured to students in a vocational high school, which is a resource of labor to industry, through STEM activities in this research. The impact of STEM activities on the academical success of the students, their motivations for mathematics learning, their interest in their area of profession and their act towards STEM were examined.

While a statistically significant difference was observed to be provided by STEM activities in the academic success of the students, a statistically significant differences was not observed in their interest in the areas of profession, their motivation for mathematics and their attitude towards STEM.

Key Words: FeTeMM, Geometry Education**,** Mathematics Education, STEM, STEM Education

REFERENCES

[1] Akgündüz, D., Aydeniz, M., Çakmakçı, G., Çavaş, B., Çorlu, M., Öner, T., & Özdemir, S. (2015). STEM eğitimi Türkiye raporu: "Günümüz modası mı yoksa gereksinim mi?" *İstanbul: İstanbul Aydın Üniversitesi STEM Merkezi*.

[2] Akgündüz, D. (Ed.) (2018). STEM eğitiminin kuramsal çerçevesi ve tarihsel gelişimi. Okul öncesinden üniversiteye kuram ve uygulamada STEM eğitimi içinde (ss. 19-49). Ankara: Anı Yayıncılık.

[3] Milli Eğitim Bakanlığı Yenilik ve Eğitim Teknolojileri Genel Müdürlüğü (MEB YEĞİTEK) (2018). STEM Eğitimi Öğretmen El Kitabı. *Ankara: Milli Eğitim Bakanlığı.*

[4] National Research Council [NRC]. (2012). A framework for k-12 science education: practices, crosscutting concepts, and core ideas. *Washington DC: National Academies Press.*

[5] Sanders, M. E. (2008). Stem, stem education, stemmania. *The Technology Teacher*, 68(4). 20-26.

[6] Türkiye Sanayiciler ve İş Adamları Derneği (TÜSİAD) (2017). 2023'e Doğru Türkiye'de STEM Gereksinimi.

[7] Yıldırım, Bekir. (2018). Teoriden Pratiğe STEM Eğitimi Uygulama Kitabı. Ankara: Nobel.

THE EFFECT OF SCHEMATIC REPRESENTATION FORMED PROBLEM SOLVING ACTIVITIES CONDUCTED WITH PRIMARY SCHOOL SECOND GRADE STUDENTS ON THEIR PROBLEM SOLVING SKILLS

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ABSTRACT

In this research, the types of representations used by second-grade primary school children in solving different types of problems, the accuracy rate of schematic representations within this type of representation, and the effect of schematic mathematical problem solving activities carried out with the students on the level of success were examined. The research was conducted with 20 students attending the 2nd grade of primary school. An achievement test prepared by the researchers, containing eight different problem types and validated by expert opinions, was used as a data collection tool. A total of 167 representations used by students in problem solving were examined. Descriptive analysis and content analysis were used in data analysis, the operational representation, pictorial representation, schematic representations and mental solutions used by the students in the problem solving process were examined, and the types of representations were evaluated in terms of the correct solution to the problem. Then, for a total of 8 class hours over 2 weeks, students were given problem-solving activities using only schematic representation forms. After the activities, a test parallel to the first test was administered to the students again. As a result of the research data, it was seen that the students used 39 operational representations, 85 pictorial representations, 21 schematic representations and made 22 mental solutions in the first problem solving process. It was determined that the accuracy rates of problem solutions were 18% for students using operational representation, 31% for students using pictorial representation, 67% for students using schematic representation and 41% for students using mental solutions. In the problem solving process applied as a post-test after the activities, it was observed that they used 20 operational representations, 51 pictorial representations, 83 schematic representations and made 13 mental solutions. It was determined that the accuracy rates of problem solutions were 15% for students using operational representation, 37% for students using pictorial representation, 73% for students using schematic representation and 44% for students using mental solutions. As a result, it was observed that the students' correct solution rate increased significantly. As a suggestion to teachers, it can be suggested to use schematic representation at an early age when solving mathematical problems and to include activities related to this.

Key Words: 2nd Grade Problem Solving, Schematic Representation Form, Early Childhood Mathematics Education

REFERENCES

[1] Çiğdem Kılıç (2009), İlköğretim Beşinci Sınıf Öğrencilerinin Matematiksel Problemlerin Çözümünde Kullandıkları Temsiller, Doktora Tezi, Anadolu Üniversitesi Eğitim Bilimleri Enstitüsü, Proje No: 070516

[2] Bal, A. P., "The Examination of Representations Used by Classroom Teacher Candidates in Solving Mathematical Problems", Educational Sciences: Theory and Practice, 14, 6, 2014, s. 2349-2365.

[3] Özdemir, Ş., İlköğretim Matematik Öğretmeni Adaylarının Çoklu Temsiller Kullanarak Problem Çözme Algılarının Açınsanması, Yayımlanmamış Yüksek Lisans Tezi, İstanbul Üniversitesi, Fen Bilimleri Enstitüsü, İstanbul, 2012.

[4] Ballard, J. W. (2000). Students' use of multiple representations in mathematical problem solving. Yayınlanmamış doktora tezi. Montana State University.

The Evaluation of Turkish Century Education Model Within Digital Literacy Framework

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ABSTRACT

This research study aims to evaluate the 2024 Turkey Century Education Model Elementary School Mathematics Curriculum within the framework of digital literacy. With this purpose, a qualitative research method was adopted to analyze the new curriculum in depth. Document analysis method was used, which allows phenomenon to be investigated by examining documents. The document examined in this research, Turkey Century Education Model Elementary School Mathematics Curriculum (5th, 6th, 7th and 8th grades), was obtained from the official website of the Ministry of Education. The document was coded under the categories of terms related to digital literacy, the types of digital and technological tools, and the purposes of using these digital tools. The obtained codes are divided into main categories using the content analysis method. When the Turkish Century Education Model Elementary School Mathematics Curriculum is examined, the most commonly used concepts in the recently introduced curriculum regarding digital literacy are mathematics software, technology, virtual manipulatives, statistical software, digital tools, digital environment, online applications, platforms (*etc*. EBA, Turkish Statistical Institute, OECD or World Health Organization) and social media platform. The digital and technological tools recommended to be used in the curriculum would be listed as mathematical software, virtual manipulatives, statistical software, online applications, digital presentation tools, online calculators, digital games, and electronic spreadsheets. In the Turkish Century Education Model Elementary School Mathematics Curriculum, frequently mentioned purposes of using digital tools are drawing geometric shapes, making presentations, using them for performance tasks, collecting data, organizing and analyzing data, measuring, demonstrating with

different representations, and creating simulations.

Key Words: Digital literacy, maarif model, technology, mathematics curriculum

REFERENCES

[1] B. Akpınar and B. Köksalan, The need for education and curriculum renewal: A theoretical analysis through the Turkish Century Education Model, Journal of History School, 17(68), (2024), 27-48.

[2] Y. Alpaydın, Milli Eğitim Bakanlığı'nın müfredat değişimi ve Türkiye yüzyılı maarif modeli, SETA Perspektif, 397, (2024), 1-4.

[3] E. Banaz, 2024 Türkiye Century Education Model examination of secondary school Turkish curriculum in terms of digital literacy, Social Sciences: Theory & Practice, 8(1), (2024), 279-290.

[4] Ş. Büyüköztürk, E. Kılıç Çakmak, Ö. E. Akgün, Ş. Karadeniz and F. Demirel, Scientific research methods in education (35th ed.), Pegem Akademi Publishing, Ankara, 2024.

[5] J. R. Fraenkel, N. E. Wallen and H. H. Hyun, How to design and evaluate research in education (8th ed.), Mc Graw Hill, 2012.

[6] Milli Eğitim Bakanlığı (MEB), Ortaokul matematik dersi öğretim programı Türkiye yüzyılı maarif modeli, Ankara, 2024.

[7] P. Üredi, To guide teacher training programs: The implementation levels of holistic assessment and evaluation practices in curricula, Journal of Education and New Approaches, 7(1), (2024), 10-28.

[8] A. Yıldırım and H. Şimşek, Qualitative research methods in the social sciences (12th ed.), Seçkin Publishing, Ankara, 2021.

THE IMPORTANCE OF THE PHİLOSOPHİCAL JUSTİFİCATİON OF MATHEMATİCAL EQUALİTY FOR MATHEMATİCS EDUCATİON: A STUDY ON THE PHİLOSOPHY OF MATHEMATİCS EDUCATİON

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ABSTRACT

Mathematical Equality has sometimes met of with equivalence and sometimes with biconditional requirement within logical relations. However, the concepts of equality, identity and equivalence have been interpreted differently, especially in the 19th century. The necessity of an identity underlying every equality is an idea that developed with modern mathematical philosophy. This necessity also explains the reason for equality. At the same time, this idea supports meaningful learning in mathematics education.

Gottlob Frege (1848-1925), a philosopher of mathematics, pioneered the problem of the foundations of arithmetic, which was one of the most heated debates of his time. With this aim, he created six basic laws (axioms) to form the basis of arithmetic. The fifth of these laws, just like Euclides' fifth law, is controversial as to whether it is a fundamental law. The Basic Law V explains the justification that makes two expressions with different spellings equal to each other. For example, there must be a justification that makes an equality such as $x^2 - 4 = (x - 2)(x + 2)$ possible. According to this law, this justification is possible with an identity that must be established between the "value range" (Wertverläufe), which is a definition belonging to Frege, of each expression. In other words, according to Frege, what makes every equality possible is the existence of an identity underlying them.

The Basic Law V with its current interpretation is as follows.

$$(\varepsilon' f(\varepsilon) = \alpha' g(\alpha)) \leftrightarrow \forall a\, (f(a) = g(a))$$

The letters ε´ and α´ expressed with the Greek softening symbols here indicate the value range of functions. The value range is also the extension of a concept for

Frege.In this respect, the equality of two functions such as $f(a)$ and $g(a)$ with different spellings is possible with the identity of their extensions. This extension is represented in class terminology as the class of ordered pairs (x, y) such that F(x) = y. Then the value range of $f(a)$ and $g(a)$ are the identities of all (ε, ξ) and all (α, γ) pairs for $f(\varepsilon) = \xi$ and $g(\alpha) = \gamma$. This situation will lead to the identity of all ordered pairs provided by these functions, all points on the graph of each function. In other words, there must be an identity under each equality. Such an idea leads to the idea that the identity underlying each equality should be sought in mathematics education, which discusses the whatness of equalities. For example, let's consider the equality $\binom{n}{r} = \binom{n}{n-r}$. Of course, the reason for this equality can be shown operationally with various operations. However, with a philosophical approach, the identity between the extension of each concept can be sought. The extension of the concept $\binom{n}{r}$ is on its whatness. This gives the answer to how many ways r elements can be selected from n elements. However, in this case, when r elements are selected, there are $(n - r)$ elements that are not selected. In that case, selecting r elements at the same time or not selecting $(n - r)$ elements is the same event and should be calculated in the same way. In this case, the extensions of the concepts on both sides of the equation express one and the same thing. It can be thought that searching for the identity under mathematical equations with philosophical approaches will contribute to making mathematics education meaningful.

Key Words: Equality, identity, equivalence, Gottlob Frege,

REFERENCES

[1] A. Asghari, Equivalence: An Attempt at a History of the Idea, Synthese, 196(11), 2019, s.4657-4677.

[2] R. T. Cook, How to read Grundgesetze, The Basic Laws of Arithmetic, New York: Oxford University Press: 2016, pp.A1-42.

[3] P. Ebert, and M. Rossberg, Presentation: The Basic Laws of Arithmetic, New York: Oxford University Press, 2016, pp. xiii- xxxix.

[4] G. Frege, Function and Concept, Collected Papers On Mathematics, Logic, And Philosophy, Ed. Brain Mc Guinness, New York, Basic Blackwell Publisher, Oxford, 1984, pp. 137-156.

[5] G. Frege, The Basic Laws of Arithmetic, Trans. Montgomery Furth, Berkeley, Los Angeles, London: University of California Press, 1964.

Trends in Studies on Out-of-School Learning Environments in Mathematics Education: A Systematic Review

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ABSTRACT

This study aims to examine the studies on out-of-school learning environments in mathematics education through a systematic review with a holistic perspective and to determine their trends. It aims to reveal the similar and different aspects of the studies in this field by comparing the general characteristics and methods of the studies on out-of-school learning environments in mathematics education in national and international literature. In this direction, the Preferred Reporting Items for the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) (PRISMA) 2020 criteria were considered. In line with the inclusion and exclusion criteria determined within the scope of the research, 36 studies were included in the scope of the research. The articles were examined within the framework of the article review template and the research data were obtained. The document review method was used for the analysis of the studies. Within the framework of the analyses performed, it was determined that although more studies were reached nationally, the first studies on out-of-school learning environments in mathematics education were carried out internationally. When the distribution of studies by year was focused on, it was seen that there were studies in the international field before 2000, but it was revealed that the increase in this field occurred significantly after 2020. It was concluded that the vast majority of studies at the national level were conducted in 2018 and later. In the studies examined, it was determined that qualitative research was generally preferred, and quantitative and mixed-type studies were not sufficient. When the study group was focused on, it was revealed that secondary school level studies were preferred more both nationally and internationally. Within the framework of the findings, it was seen that studies were generally put forward to determine the situation regarding out-of-school learning environments in mathematics education and that there was a greater need for studies on how to design out-of-school learning environments in mathematics education and what kind of activities would make learning more effective in mathematics education.

Key Words: Out-of-school learning environments, mathematics education, systematic review

REFERENCES

[1] Alisinanoğlu, F., Kesicioğlu, O. S., & Mart, M. (2013). Evaluation of pre-school children's development of geometric thought in the UK and Turkey according to van Hiele model. International Journal of Education and Research, 1(10), 1-10.

[2] (*) Andreev, C. (2021). On thenature of mathematics and mathematics in nature-A tale of symmetry with a sequel for young and old. Mathematics and Informatics, 64(4), 425-437.

[3] (*) Andrews, J., &Yee, W. C. (2006). Children's 'funds of knowledge'and their real life activities: Two minority ethnic children learning in out-of-school contexts in the UK. Educationalreview, 58(4), 435-449.

[4] (*) Ata Doğan, S., & Akman, B. (2023). Okul öncesi öğretmenlerinin matematik etkinliklerinde iç ve dış mekân kullanımına ilişkin görüşleri ve uygulamaları. Journal of NationalEducation, 52(238).

[5] (*) Aydoğdu, M. Z., & Aydoğdu, A. S. (2023).Okul dışı öğrenme ortamlarında geometri öğretimi. Retrieved from https://www.researchgate.net/profile/Mustafa-Aydogdu-5/publication/377117460_OKUL_DISI_OGRENME_ORTAMLARINDA_GEOMETRI_OGRETIMI_GEOMETRY_TEACHING_IN_OUT_OF_SCHOOL_LEARNING_ENVIRONMENTS/links/65968a370bb2c7472b320eee/OKUL-DISI-OeGRENME-ORTAMLARINDA-GEOMETRI-OeGRETIMI-GEOMETRY-TEACHING-IN-OUT-OF-SCHOOL-LEARNING-ENVIRONMENTS.pdf

[6] (*) Aydoğdu, A. S. E., Aydoğdu, M., & Aktaş, V. (2022). Matematik dersinde bir eğitim aracı olarak sanal müze kullanımı. International Journal of Social Science Research, 11(1), 51-70.

[7] (*) Aydoğdu, A. S. E., Aydoğdu, M. Z., & Aktaş, V. (2023). Okul dışı öğrenme ortamlarıyla ilgili matematik öğretmenlerinin görüşleri. Dokuz Eylül Üniversitesi Buca Eğitim Fakültesi Dergisi, (55), 60-78.

[8] (*) Aydoğdu, M. Z., & Aydoğdu, A. S. (2024). Matematik öğretmen adaylarının okul dışı öğrenme ortamlarına ilişkin görüşleri. Cumhuriyet International Journal of Education, 13(1): 1-15.

COMPUTER SCIENCE

A Blockchain-Based Approach to Enhancing Transparency and Accountability in Cyber Security Incident Response and Digital Forensics

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ABSTRACT

Preserving the traces of cyberattacks in incident response and maintaining a chain of custody in digital forensics processes is crucial to determine the root-cause of the attacks and to hold the perpetrators accountable in justice. Traditional incident response processes often lack proactive evidence collection, hindering forensic investigations. The secure storage of evidence and chain-of-custody records is critical for legal proceedings but susceptible to tampering or destruction. In this study, we propose a re-modelling of current enterprise cyber incident response systems on blockchain called SecOpsChain. The proposed model records all evidence and actions related to cyber incidents in a transparent and immutable manner. The model increases the forensic readiness of organizations by allowing all stakeholders to access complete and accurate information about incidents. By leveraging the tamper-proof and transparent nature of blockchain, the model aims to enhance forensic readiness within organizations. The SecOpsChain model, developed on the Hyperledger Fabric blockchain platform, integrates with Security Operations Centers (SOCs) to record critical security alerts and incident details. This immutable record ensures the integrity of evidence for potential future investigations. The model offers enhanced forensic readiness, improved collaboration among stakeholders, and increased accountability. Future research will focus on real-world implementation and performance evaluation.

Key Words: cyber security incident response, forensics, blockchain technologies, chain of custody.

REFERENCES

[1] Moreno, J., Serrano, M. A., & Fernandez, E. B. (2019). Improving Incident Response in Big Data Ecosystems by Using Blockchain Technologies. Applied Sciences, 10(2), 724. https://doi.org/10.3390/app10020724

[2] Valjarevic, A., Venter, H. (2013). A Harmonized Process Model for Digital Forensic Investigation Readiness. In: Peterson, G., Shenoi, Advances in Digital Forensics IX. DigitalForensics 2013. IFIP Advances in Information and Communication Technology, vol 410. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-41148-9_5

[3] Jaquet-Chiffelle, D., Casey, E. (2020), Bourquenoud, J., Tamperproof Timestamped Provenance Ledger Using Blockchain Technology, FSI Digital Investigation 33, https://doi.org/10.1016/j.fsidi.2020.300977

[4] Lone, A. H., & Mir, R. N. (2018). Forensic-chain: Ethereum blockchain based digital forensics chain of custody. Sci. Pract. Cyber Secur. J, 1, 21–27.

[5] Li, S., Qin, T., Min, G. (2019), Blockchain-based digital forensics investigation framework in the internet of things and social systems. IEEE Trans. Comput. Soc. Syst., 1433–1441.

[6] Moreno J., Serrano M.A., Fernandez E.B., Fernández-Medina E. (2020), Improving Incident Response in Big Data Ecosystems by Using Blockchain Technologie, Applied Sciences,. https://doi.org/10.3390/app10020724

[7] Gürfidan, R., & Tatli, M. (2023), Performance Comparison Of Secure Storage Methods For Digital Forensic Evidence. Uluslararası Sürdürülebilir Mühendislik ve Teknoloji Dergisi, 7(2), 131-138

[8] Schneier, B., Kelsey., J. (1999), Secure audit logs to support computer forensics. ACM Trans. Inf. Syst. Secur. 2, 159–176. https://doi.org/10.1145/317087.317089

Anomaly Detection in Computer Networks with Machine Learning Methods

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ABSTRACT

Today, computer networks connect countless devices and enable the exchange of large amounts of data. However, this large data flow is exposed to cyber threats such as unauthorised access and data breaches. The most important issue to take precautions against these threats is anomaly detection. Many methods are used to detect anomalies in computer networks. Machine learning classification methods are frequently used by researchers in the literature for anomaly detection [1], [2].

In this study, Space Vector Machine (SVM) and Naive-Bayes, which are widely used machine learning classification methods for anomaly detection, are used. SVM is a machine learning classification technique that is based on mathematical topics such as calculus, vector geometry and constrained optimisation and allows to perform linear and non-linear classification problems. The main purpose of classification with SVM is to provide the most appropriate classification by separating the hyperplanes well in the high-dimensional attributes space, as well as classifying in a two-dimensional space. Naive Bayes is a classification algorithm in the supervised learning category and is based on a probabilistic approach. Naive Bayes calculates the conditional probabilities of the features under observation for each class and predicts to which class a new observation may belong by combining these probabilities through Bayes theorem. In this study, an open source dataset presented in the literature was also used for anomaly detection [3]. SVM and Naive Bayes methods were used to classify anomaly in this dataset. According to the findings obtained as a result of the experimental study, a high prediction accuracy rate was achieved.

Key Words: Machine Learning, anomaly detection, space vector machine, naive bayes.

REFERENCES

[1] F. Onur, S. Gönen, M. A. Barışkan, C. Kubat, M. Tunay, E. N. Yılmaz, Machine learning-based identification of cybersecurity threats affecting autonomous vehicle systems, Computers & Industrial Engineering, (2024), 190, 110088.

[2] T. Shon, J. Moon, A hybrid machine learning approach to network anomaly detection, Information Sciences, (2007), 177(18), 3799-3821.

[3] B. S. Sharmila, R. Nagapadma, Quantized autoencoder (QAE) intrusion detection system for anomaly detection in resource-constrained IoT devices using RT-IoT2022 dataset, Cybersecurity, (2023), 6(1), 41.

Biometric Data Systems and Cybersecurity

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ABSTRACT

Biometric data can be processed in accordance with the Personal Data Protection Law, considering personal data, privacy and fundamental rights and freedoms of individuals. With Industry 4.0, very large and critical systems work for the benefit of humanity. However, all these systems must operate without interruption due to their importance. At this point, unique patterns, remain unchanged, individual characteristics and classifiable biometric datas and the cyber security of these systems can be an inseparable partner. Our study focus on the use of biometric data for logging into a critical system. In this context, verification, system entry stages and cyber security of these stages with biometric data such as fingerprint, face, iris, retina are discussed. First of all, the usability of biometric data in the 2fa (two factor authentication) security step will provide great benefits to all system users. Because biometric data cannot be forgotten or captured by someone else like classic passwords. For this reason, it is a perfect step for the verification process. Is it possible to log into the system safely with the verification process? When the answer to this question is investigated, it is seen that the cyber security of verification systems may be under threat at certain points. When these points are examined, it is predicted that 7 points could be possible cyber vulnerability points. The first of the vulnerability points is the user login step. The second point is the biometric data acquisition step, the third point is the stage of sending the received data to the feature extraction server, the fourth point is the feature extraction server operation stage, the fifth point is the recording of the extracted biometric data to the database, the sixth point is the query stage made in the database, and the seventh and last point is the stage of returning the query result to the user. The points are possible cyber vulnerability points. Future studies will focus on cyber security software packages of biometric verification systems and the production of biometric data and fake data.

Key Words: biometric data, verification, cyber security, digital forensics

REFERENCES

[1] Ratha N.K., Connell J.H. & Bolle R.M. (2001). An Analysis of minutiae matching strength, International Conference on Audio and Video-Based Biometric Person Authentication (AVBPA), 223-228.

[2] Jain A.K., Ross A. & Prabhakar S. (2004). An introduction to biometric recognition, IEEE Transactions on Circuits and Systems for Video Technology, 14(1), 4-20.

[3] H. M. Alzoubi et al., "Cyber Security Threats on Digital Banking," 2022 1st International Conference on AI in Cybersecurity (ICAIC), Victoria, TX, USA, 2022, pp. 1-4, doi: 10.1109/ICAIC53980.2022.9896966. keywords: {Computer hacking;Authentication;Banking;Mobile applications;Encryption;Computer crime;Artificial intelligence;Cyber Security;Threats;Digital Banking},

[4] Taşçı H.B., Gönen S., Barışkan M.A. & Yılmaz E.N. (2021). Password Attack Analysis Over Honeypot Using Machine Learning Password Attack Analysis, Turkish Journal of Mathematics and Computer Science, 13(2), 388-402.Li, S., Qin, T., Min, G. (2019), Blockchain-based digital forensics investigation framework in the internet of things and social systems. IEEE Trans. Comput. Soc. Syst., 1433–1441.

[5] Alaswad A.O., Montaser A.H. & Mohamad F.E. (2014). Vulnerabilities of Biometric Authentication "Threats and Countermeasures", International Journal of Information & Computation Technology, ISSN 0974-2239 Volume 4, Number 10 (2014), 947-958.

[6] Dmitrienko A.,Liebchen C.,Rossow C. &Sadeghi A.R. (2014). On the (In)Security of Mobile Two-Factor Authentication, International Financial Cryptography Association, DOI: 10.1007/978-3-662-45472-5 24, 365-383.

Creating a Model for Improving the Effectiveness of Foreign Language Education with Artificial Intelligence

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ABSTRACT

Education has been an important element of civilizations with a great impact on individuals and societies. Improving existing educational activities, materials and systems has a significant impact on social development. Nowadays, artificial intelligence has started to be used extensively in educational activities as in every field of technology. Language learning is one of the fields affected by these developments. With personalized language learning applications with artificial intelligence, content can be produced according to the level, learning speed and preferences of the individual. With an AI-based language learning application, pronunciation errors can be detected with instant feedback and the development of the individual can be supported by making corrections. Artificial intelligence can produce personalized language learning materials, i.e. personalized tests, and study texts. Thus, the diversity and effectiveness of language learning resources can be increased.

With the personalized learning method, methods and materials are designed for each individual. In foreign language teaching, the development of content and methods in accordance with the interests of individuals increases the effectiveness and success of the learning process. For this purpose, computer-based training models are developed using machine learning algorithms and machine learning methods are used to improve student performance. Personalized learning process: virtual reality, game-based learning using augmented reality, communities of practice, adaptive technologies, learning analytics and e-assessment techniques are used. One of the important processes of foreign language education is assessment and evaluation. The assessment and evaluation process provides continuous feedback to individuals about how they are learning, the support they need and the progress they are making

towards their learning goals.

In this study, a model was designed for determining the interests of the individual (technology, sports, health, etc.) and developing learning materials suitable for these interests, determining topics for listening activities and assessment and evaluation processes with machine learning methods. For this purpose, algorithms using artificial neural networks, support vector machines, k-nearest neighbors and decision trees were developed, and their performances were compared. With the most successful model, it will be possible to develop applications that offer content and measurement questions suitable for the interests of individuals and evaluate the level of achievement.

Key Words: Artificial Intelligence, foreign language education, assessment and evaluation, individual language education

REFERENCES

[1] R.R. Divekar, J. Drozdal, S. Chabot, Y. Zhou, H. Su, Y. Chen, J. Braasch. Foreign language acquisition via artificial intelligence and extended reality: design and evaluation. *Computer Assisted Language Learning*, (2021), *35*(9), 2332–2360. https://doi.org/10.1080/09588221.2021.1879162

[2] O. Yunina, Artificial Intelligence Tools in Foreign Language Teaching in Higher Education Institutions. *The Modern Higher Education Review*, (2023), (8)77–90. https://doi.org/10.28925/2617-5266.2023.85

[3] T. Liang, N. Duarte, and G. Yue. An Evolutionary Study of the Impact of Artificial Intelligence Technology on Foreign Language Education. *International Journal of Emerging Technologies in Learning (iJET), (2024), 18*(19), 190-204. Kassel, Germany: International Journal of Emerging Technology in Learning. https://www.learntechlib.org/p/223763/

Evaluating the Video Communication Platforms with Fuzzy Linguistic based Mathematical Analysis

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ABSTRACT

With their versatility and convenience, video calling and conferencing applications have increased significantly in recent years and have become an integral part of modern life. These platforms have evolved into indispensable tools for various purposes, from personal interactions with distant friends and family to professional conditions, like collaborative work sessions and business negotiations.

The COVID-19 pandemic, which took the world by storm in late 2019, introduced unprecedented disruption to daily life, compelling governments to enforce strict measures such as lockdowns and curfews to curb the spread of the virus. As a result, the remote working culture, previously a minority practice in a few industries, quickly achieved general acceptability across various sectors. Companies and organizations were forced to adapt to this new reality, seeking digital solutions that would allow them to continue their operations while adhering to social distancing guidelines. In this context, video communication platforms emerged as a critical tool, enabling individuals and teams to maintain contact and collaborate effectively despite the physical barriers imposed by the pandemic. However, the surge in demand for these platforms also brought to light the varying levels of performance and reliability among different video communication tools. While some platforms excelled in delivering high-quality video and audio experiences, others struggled to keep up with the increasing user demands. The differences in features, user interfaces, and overall user experience led to the need for a comprehensive evaluation of these platforms to determine which ones indeed met the needs of their users.

This study seeks to undertake a detailed evaluation of various video communication platforms using a decision analysis tool. The methodology employed in this analysis

is based on fuzzy sets, a mathematical approach that is particularly efficient in uncertainty and imprecision results from subjective opinions and preferences. A panel of experts was consulted to conduct this evaluation, each providing ratings for the selected video communication platforms based on a linguistic measurement system. Initially collected verbally, these ratings were then processed using fuzzy sets to translate the subjective assessments into objective data. This transformation from qualitative to quantitative analysis enables a more structured and rigorous comparison of the platforms, identifying strengths and weaknesses across different criteria.
Conclusively, the results of this analysis will provide valuable insights into which platforms are best suited to meet the diverse needs of users, whether for personal, educational, or professional use. It demonstrates the practical application of fuzzy sets in decision analysis, offering a model that can be applied to other areas of technology assessment.

Key Words: Fuzzy sets, remote working, decision analysis.

REFERENCES

[1] B. Eröz-Tuğa and R. Sadler, Comparing six video chat tools: A critical evaluation by language teachers, Comput. Educ. 53(3) (2009), 787-798.

[2] C.-L. Hwang and K. Yoon, Multiple Attribute Decision Making: Methods and Applications, Springer-Verlag Berlin, 1981.

[3] L. A. Zadeh, Fuzzy sets, Inf. Control 8(3) (1965), 338-353.

[4] R. E. Bellman and L. A. Zadeh, Decision-making in a fuzzy environment, Manag. Sci. 17(4) (1970), B-141.

[5] S.-M. Chen and L.-W. Lee, Fuzzy multiple attributes group decision-making based on the interval type-2 TOPSIS method, Expert Syst. Appl. 37(4) (2010), 2790-2798.

[6] B. Meniz, An advanced TOPSIS method with new fuzzy metric based on interval type-2 fuzzy sets, Expert Syst. Appl. 186 (2021), 115770.

Software Approaches To Mathematics

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ABSTRACT

Throughout history, mathematics and software technologies have played an important role in solving complex problems in science and engineering. Today, software is used as an effective tool for solving mathematical problems, performing fast calculations, big data analysis and data visualization. The historical development of mathematics covers a wide range of topics, from the first discoveries in Mesopotamia and Egypt to advances in Ancient Greece, the Islamic world of the Middle Ages, the Renaissance and the modern era. The historical development of software started with the invention of the first electronic computers, accelerated with the development of high-level programming languages, and has reached the present day with advances in areas such as the internet, mobile technologies, cloud computing and artificial intelligence. The combination of mathematics and software has led to significant advances in many fields, including algorithms and computational mathematics, cryptography and security, machine learning and data science, optimization and computational research, computer graphics and image processing. The software tools used in these areas enable mathematical concepts to be applied and used at large scales. Software such as Mathematica, Maple, Matlab, Python and R have become indispensable for mathematical operations and analysis thanks to their extensive libraries and strong community support. Excel and other spreadsheet software are also effective tools for data analysis and solving mathematical problems.

Key Words: Mathematics, software, algorithms, data science

REFERENCES

[1] Öztürk, B., Tutkun, Ö. F., & Demirtaş, Z. (2011, May). Matematik öğretiminde matematik yazılımları: Dinamik geometri ve bilgisayar cebiri sistemleri [Mathematical software in mathematics teaching: Dynamic geometry and computer algebra systems]. In 11th International Educational Technology Conference (p. 1756).

[2] Görgüt, R. Ç. (2024). Yapay zeka ve matematik eğitimi. Eğitim & Bilim 2023-IV, 43.

Smoothing Conjugate Gradient Method Associated with New Generation Techniques to Solve Image Restoration Problems

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ABSTRACT

Numerous applications in engineering, finance, medicine, and other scientific fields have led to an increasing interest in optimization problems. Many practical problems have been converted into popular optimization problems, including regularization, eigenvalue, min-max, and min-sum-min problems. The use of nonsmooth, nonconvex regularization offers significant benefits in the restoration of images. Therefore, problems that arise during image restoration are frequently transformed into large-scale optimization problems that are not smooth and are not convex. The majority of the currently available minimization approaches are not effective in handling situations of this nature. It is common knowledge that nonlinear conjugate gradient methods are the most popular choice for solving large-scale smooth optimization problems. This is because these methods are simple to use, need little storage space, are efficient in terms of practical computation, and have great convergence properties. This paper investigates norm regularization-based non-smooth image restoration problems. We propose a smoothing conjugate gradient method (SCGM) by combining the conjugate gradient method with two new generation smoothing techniques. The SCGM method is created as a numerical algorithm. Furthermore, numerical applications of the algorithm on test images with various types of noise are demonstrated, and the results are compared with similar algorithms. Experimental results show the efficacy of the proposed algorithm.

Key Words: Non-smooth optimization, image restoration, conjugate gradient method.

REFERENCES

[1] X. Chen, Smoothing methods for nonsmooth, nonconvex minimization, Math. Prog. Ser. B, 134(2012), 71-99.

[2] X. Chen and W. Zhou, Smoothing nonlinear conjugate gradient method for ımage restoration using nonsmooth nonconvex minimization, SIAM J. Imaging Sci. 3(2010), 765-790.

[3] A. Sahiner and N. Yilmaz, Double stranded smoothing technique in energy minimization problems, Numer. Algebra Control Optim. 14(2024), 80-88.

Using Artificial Neural Networks in Evaluating Biometric Data in Paleontology

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ABSTRACT

Since artificial neural networks have the ability to generalize information and data, they are frequently used in solving problems for different disciplines. Since artificial neural networks can continuously learn rapidly, they can solve problems at high levels of accuracy. In addition, artificial neural networks do not need to wait in their relationships to produce inputs and outputs so that the appropriate outputs can be presented to the network. Artificial neural networks can be learned linearly or nonlinearly between predefined inputs and outputs by using the parameters of the defined problem. In addition, artificial neural networks can also produce output for classified network structures whose output values cannot be seen. With these features, it offers effective solutions to quite complex problems. In order to demonstrate the contribution of this software of artificial neural networks in solving paleontological problems, the pattern completion processes of biometric parts that were not damaged or measured during fossilization, transportation or sample preparation in the laboratory were predicted.

Key Words: Artificial neural networks, paleontology, pattern completion.

The Power of Software in Big Data:Diabetes Analysis and Prediction

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ABSTRACT

This study examines the analysis and prediction of diabetes using big data analytics techniques. The focus of the research is to highlight the importance of software and algorithms for in-depth analysis of health data. In the healthcare sector, data analytics plays a critical role in preventing diseases, improving treatments and improving patient outcomes. Using the Pima Indian Diabetes dataset, we aimed to predict diabetes and identify risk factors. The data set includes 8 independent and 1 dependent variable for 768 individuals. Powerful big data processing tools such as Apache Hadoop and Apache Spark were used in the study. These tools are critical for efficient processing and analysis of data. After loading the dataset and performing pre-processing steps, machine learning models such as logistic regression, decision trees and support vector machines were created and trained. The performance of these models is evaluated using cross-validation methods and different algorithms are compared. Apache Hadoop enables large data sets to be stored and processed in parallel in a distributed environment. The HDFS component distributes the data, while the MapReduce algorithm processes it. Apache Spark accelerates data analytics, machine learning and graph processing with in-memory processing capabilities and extensive libraries. After loading the dataset and preprocessing steps, machine learning models such as logistic regression, decision trees and support vector machines were created and trained. The performance of these models was evaluated by cross-validation methods. The results highlight the importance of big data analytics and software tools in diabetes prevention and management.

Key Words: Big Data Analytics, Pima Indian Diabetes Dataset, Disease Prediction, Data Processing, Decision Trees, Data science

REFERENCES

[1] UCI Machine Learning Repository. (2023). Pima Indians Diabetes Dataset. Retrieved from https://archive.ics.uci.edu/ml/datasets/Pima+Indians+Diabetes.

[2] Brown, C. A., Lilford, R. J., & Johnson, R. (2019). The Use of Machine Learning in the Analysis of Diabetes Mellitus. Diabetes Research and Clinical Practice, 123, 150-158.

Voice Cloning for Turkish Speech: High-Fidelity Voice Synthesis Using Deep Learning Techniques

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ABSTRACT

Recent advancements in artificial intelligence technology have given rise to crucial changes in content creation processes. The popularization of human-like work by artificial intelligence has provided AI to mimic human voices, especially by being trained with voice data. Voice data is a particular ideal resource for training AI algorithms to analyze, learn, and reproduce human speech. The voice data used for this purpose is often collected through social media and other digital platforms.

Voice cloning, the process of replicating a specific individual's voice [1, 2, 3] using advanced machine learning (ML) and deep learning (DL) techniques [4, 5, 6]. This process has been revealed as a viable mechanism in different applications such as personalized voice assistants, content creation and assistive technologies for the visually impaired individuals. Main advantages of voice cloning are its ability to produce highly realistic and personalized synthetic voices, which can crucially enhance user experience and accessibility in digital interfaces and to provide the preservation of a person's voice for future use, which is especially valuable for individuals suffering from degenerative diseases that affect speech.

In this work, we present a comprehensive voice cloning approach [7] for Turkish speech by utilizing state-of-the-art natural language processing (NLP) and deep learning models. The proposed system integrates a robust sequence-to-sequence model for spectrogram generation and an advanced vocoder for high-fidelity audio synthesis. The pipeline begins with the preprocessing of textual input, converting it into phonemes by using text normalization, tokenization and language-specific phonemization techniques. The phoneme sequences are then fed into a Tacotron and FastSpeech based model to generate a mel-spectrogram, effectively capturing

the prosody and acoustic features of the target speech.

For the final waveform synthesis, we employ WaveRNN, UnivNet and finally HiFi-GAN model which is a generative adversarial network-based vocoder, known for its ability to produce high-quality and realistic audio waveforms. This vocoder translates the mel-spectrogram into a natural-sounding voice that closely mimics the original speaker's vocal characteristics.

We also conduct several visual evaluations to validate the effectiveness of the proposed system. Visual analyses of input-output (synthesized) waveforms, spectrograms and mel-spectrograms below indicate that the proposed approach is capable to accurately replicate the temporal structure, spectral content, and harmonic characteristics of the input speech. The related visual results also show a noticeable degree of consistency between the synthesized output and the original input across various metrics. Especially, we can observe that the output waveform preserves the amplitude and timing patterns of the original, while the spectrogram analysis demonstrates closely aligned frequency distributions by maintaining the essential formant structures critical for speech intelligibility and naturalness.

Input Speech Text for Experiment: "Bilgisayar, aritmetik veya mantıksal işlem dizilerini (berim) otomatik olarak yürütmek üzere programlanabilen dijital bir elektronik makinedir."

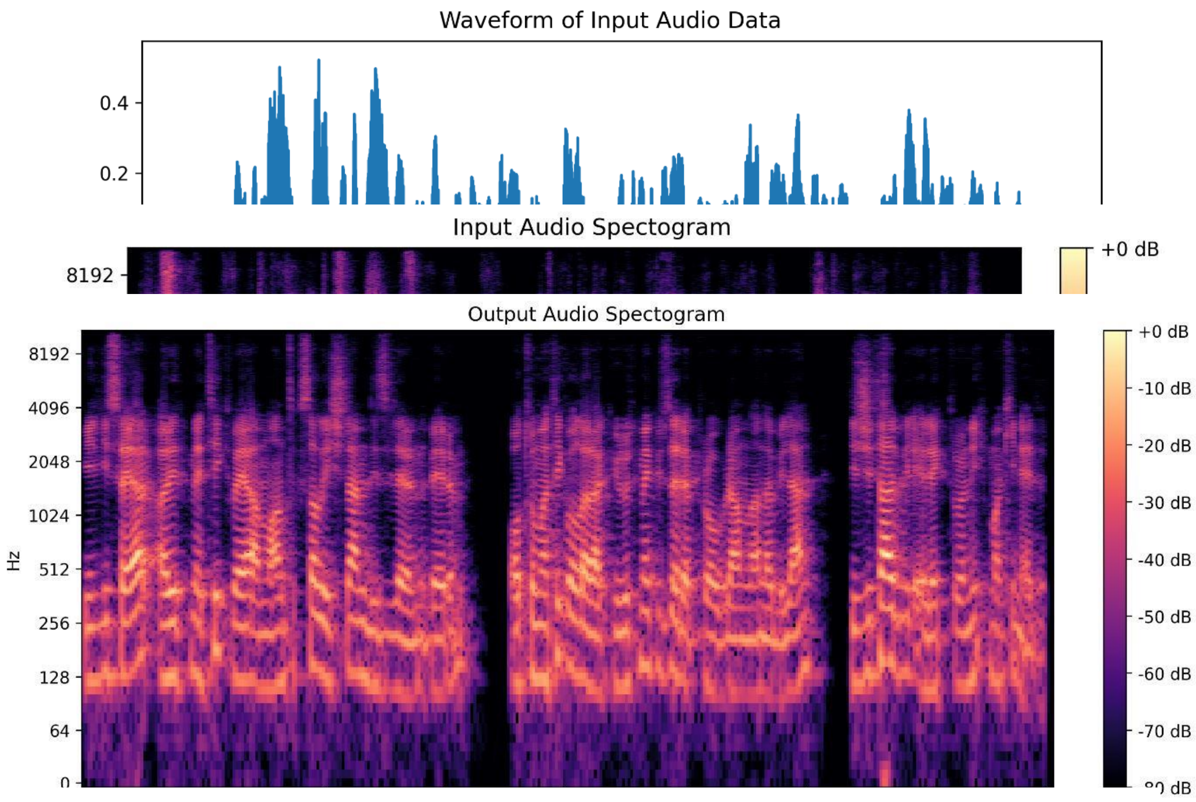

Figure 4: Output Audio Data Spectogram Visualization

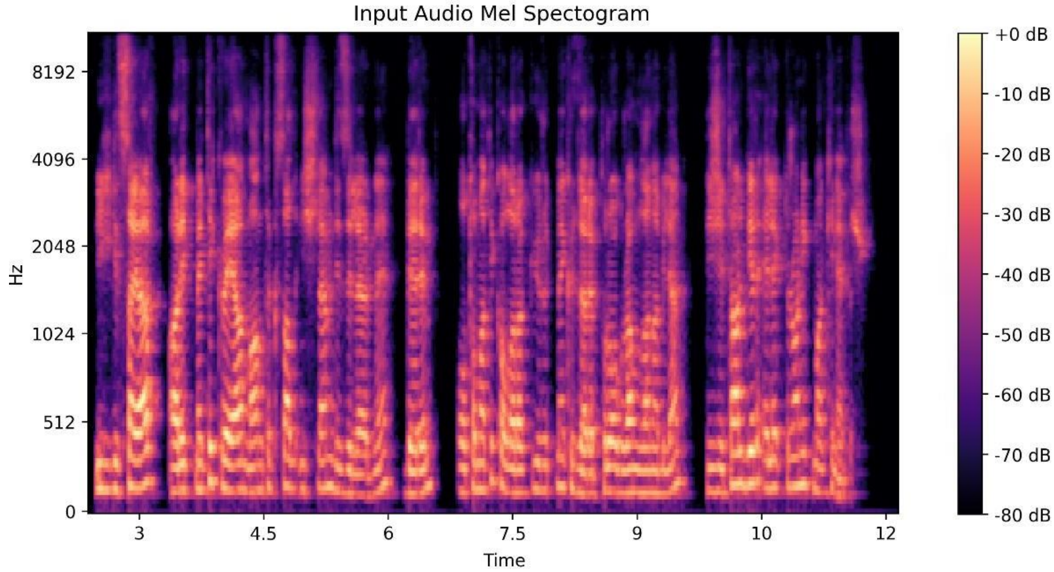

Figure 5: Input Audio Data Mel-Spectogram Visualization

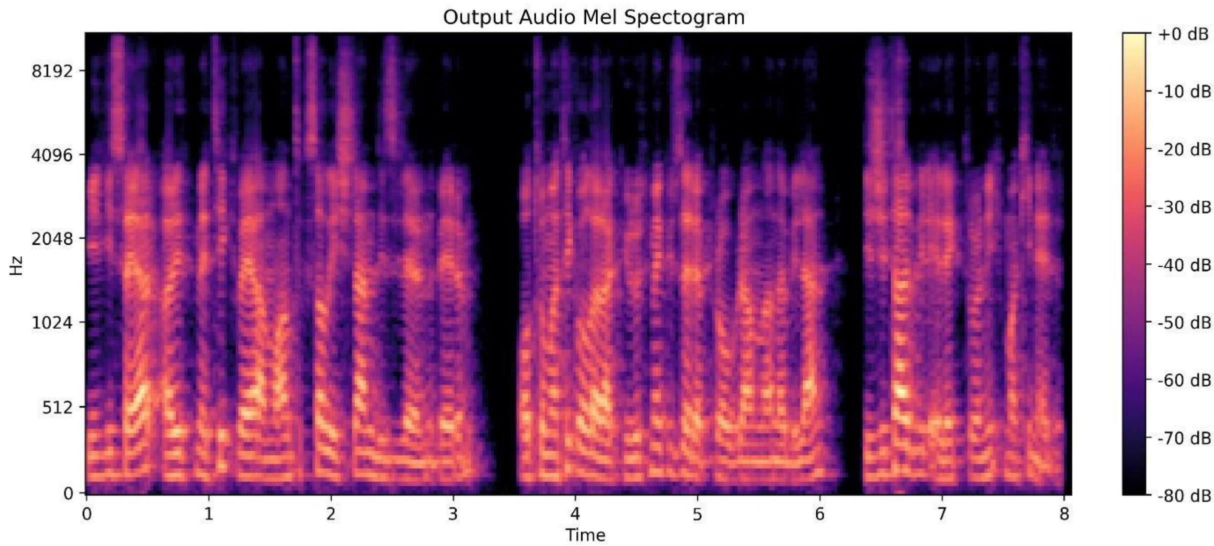

Figure 6: Output Audio Data Mel-Spectogram Visualization

Key Words: Voice Cloning, Deep Learning, Turkish Speech.

REFERENCES

[1] Inamdar, F. M., Ambesange, S., Mane, R., Hussain, H., Wagh, S., & Lakhe, P. (2023). Voice Cloning Using Artificial Intelligence and Machine Learning: A Review. Journal of Advanced Zoology, 44.

[2] Ramu, S. C., Saxena, D., & Mali, V. (2024, March). A Survey on Voice Cloning and Automated Video Dubbing Systems. In 2024 International Conference on Wireless Communications Signal Processing and Networking (WiSPNET) (pp. 1-5). IEEE.

[3] Tamilselvan, G., & Biswal, M. (2024). Voice Cloning & Deep Fake Audio Detection Using Deep Learning. International Journal of Advanced Research and Interdisciplinary Scientific Endeavours, 1(2), 22-26.

[4] Arik, S., Chen, J., Peng, K., Ping, W., & Zhou, Y. (2018). Neural voice cloning with a few samples. Advances in neural information processing systems, 31.

[5] Jemine, C. (2019). Real-time voice cloning.

[6] Neekhara, P., Hussain, S., Dubnov, S., Koushanfar, F., & McAuley, J. (2021, November). Expressive neural voice cloning. In Asian Conference on Machine Learning (pp. 252-267). PMLR.

[7] https://huggingface.co/spaces/Saadin/Voice_Cloning_TR